FALL 2024 | VOLUME 15, ISSUE 2

JOURNAL OF MATHEMATICS EDUCATION AT TEACHERS COLLEGE

A Century of Leadership in Mathematics and Its Teaching

Historical and Current Reflections on Mathematics Education

Published Semiannually by the Program in Mathematics Education
TEACHERS COLLEGE | COLUMBIA UNIVERSITY

AIMS AND SCOPE

The *Journal of Mathematics Education at Teachers College (JMETC)* is a recreation of an earlier publication by the Program in Mathematics Education at Teachers College, Columbia University. As a peer-reviewed, semiannual journal, it is intended to provide dissemination opportunities for writers of practice-based or research contributions to the general field of mathematics education. Although many of the past issues of *JMETC* focused on a theme, the journal accepts articles related to any current topic in mathematics education, from which pertinent themes for future issues may be developed.

JMETC readers are educators from pre-kindergarten through twelfth-grade teachers, principals, superintendents, professors of education, and other leaders in education. Articles appearing in the *JMETC* include research reports, commentaries on practice, historical analyses, and responses to issues and recommendations of professional interest.

COPYRIGHTS AND PERMISSIONS

JOURNAL SUBSCRIPTION

The journal is converting primarily to electronic publication. All *JMETC* issues—including all future and archived issues—will be accessible and available for free electronically online at: jmetc.columbia.edu. To receive an email notification when new issues are published, please register on the journal website.

LIBRARY OF CONGRESS CATALOGING-IN-PUBLICATION DATA

Journal of Mathematics Education at Teachers College
p. cm.
Includes bibliographical references.
ISSN 2156-1397
EISSN 2156-1400
1. Mathematics—Study and Teaching—United States—Periodicals
QA11.A1 J963

AVAILABLE ONLINE

jmetc.columbia.edu

JOURNAL OF MATHEMATICS EDUCATION AT TEACHERS COLLEGE

A Century of Leadership in Mathematics and Its Teaching

Historical and Current Reflections on Mathematics Education

TABLE OF CONTENTS

PREFACE

To best support student learning, it is important for mathematics educators both to reflect on the past and to stay connected to current research and practices in their ever-changing educational communities. The Fall 2024 issue of the Journal of Mathematics Education at Teachers College presents two collections of papers, written by Teachers College graduate students, that highlight both historical and contemporary ideas in mathematics education.

The collection of research articles includes six papers that were conceptualized in a History of Mathematics Education course. This collection uses historical analysis methodologies to explore textbooks from various countries and epochs, paying specific attention to both the text within these books, and the social, political, and historical contexts in which they were written.

The collection of short reports consists of three pieces that were inspired by a mathematics education "study tour" across New York City. During this study tour, students explored the abundant centers of mathematics learning within their own communities. From colleges and universities, to museums, to specialized high schools, New York City provides a unique opportunity to experience and reflect on the teaching and learning of mathematics in a wide variety of settings.

The Journal of Mathematics Education at Teachers College is proud to feature these collections, written by students, guided by faculty, and motivated by classes in the program.

Emma LaPlace
Molly Stern

ON STUDYING OLD TEXTBOOKS

Dr. Alexander Karp,
Teachers College, Columbia University

The five papers presented to the reader's attention in this volume are the fruits of a course in the history of mathematics education given in spring 2024. Its participants focused on the analysis of old textbooks, and this analysis is represented in the papers published here. The five papers have much in common, including their similar aims and methodologies, and I will try to address these briefly below.

Why should old textbooks be studied and how can this be done? It is sometimes said that in old textbooks one can find certain techniques that could be made use of in today's classrooms. This assertion may be justified to a certain extent, but direct transfer from a different time and a different culture usually proves impossible, so to hope that the answers to today's questions lie in the past is naive.

Schubring (2022) devotes a whole chapter to discussing the questions formulated above, noting that "Textbooks constitute a major source for investigating the interfaces between research and teaching" (p. 4). Indeed, textbooks contain routine knowledge, as it were, and what such knowledge is considered to consist of specifically is telling in and of itself. It is also important to note, however, that very often textbooks turn out to be the main, and sometimes almost the only, source of our knowledge about what went on in classes in school. Of course, there are also various accounts, memoirs, letters, newspaper articles, students' notebooks, recommendations written by all kinds of administrators and supervisors, including official curricula and much else; but, first, these documents do not by any means always survive into our time, and second, all such sources usually give only a very fragmentary picture. Clements and Ellington (2012), for example, recommended students' notebooks (cyphering books) as a main source of knowledge about the school mathematics of the past, arguing that it was precisely they that preserve what was actually studied in school, while textbooks reflect only the so-called "intended curriculum." One cannot but agree that textbooks are by no means always read in school in their entirety. Nonetheless, it is specifically textbooks that lie on the teacher's desk, and since the time when they started being widely distributed, on the desk of every student as well, while students' notebooks (unquestionably an important source) nonetheless inevitably reach us only in part–usually, we do not even know how many notebooks students had. It is a different matter that, as we will discuss below, one must not confine oneself to the analysis of textbooks alone. In order to understand how mathematics was taught in the past and why it was taught the way it was, one must make use of the most varied sources.

But at this point, we might ask: what's the point of knowing how people taught previously? This question is a rhetorical one, since everyone always makes references to the past–it is hardly possible not to–but nonetheless, we should say that without understanding the past, it is impossible to understand the present. A person who sincerely thinks that it is only now, for instance, that people have started talking about the need to study geometric transformations turns away from all experience, successful and unsuccessful, accumulated by humanity in studying this topic; such a person considers what happens today as something isolated and separate, not as part of a long and complicated process, and this is hardly a way to accomplish anything.

But once we have agreed that researching the mathematics education of the past in general and that focusing on textbooks as a means and object of such research in particular is a useful task, we must address the methodology of such research, especially since, although the study of old textbooks began very long ago (the first dissertation in mathematics education defended in the United States, Stamper, 1906, was at least partly devoted to the analysis of textbooks), the methodology of such research is still in its formative stages.

It should be said that mathematics textbooks have lately been recognized as such an important aspect of education that special conferences are devoted to them (the first was held in 2014, see Jones et al., 2014). Studies of textbooks may focus on the most varied topics, from the relative numbers of pages allocated to different sections, to characterizations of the figures that appear in a textbook. The study of historical textbooks becomes substantive when it is connected with temporal changes and social history.

Schubring (1987) proposed analyzing textbooks in terms of three dimensions, the first of which "consists in analyzing the changes within the various editions of one textbook," the second in "finding corresponding changes in other textbooks belonging to the same" group, and the third of which "relates the changes in the textbooks to changes in the context (changes in the syllabus, ministerial decrees, …etc.)" (p. 45). In all three dimensions, the factor of time–of processes that took place in the past–is emphasized.

The second dimension, it seems to us, would also include the simple comparison of textbooks of different generations from one angle or another. As examples of such studies, I would cite my own papers Karp (2015; 2023), which analyze problem sets from Russian and American textbooks of different periods, making it possible to observe transformations in the understanding of what the actual outcome of education was supposed to be, and consequently, of the very process of teaching, the recognition of which in turn allows us to ask what it was that brought these transformations about.

We should point out here that the historical method used in historical research, including research in mathematics or mathematics education, is based on juxtaposing and comparing documents, not statistical data. Of course, if it is discovered that, for example, until a certain time there were tens of times fewer computation problems in geometry textbooks than there are in textbooks today, such numerical findings are useful and meaningful. But usually, studies of this kind involve less computing than determining which solutions to problems students were intended to find, comparing problems with the theoretical texts in the textbook, and so on. Such methods of analysis, in our view, can be quite rigorous and well-grounded.

Studying a single textbook and even only a single edition of it, although inevitably poorer than studying a group of textbooks, nonetheless is also possible and useful. It is only important, as we have already said, not to remain confined to what is printed in the textbook. The textbook was written for a specific group of learners, under specific political circumstances, in light of a specific labor market, in the context of a specific philosophy and tradition of education, a specific technical basis for publication, and finally, by specific individuals with their specific characteristics. All of this forms what Schubring calls "context," and it is precisely the textbook's relation to this context that must constitute a part of its study (given that the objective is something broader than simply a short resume of a forgotten volume–which might be helpful to researchers, but can hardly be considered a study on its own).

How, then, should the study of the context and the textbook itself be related? Once again, Schubring (1987), noting the complexity and multidimensionality of the processes connected with the writing and use of textbooks, urged researchers to study "units" in which various aspects or dimensions interact. One such "unit" might be the life of the author of a textbook, his book, and its distribution. Such an approach can reveal the author's interaction with the textbook market, with those who make use of the textbook, and with those who approve and finance the textbook. But other unifying ideas are also possible, and indeed, the text of a textbook itself suggests a specific "unit"–how and why it was written the way it was, and what this was connected to.

The first thing one must determine in studying a textbook is what exactly one wishes to find out and understand. Then, one must establish the circumstances under which it was written. This requires, on the one hand, a general historical study of the time when it was written, and on the other hand, if possible, of the author's biography–for which it turns out to be expedient to work with various dictionaries and encyclopedias, search for other publications from the author's time, and analyze archival materials and various currently offered databases. But the main part of the investigation consists in the critical reading of the textbook itself and the selection of topics, sections, and other features that are in one way or another connected with the questions that have been posed.

It should be said at once that such a connection may be anything but simple. In one of the papers published below, Christine Gao set herself the task of studying how Nazi ideology was expressed in textbooks of the time. Evidently, it had to manifest itself in "general" sections – sections on the history of mathematics and its significance, or in the descriptions of a textbook's aims. Naturally, it could also manifest itself in word problems. Siegmund-Schultze (n.d.) cites a monstrous problem in the process of solving which schoolchildren had to reach the conclusion that a great deal of money is spent to make the lives of criminals and the mentally ill more comfortable, which could be used to improve the lives of those who are healthy and law-abiding. But even word problems do not tell the whole story: changes in the curriculum may also reflect changes in ideology. In this respect, a mathematics textbook, of course, is in somewhat lesser danger than, say, a history textbook, from which one can simply exclude certain sections, while, on the other hand, adding chapters that are politically expedient to the government. A course in mathematics is relatively stable. And simply to deny the facts, as this is done in history textbooks, is also not possible here. But to expand one section or another, to add material where it seems to be useful to the regime, is possible, and such changes must also be taken into account.

Let us repeat that reading with close attention, not only to the content of what is presented in the textbook, but also to the language of the presentation, to the structure of the text, to how the theoretical material is laid out and to what kind of problems are offered to the students, to the intended organization of the textbook's use, and to many other parameters–all carried out against the background of the study of the textbook's context–is precisely what constitutes the main method for studying historical textbooks. It should be remarked that such a method might appear imprecise to someone trained in doing statistical research, but it is just such a methodology or one like it that is used to conduct research in numerous disciplines.

Perhaps the most natural thing to recall in this connection is research in literary criticism, in which what is studied is some specific work of literature. As an example of successful analysis, let me cite Vygotsky (1971), who demonstrates that the order of events as they are presented in a short story that he analyzes by no means coincides with the order in which these events are actually supposed to have happened, and that by this means a certain effect is achieved. There exists a vast literature, for example, about the use of specific aspects of language, about what is achieved through the use of "special" language, for example, common colloquial language, or on the contrary, elevated literary language. Without adding any more examples, let us say that the text of a mathematics textbook, although it does not claim to be a work of literature, nonetheless also reveals a great deal to an attentive reading.

Text and context–these are the main objects of analysis in the study of historical textbooks. Let us say a few words about how these general principles are put into practice in the papers published here. Christine Gao's paper has already been mentioned; we would just add that mathematics textbooks from Nazi Germany have not been sufficiently studied, and literature about them in English is altogether scarce.

Three other papers are also devoted to non-American textbooks; or more precisely, two, since Wilson's paper is devoted to a periodical–although this periodical, too, was published in Romania, at the end of the nineteenth century, in order to carry out certain functions that were not sufficiently met by ordinary education, in the opinion of the publishers. Nonetheless, from a methodological point of view, although the study of this periodical has features in common with the study of textbooks, it still differs in many respects, and the author discusses these differences.

The paper by Daria Chudnovsky offers parallel analyses of the text of a textbook and its context, namely, the biography of its author, the famous Russian mathematician Nikolai Bugayev, and his overall philosophical views. Omar Faruque's paper, more descriptive in character, is devoted to a popular Bengali arithmetic textbook written at the end of the nineteenth century. In view of the fact that not enough is known about the teaching of mathematics in European colonies in general and in India in particular, the relatively detailed analysis of this textbook appears quite useful.

One more paper is devoted to American textbooks. Moroney analyzes a textbook in arithmetic, one of whose authors was David Eugene Smith, the founder of the Program in Mathematics at Teachers College. This analysis is supplemented by archival findings: the surviving correspondence between Smith and his coauthor. But the very description of this textbook by itself helps us to understand much about the mathematics education of that time.

This last remark, as it seems to us, applies to the other papers as well. Of course, they do not exhaust the problems they pose, and the studies initiated here may be continued, expanded, and deepened, but a definite step forward has been taken, and people who are interested in the history of mathematics education–and this subject should be of interest to all mathematics educators–will be interested to find out about what is new and little known.

And here yet another, perhaps most important, observation must be made. The study of historical textbooks must be continued. I know of only one book (the already cited Schubring, 2022) that is entirely devoted to historical mathematics textbooks. Meanwhile, an enormous number of textbooks remain unanalyzed, and while in the United States and certain other countries lists of mathematics textbooks (of different degrees of completeness) have at least been compiled, in many countries even this is lacking, and one might conclude that in the "old days" people did not study at all, or studied very badly. By no means proposing to borrow anything from the past, let us say once again that by studying old textbooks, we can better understand what role mathematics played in the life of people in the society of their time, how that life and the attitude toward mathematics developed and changed, how the conception of what should be taught and how it should be taught became transformed, and how methodological and mathematical ideas spread through the world, influencing other countries. There is much room here for new and useful studies. We would like to hope that such studies will be continued.

References

Ellerton, N. F., & Clements, M. A. (2012). *Rewriting the history of school mathematics in North America 1607–1861.* Springer.

Jones, K.; Bokhove, C.; Howson, G.; Fan, L. (2014). *Proceedings of the International Conference on Mathematics Textbook Research and Development.* Southampton.

Karp, A. (2015). Problems in old Russian textbooks: How they were selected In K. Bjarnadottir et al. (Eds.), *"Dig where you stand" 3* (pp. 203-218). Uppsala University.

Karp, A. (2023). Problem sets in school textbooks: Examples from the United States. In R.Leikin (Ed.), *Mathematical Challenges For All* (pp. 459-479). Springer.

Vygotsky, L. (1971). *The psychology of art.* MIT Press.

Schubring, Gert. (1987). On the methodology of analysing historical textbooks: Lacroix as textbook author. *For the learning of mathematics 7:* 41- 51.

Schubring, Gert. (2022). *Analysing historical textbooks.* Springer.

Siegmund-Schultze, R. (n. d.). Nazi Rule and teaching of mathematics in the Third Reich, particularly school mathematics. Retrieved from https://publimath.univ-irem.fr/numerisation/ACF/ACF08076/ACF08076.pdf

Stamper, A. W. (1906). *A history of the teaching of elementary geometry with references to present-day problems* (Doctoral dissertation). Teachers College, Columbia University.

JOURNAL OF MATHEMATICS EDUCATION AT TEACHERS COLLEGE | FALL 2024 | VOLUME 15, ISSUE 2

Nikolai Bugaev's Philosophy of Education: *Arithmetic of Whole Numbers* Textbook Analysis

Daria Chudnovsky
Teachers College, Columbia University

ABSTRACT This article explores the educational and philosophical contributions of Nikolai V. Bugaev, a prominent 19th-century Russian mathematician and founder of the Moscow philosophical-mathematical school. The study specifically focuses on Bugaev's textbook, Arithmetic of Whole Numbers, analyzing Bugaev's pedagogical approaches within the broader context of Russia's educational reforms during that era. Bugaev's work can be seen as a response to the evolving needs of a rapidly industrializing society, in which he emphasizes three fundamental components of mathematical education: integration of theory, calculation mechanisms, and practical problem-solving. While Bugaev's textbook may not have achieved the widespread popularity of other contemporaneous works, it played a crucial role in fostering mathematical thinking and underscored his vision of mathematics as a tool for intellectual development and its interconnectedness with other fields of knowledge.

KEYWORDS: *Russian mathematics education, textbook analysis, philosophy and pedagogy*

Nikolai V. Bugaev (1837-1903) was a distinguished Russian mathematician, chairman of the Moscow Mathematical Society, and founder of the Moscow philosophical-mathematical school. His life's work coincided with the second half of the 19th century, a period of significant social, political, and economic changes in Russia, all of which influenced the rapid development of the country's education system. Bugaev significantly contributed to the field of education, specifically mathematics education, giving numerous speeches about the Russian education system and writing a series of school mathematics textbooks, from arithmetic to geometry. Although Bugaev is recognized as an outstanding mathematician, his name is rarely mentioned in the context of school mathematics education. This article aims to contribute to a better understanding of mathematics education, specifically arithmetic, in Russia during this period, and to examine Bugaev's pedagogical and philosophical approaches to school mathematical education through the example of his textbook, *Arithmetic of Whole Numbers*. In what follows, the author will refer to original documents written by Bugaev, or studies about the mathematician and his textbooks.

Late 19th Century: Development of School Education in Russia

In the 19th century, Russia saw four major reforms in its educational system, drastically influencing mathematics education. After the first two statutes of 1804 and 1828, quality school education (i.e., gymnasia) was limited to a select group of individuals, typically those from families with a certain level of economic and social standing, though not necessarily a very high level. Mass education in Russia began with the third reform, the 1864 Statute on Secondary School, initiated under Emperor Alexander II, following the 1861 abolition of serfdom in Russia. Restrictions on admission to classical gymnasia were mainly lifted, extending to a larger circle of students, and establishing various forms of gymnasia. One of the types was classical gymnasia, with a mission of learning ancient languages and mathematics to promote the growth of intellectual abilities, while the other type was real schools (from the German *Realschulen*), focusing on technical and natural sciences rather than classical languages (Karp, 2013). During this reform period, mathematics played an important role in both types of gymnasia, as well as in other institutions such as commercial, eparchial, and military schools. In classical gymnasia, mathematics was regarded as a formal subject that promoted cognitive growth and remained unaffected by immediate political trends, whereas in real schools, it was essential for aspiring technical specialists and natural scientists (Karp, 2013).

Throughout the 1860s, the mathematics curriculum addressed different objectives depending on the type of school system and the varying needs of each system's graduates. The divergence in the goals of mathematics education became even more pronounced in the early 1870s with the issuance of new regulations by Dmitry Tolstoy in 1871 and 1872. The 1871 statute and 1872 programs under Dmitry Tolstoy diminished the privileges of real school graduates in favor of gymnasium graduates and reduced lower-class student representation from 53% to 44% over the next decade (Karp, 2013). Nevertheless, by the end of the 19th century, gymnasia remained Russia's most prevalent school system. Despite the slow expansion of the mass education system, the overall socio-economic development and industrialization of the country spurred significant growth in new mathematical education, methodologies, programs, and teaching materials for the emerging mass Russian school system. Developing new courses in arithmetic was central to the expansion of mass education, as arithmetic was seen as the foundation of cognitive development.

This development attracted the attention of professionals from various fields, from teachers to university mathematicians. These included diverse individuals such as secondary education teachers A.P. Kiselev, F.I. Simashko, and A.F. Malinin, as well as university professors like A.Y. Davydov, V.A. Yevtushevsky, and N.V. Bugaev (Karp, 2012). It was precisely in this environment that Bugaev's textbook, *Arithmetic of Whole Numbers*, was published in 1875. His approach not only addressed the educational demands of the time, but also anticipated modern pedagogical trends that emphasize critical thinking and problem solving–ideas that align with his broader views on mathematics education, which are explored later in this paper.

Bugaev's Life and Career

Nikolai Vasilyevich Bugaev was born on September 14, 1837, in Dusheti, located in the Tiflis Governorate, an area that is now part of modern-day Georgia. Bugaev was the son of a military doctor, and the father of famous Russian writer, Andrey Bely. Bugaev's educational journey began at the prestigious First Moscow Gymnasium, where he graduated with a gold medal in 1855. He then entered Moscow University, completing his degree in 1859 in the Department of Physics and Mathematics, where Bugaev studied under the guidance of leading Russian professors and educators, N.E. Zernov, N.D. Brashman, and A.Y. Davydov (Kolyagin & Savina, 2009). These scholars organized the Moscow Mathematical Society in 1867, intending to promote the development of mathematical sciences in Russia. From 1859 to 1861, Bugaev studied at the Nikolaevskaya Engineering Academy, seeking to gain practical experience in how mathematics is applied to military engineering, where he attended lectures by the renowned Russian mathematician, M.V. Ostrogradsky (Kolyagin & Savina, 2009).

Following the defense of his master's thesis at Moscow University in 1863, Bugaev spent from 1863 to 1865 in Germany and France, preparing his doctoral dissertation. During this period, Bugaev attended lectures by some of the best European mathematicians of the time, such as Betrand, Kronecker, Kummer, Liouville, Weierstrass, and others, greatly influencing his scientific interests (Kolyagin & Savina, 2009). Initially, Bugaev was quite fascinated with mathematical analysis. After his trip to France, Bugaev's mathematical interests shifted to number theory, and his doctoral dissertation, which he defended in 1866, was devoted to number theory (Kolyagin & Savina, 2009). From 1867 onward, Bugaev worked as a professor at Moscow University, serving twice as dean of the Department of Physics and Mathematics. In 1891, he was elected president of the Moscow Mathematical Society, and in 1897, he was elected a corresponding member of the St. Petersburg Academy of Sciences (Kolyagin & Savina, 2009).

Bugaev developed a whole new perspective of studying functions by creating a new approach, which he initially named the 'theory of discontinuous functions' and later renamed 'arithmology.' According to Bugaev, arithmology represents the discontinuity of the surrounding world and allows for the application of mathematics in understanding all fields of knowledge (Taube, 1907). This paper does not seek to offer a deep analysis of Bugaev's philosophical concepts; rather, it focuses on how these ideas shaped his overall approach to mathematics education, with particular emphasis on their influence in his textbook, *Arithmetic of Whole Numbers*.

Bugaev's Views on Mathematics Education

Bugaev's pedagogical involvement began immediately after his return from his time abroad in 1865. His new position as an associate professor at Moscow University required him to deliver an inaugural lecture on number theory. At the beginning of his speech, Bugaev emphasized the importance of defining any scientific theory from a historical perspective of the theory's development and understanding its role within the overall system of mathematical sciences (Bugaev, 1877). Although Bugaev expressed these ideas in relation to number theory, they reflected his general philosophical stance on the process of mathematical cognition.

Bugayev's philosophical views changed throughout his life–in the 1860s, he was still a follower of classical positivism, a theory by French philosopher, Auguste Comte, asserting that true knowledge can only be derived from empirical evidence obtained through scientific methods and dismissing metaphysical and theological approaches as ineffective. Yet, in subsequent years, Bugaev distinguished his understanding from the positivist view:

> The so-called positivist worldview seeks to answer only the question: 'How do these phenomena occur?' The prevailing analytical worldview attempts to address both, 'How?' and 'Why?'... The true scientific-philosophical worldview strives to respond, to the extent possible, not only to the questions of 'how and why,' but also to the questions: 'To what extent and for what purpose?' (Bugaev, 1898, p. 715)[1]

Bugaev viewed the process of cognition as a collection of infinitely many discrete parts into a coherent whole, with a conscious understanding of how these discrete parts transition into one another, and contribute to the overall outcome (hence the 'to what end' and 'for what purpose' questions). Based on this, Bugaev formulated his approach to teaching mathematics consisting of three components: theory, calculation mechanisms, and apply theory to solving practical problems. In his 1869 speech "Mathematics as a Scientific and Pedagogical Tool," he explains:

> Theory acts in a developing way on thought, forcing one to rethink in a systematic form what humanity has discovered after a long series of efforts… The mechanism of calculation is the language by which the mathematician expresses his ideas, poses, and solves his questions. Not being able to master the mechanism means not being able to master this great tool of our civilization, not being able to express one's thoughts in the mathematical language… Finally, the application of theoretical principles and the developed mechanism to solving practical problems constitutes the third most important aspect of the pedagogical influence of mathematics on the development of intellectual abilities... The educational power of mathematical exercises in solving various problems is manifested in the development of independence. This aspect in the best way completes the influence of mathematics on the development of reasoning and is the best measure and means for developing mathematical abilities. In teaching, these three aspects should follow in the very order in which we have presented them, and only their complete combination has the most beneficial effect on the mind (as cited in Kolyagin & Savina, p. 221)

The practical realization of these principles and concepts was Bugaev's publication of school textbooks over the next two decades. Beginning the series with his *Arithmetic of Whole Numbers* textbook, Bugaev executed the foundation of his educational views–integrating the parts of school courses with each other, providing a holistic view of mathematics and its connections with the overall process of cognition and other related fields.

Bugaev's Textbook

Arithmetic of Whole Numbers is part one of Bugaev's textbook series on arithmetic, with the second part being *Arithmetic of Fractional Numbers*. The textbook consists of two parts: The *rukovodstvo* (manual guide), first attempted to be published in 1874, and a *zadachnik* (problem book), which followed later in 1875. Bugaev's textbook was not easily published–it was rejected in 1874 by the Ministry of Public Education. Below is a direct quote from a letter written to Bugaev by a notable mathematician of the time, Aleksandr Korkin, expressing his opinion about the rejection of the textbook:

1 All translations from Russian are by the author.

> In Russian pedagogical literature of today's time, guides by people competent in their subject have rarely started to appear… Imagine my surprise when I learned that it was not approved by Dmitriev as a guide at the gymnasia of the Ministry of Public Education. Having read his review, I was outraged by the remarks, from which it all consists. Some of them cannot be explained by the mere ignorance of Dmitriev-the-mathematician; it is evidently a desire to disapprove the book at all costs (as cited in Kolyagin & Savina, 2009, p. 165).

It was finally published in 1875 as a manual and problem book together. Bugaev's arithmetic books proved to be successful for decades after. Although the exact number of editions is unknown, the books were reprinted at least dozens of times, with editions 11 and 12 being the most popular, and approved by the Ministry of Public Education for use in secondary schools (Kolyagin & Savina, 2009).

Although both parts of the book could be used independently and written by Bugaev in separate years, they are designed to be absolutely interconnected as parts of an organic whole, in accordance with Bugaev's conceptual vision. Bugaev believed that it is more useful to collect practice problems in dedicated problem books rather than scatter them throughout a textbook, since mixing problems with the theory itself disrupts the integrity and logical flow of the material (Bugaev, 1881). Conceptually, the manual clearly corresponds to the first two elements of Bugaev's approach mentioned earlier–theory and calculation mechanisms. In contrast, the problem book corresponds to the third component of application of theory to solving practical problems.

Manual Guide

Bugaev presents the manual as a guide for students entering gymnasia at age 10. Before enrolling in the first year of gymnasia, these children have already acquired basic arithmetic skills, such as performing simple operations and knowing the multiplication table. He also claims that it is not a self-study textbook in any way, nor is it his intention to replace the teacher's role with this manual. Rather, he focused on providing a clear, concise, and systematic presentation of arithmetic truths (Bugaev, 1898). The topics Bugaev includes in the manual are: numeration/counting; basic arithmetic operations (addition, subtraction, multiplication, and division); checking operation results; order of operations (with parentheses); units, conversions, and operations with units; applications of arithmetic operations to problem solving; historical details on numeration; and counting in various bases.

With Bugaev's goal of providing students with the whole, systematic approach of arithmetic truths, Bugaev guides students through their process of cognition, and helps them to answer the questions, 'To What Extent?' and 'For What Purpose?' This is reflected in Bugaev's strictly organized structure of writing in the manual–definitions of terms and concepts are introduced in each chapter, followed by informal explanations resembling those that a student would encounter in a classroom setting from the words of a teacher. For example, in Chapter 2, Bugaev explains the notion of counting and numeration, including an informal proof of why the set of whole numbers is an infinite set:

> **Counting:** When counting objects, we enumerate whole numbers in sequential order. In doing so, we move from one whole number to the next larger number by adding one each time.
>
> **The number of whole numbers:** There are an infinite number of whole numbers because no matter how large a whole number is, it is always possible to obtain the next larger whole number by adding one to it. Each whole number must have a specific name and symbol so that it can be distinguished from other numbers both verbally and in writing…
>
> Whole numbers are an infinite set. If there were a separate word for each whole number, it would be impossible to remember them. This difficulty is avoided by using special methods of verbal expression, which constitutes the subject of verbal numeration (Bugaev, 1898, p. 6).

Another example of his textbook style can be taken from the last chapter on problem solving:

> **Problem 20.** A certain person, having a capital of 8998 rubles, bought 15 dessiatins [unit of measure] of arable land at 125 rubles each, 37 dessiatins of meadow at 112 rubles each, and 5 horses at 147 rubles each. With all the remaining money, he bought forest land at 132 rubles per dessiatin. How many dessiatins of forest land did he buy? (Bugaev, 1898, p. 116)

After providing the student with the problem above, Bugaev guides them through a plan, or algorithm, of problem solving the students should take when attempting to approach the problem:

> **Problem Composition.** It is easy to determine the composition of this problem. Our complex problem breaks down into the following 6 simple problems, among which:

The first problem determines how much he paid for the arable land and is solved by multiplication.

The second problem determines how much he paid for the meadow and is solved by multiplication.

The third problem determines how much he paid for the horses and is solved by multiplication.

The fourth problem determines how much money he spent on all these purchases and is solved by addition.

The fifth problem determines how much money he had left after these purchases and is solved by subtraction.

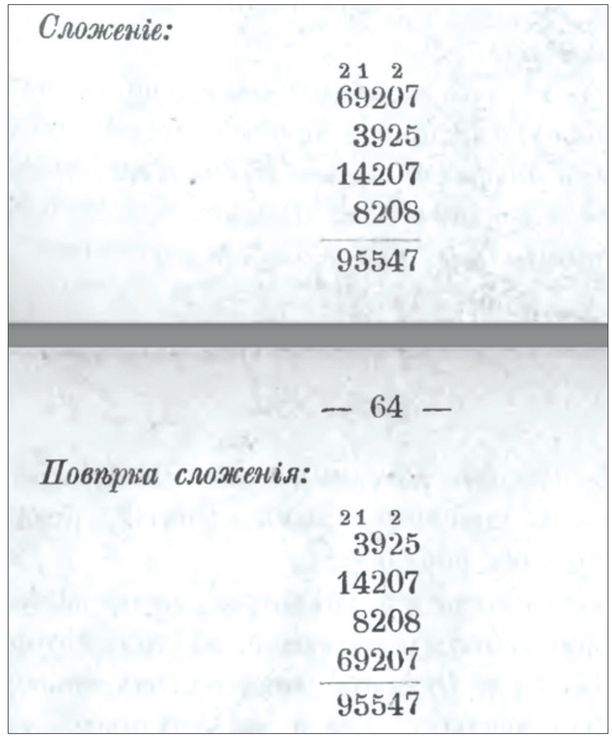
The sixth problem determines how many dessiatins of forest land he bought with the remaining money and is solved by division (Bugaev, 1898, p. 117).

Although Bugaev didn't intend to replace school teaching with his manual guide, the provided guidance to problem solving encourages independence, which Bugaev emphasized was a significantly important skill in his 1899 address titled, "On the question of secondary school": "In my opinion, a person's ability to independently, actively, and energetically acquire knowledge should be valued more highly than the knowledge itself" (as cited in Kolyagin & Savina, 2009, p. 253.) As such, Bugaev also includes a chapter on checking operation results, in which he encourages the students to confirm that they got the right answer and increases their confidence by providing numerous methods of checking their work with the four operations. He claims that it is not enough to simply repeat the operation another time, but that "our confidence increases if we verify the correctness of a result by another method" (Bugaev 1898, p. 63). Bugaev then introduces two distinct ways of checking the completed operation: one using the same operation but in a different order, and the other using the opposite operation (see Figures 1 and 2).

Bugaev dedicates the concluding chapter (appendix) of the manual to exploring the historical evolution of counting and numeration systems. He places considerable emphasis on the foundational component of theory, which systematically enriches understanding by revisiting the significant discoveries made by humanity throughout history. Bugaev provides a detailed examination of the development and interconnections of numeration methods in Chinese, Finnish, Greek, Roman, and Church Slavonic languages, tracing their dissemination across Europe (Bugaev, 1898). See Figure 3.

Figure 1

Checking an addition problem by using the same operation (Bugaev, 1898, pp. 63-64)


Сложенiе:

```
  21 2
  69207
   3925
  14207
   8208
  -----
  95547
```

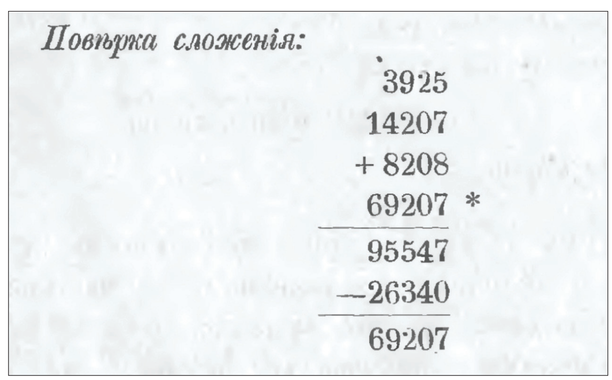
— 64 —

Повѣрка сложенiя:

```
  21 2
   3925
  14207
   8208
  69207
  -----
  95547
```

Figure 2

Checking an addition problem by using the inverse operation (Bugaev, 1898, pp. 66)


Повѣрка сложенiя:

```
    3925
   14207
  + 8208
   69207 *
  ------
   95547
  -26340
  ------
   69207
```

Problem Book

Bugaev's problem book is structured to match the manual completely, with each chapter in the problem book corresponding to a chapter in the manual and providing two sections–questions and problems–which complement the theory provided in the manual. According to Bugaev, the questions in each chapter emphasize aspects of the theory that "compose their systematic

Figure 3

Comparison of different number systems (Bugaev, 1898, p. 122)

Числа	Греческія,	Славянскія.	Числа	Греческія,	Славянскія.
1 —	α —	а҃	60 —	ξ —	ѯ҃
2 —	β —	в҃	70 —	ο —	о҃
3 —	γ —	г҃	80 —	π —	п҃
4 —	δ —	д҃	90 —	η —	ч҃
5 —	ε —	є҃	100 —	ϱ —	р҃
6 —	ς —	ѕ҃	200 —	σ —	с҃
7 —	ζ —	з҃	300 —	τ —	т҃
8 —	η —	и҃	400 —	υ —	у҃
9 —	ϑ —	ѳ҃	500 —	φ —	ф҃
10 —	ι —	і҃	600 —	χ —	х҃
20 —	κ —	к҃	700 —	ψ —	ѱ҃
30 —	λ —	л҃	800 —	ω —	ѡ҃
40 —	μ —	м҃	900 —	з —	ц҃
50 —	ν —	н҃			

understanding of arithmetic as a subject" (Bugaev, 1876, Preface). These questions strive to make students think deeper about the concepts that they have read about in the manual, while the problems that follow are meant for practical application problem solving. In the introduction to the problem book, Bugaev claims that the problems in the book are split into two types: Those with a goal of developing skills in calculation, and those with a goal of reaching an understanding and meaning of arithmetic operations and their applications. In this way, rather than just blindly applying learned skills to solve elementary arithmetic problems, Bugaev encourages students to consciously apply the learned concepts to, again, be able to answer the two main questions 'To What Extent?' and 'For What Purpose?'

Let's give a few examples of questions and problems from the first chapter (Foundational arithmetic concepts).

Questions:

-How are numbers classified by their relationship with their unit?

-What relationship exists between a whole number and counting?

Problems:

-Name some whole numbers with units.

-Place the following whole and fractional numbers into a sequence of numbers: five, one fourth, twenty, one seventh, three fourths, eight, eleven, twenty, three eighths, one eighth (Bugaev, 1876 pp. 1-2).

Note that as mentioned previously, although Bugaev dedicated this textbook entirely to whole numbers, he includes fractional concepts in the practice problems from the subsequent step of mathematics to achieve the wholeness and continuity of mathematics teaching.

Additionally, rather than practical applications of mathematics such as sales and agriculture, which were common in math textbooks at the time, Bugaev's word problems throughout the problem book can be categorized under scientific fields, such as astronomy, geography, history, and biology (Gavrilova, 2017). In his

1869 speech, Bugaev said that "...the degree of development of mathematical deduction primarily determines the nature and level of our knowledge of the external world" (as cited in Kolyagin & Savina, 2009, p. 216). Viewing the seemingly simple arithmetic concepts through various drastically different scientific lenses demonstrates the role of mathematics and its relationship to science, as well as motivates students to think and learn about the universe through mathematics, aiding their process of cognition. As an example, below are a few problems from Chapter 2 (Numeration):

Problems:

-Write the italicized words in following statements as digits:

(a) There are *eight* major planets.
(b) The earth has *one* satellite.
(c) There are *five* continents.
(d) There are *four* cardinal directions.
(e) Russia has *two* capitals.
(f) A week has *seven* days.
(g) There are *nine* significant digits.
(h) There are *six* male gymnasiums in Moscow.

-Write the italicized word in the following statement as a number: A child human heart beats *one hundred and forty* times a minute, and an elderly human heart beats sixty times a minute (Bugaev 1876, pp. 5, 8).

a few examples from Chapter 3 (Basic arithmetic operations with whole numbers):

Problems:

-Blood circulates in the body 350 times in one day. How many full times will blood pass through the body in 3 weeks?

-Sound travels 1107 feet in one second. Thunder will be heard 13 seconds after seeing lightning. How far from us does the thunder occur? (Bugaev, 1876, pp. 23, 25)

Through solving these problems, the student engages with various scientific fields of knowledge. It can help to consciously choose a life's career. In this way, Bugaev attempted to address a major problem of secondary education that he formulated below in his 1899 article "On the question of secondary school":

> ...gradually, they did not know where to turn, and, barely finishing gymnasia, they filled the universities. Their low level of intellectual and moral development adversely affected the entire course of their university education. In the gymnasium, they often developed a distaste for all sciences and cared only about obtaining a university diploma. From them emerged poor doctors, lawyers, and educators (as cited in Kolyagin & Savina, 2009, p. 249).

The problem book is majorly composed of challenging problems; to effectively solve challenges, it is crucial for students to fluently 'speak' the language of mathematics, both in symbols and words.

Discussion and Conclusion

Nikolai V. Bugaev wrote his arithmetic textbooks during a time of significant transition from elite to mass education. As Russia modernized, the demand for accessible and rigorous mathematics education grew, and Bugaev's textbooks attempted to meet these evolving needs. While his works did not achieve the same widespread adoption as others, such as the arithmetic textbooks by A.P Kiselev, or those by A.F Malinin and K. P Burenin, they were, nonetheless, innovative for their time, emphasizing the integration of theoretical knowledge, practical application, and cognitive development. Bugaev's philosophy of teaching mathematics to develop independent problem solving and logical reasoning remains relevant today. In an era where critical thinking and analytical skills are increasingly valued, Bugaev's educational methods offer insights into the ongoing need for a holistic approach to mathematics education. His textbook serves as a reminder that mathematics education is not merely about computation, but also about shaping intellectual capacity and adaptability, skills that continue to be essential in modern education systems.

Bugaev's textbooks remain an example of a textbook written by a renowned mathematician who thought about both the philosophical foundations of knowledge and the pedagogical aspects of its acquisition. The philosophical studies of the Moscow School of Mathematics have attracted attention globally (e.g., Svetlikova, 2013), where Bugaev's influence is acknowledged not only in mathematics but also in literature (as the prototype of the heroes of all his son Andrei Bely's novels), and politics. In considering this complex and multifaceted cultural phenomenon, Bugaev's pedagogical contributions–both theoretical and practical–must not be overlooked, as they influenced the education of hundreds, if not thousands, of students.

References

Bugaev, N. V. (1876). *Arithmetic of Whole Numbers. Zadachnik.* (2nd ed.). Tipografia A. I. Mamontova [in Russian]

Bugaev, N. V (1877) Vvedenie v teoriyu chisel N.V. Bugaeva [Introduction to Number Theory by N. V Bugaev]. Lecture in Moscow University [in Russian].

Bugaev, N. V. (1881). *Introduction to Algebra* (2nd ed.). Izdanie N. N. Mamontova [in Russian]

Bugaev, N. V. (1898). *Arithmetic of Whole Numbers. Rukovodstvo.* (10th ed.). Izdanie N. I. Mamontova [in Russian]

Bugaev, N. V (1898). Matematika i nauchno-filosofskoye mirosozercanie [Mathematics and Scientific-Philosophical Worldview]. *Voprosi filosofii i psihologii*, 45 [in Russian].

Gavrilova, M. S. (2017). Vospitatelniye imperativi dorevolucionnih uchebnikov matematiki N. V. Bugaeva [Educational imperatives of pre-revolutionary matehmatics textbooks of N.V Bugaev]. *Zhurnal pedagogicheskih issledovanii,* 3, 87-93 [in Russian].

Karp, A. (2012). Andrey Kiselev: the Life and the Legend. *Educação Matemática Pesquisa,* 14(3). https://tc.idm.oclc.org/login?url=https://www.proquest.com/scholarly-journals/andrey-kiselev-life-legend/docview/1506074515/se-2

Karp, A. (2013). Mathematics Education in Russia. *Handbook on the History of Mathematics Education*, 303–322. https://doi.org/10.1007/978-1-4614-9155-2_15

Kolyagin, Y. M., & Savina, O. A. (2009). *Matematiki-pedagogi Rossii. Zabitie Imena. Kniga 4.* Nikolai Vassilievich Bugaev. [Mathematicians-educators of Russia. Forgotten Names. Book 4. Nikolai Vassilievich Bugaev] (Vol. 4) [in Russian].

Svetlikova, I. (2013). *The Moscow Pythagorians.* Palgrave Macmillan.

Taube M. F. (1907). Moskovskaya filosofsko-matermaticheskaya shkola, osnovannaya prof. Bugaevim i slavyanofilstvo Homyakova [Moscow philosophical-mathematical school, founded by prof. Bugaev and Khomyakov's slavophilism]. *Mirnii Trud Journal*, 10, 13-30 [in Russian].

JOURNAL OF MATHEMATICS EDUCATION AT TEACHERS COLLEGE | FALL 2024 | VOLUME 15, ISSUE 2

School Mathematics in Colonial India: Analyzing an Arithmetic Textbook from the Late Nineteenth Century

Omar Faruque
Teachers College, Columbia University

ABSTRACT This article analyzes a popular arithmetic textbook, *Arithmetic: For the Use of Schools and Colleges*, authored by Jadav Chandra Chakravarti, to examine the contents and pedagogies in the text for how mathematics was taught and learned from the end of the nineteenth century through the first half of the twentieth century in colonial India. The study findings provide us with a glimpse of the teaching and learning of mathematics in colonial India and help us to better understand current school mathematics and its curriculum evolution and development in the subcontinent. Even though Chakravarti's textbook is mainly based on rule-method pedagogy, it has simple descriptions of rules and procedures, and a gradual progression from simple to complex in exercises and topics, which was unique at the time. The textbook maintains coherence in its style and presentation of the contents, and engages students and teachers by reflecting social life and cultural traditions in colonial India, which perhaps contributes to a long period of popularity of this book in the Indian subcontinent. The textbook has its legacy in modern education in the Indian subcontinent. Still, in the subcontinent, textbooks are one of the main sources of mathematics teaching and learning, as well as test preparation for high-stakes exams.

KEYWORDS: *Colonial India, Indian subcontinent, Chakravarti, textbook anlaysis, school mathematics*

Introduction

Mathematics takes an important place in Indian civilization as well as in the current educational system in the Indian subcontinent. The current education system in the subcontinent, including mathematics education, mainly started in the time of Colonial India. The colonial roots of education and old textbooks made its impact and imprint on current curricular design and educational practices in India (Jain, 2015; Ghosh, 2015). In depicting the history of mathematics education in modern India, Raina (2014) refers to the popularity of using mathematics textbooks in colonial India. Aggarwal (2007) surveyed a list of colonial mathematics textbooks. Textbooks play central and authoritative roles in classrooms in India, which can be characterized as "syllabus society" or "textbook culture," for the contents and the pedagogical styles that textbooks use for the subject materials in classrooms (Jain, 2015; Pinar, 2015; Thapan, 2015). However, there exists a research gap about mathematics textbooks and their relation to mathematics teaching and learning in colonial India.

Analyzing historical mathematics textbooks would provide us with a glimpse of the teaching and learning of mathematics in colonial India and help us to better understand current school mathematics and its curriculum evolution and development in the subcontinent. The purpose of this article is to analyze a popular arithmetic textbook, *Arithmetic: For the Use of Schools and Colleges*, authored by Jadav Chandra Chakravarti, to examine the contents and pedagogies in the text for how mathematics was taught and learned from the end of the nineteenth century through the first half of the twentieth century in colonial India (Chakravarti, 1911).

Colonial India and its education system originated when the East India Company (EIC) came to India in 1600. In its earliest years, the company had no intention of being involved with the country's local administration and education system. Their focus consisted mainly of trading and business. However, as they extended their business and power, they became interested in the country's political affairs and policies, and later introduced colonial education in India. Macaulay's report in 1835 advanced the policy for English as a medium of instruction (Majumdar et al., 1946; Nurullah & Naik, 1943). Wood's dispatch in 1854 formalized the colonial education system (Majumdar et al., 1946; Nurullah & Naik, 1943). Then, a great number of mathematical textbooks were printed locally in India or imported from England (Raina, 2014). Chakravarti's arithmetic textbook was first published in English in 1890 in this formal colonial education system.

Research Methodology

On the methodology of analyzing historical mathematics textbooks, Schubring (1987) developed a conceptual framework, in which a popular textbook is selected, (which is Chakravarti's textbook, in this case), to examine for the representation of school mathematics during that period. Then the framework needs to consider the textbook author's biography and changes in books' multiple editions and relate those changes to the country's social contexts and educational policies. My research questions are: (1) What is the scope and sequence of the contents and the evidence of pedagogy embedded in the textbook? (2) How can the textbook contents and the evidence of pedagogy be related to school mathematics practice? (3) How can the textbook contents and the evidence of pedagogy be related to the mathematics curriculum in the sociocultural contexts in Colonial India?

Textbook Author's Biography

Almost nothing is known about Chakravarti's early life and biography. Ray (2023) recognizes him as one of the great mathematicians in India. Chakravarti was born in 1855 in Sirajganj district in present day Bangladesh. He completed his entrance examination in 1876. He then moved to Calcutta City to complete a Master of Arts degree in mathematics from the Presidency College, University of Calcutta in 1882. While he was in college, he taught physics and chemistry part-time at St. Paul Cathedral Mission College in Calcutta, in order to be able to pay his tuition. Later he took a job as a mathematics teacher in Calcutta City College. He joined as a mathematics professor at Muhammadan Anglo-Oriental (MAO) College at Aligarh (Aligarh Muslim University) in 1888. Before he joined MAO college, he also served as a colonial government officer in Cooch Behar (Ray, 2023; Pathan, 1984).

Chakravarti's biography provides us some colonial sociocultural contexts under which the textbook originated and developed, and his motivation for the choice of pedagogical styles observed in the textbook. When he was a mathematics teacher at City College, he wrote a mathematics textbook, as he did not find any mathematics textbook which could engage students and make mathematics interesting to them. During his professorship at MAO college, he finished and published the arithmetic textbook in 1890, which he began writing while at City College. He authored other textbooks on topics including geometry and algebra, but the arithmetic textbook brought him fame and popularity as a mathematics textbook author. Pathan (1984) praised the arithmetic textbook for its simplicity, easy accessibility, and engaging, yet difficult, mathematics for students and teachers. The textbook's effectiveness in making connections between mathematics and the world outside of the classroom is significant.

When Chakravarti joined MAO College, there was an anti-Bengali sentiment at Aligarh. However, with his social and academic reputation, he took the Registrar position of the college in 1899 in *addition* to the role of mathematics professor (Lelyveld, 1975; Pathan, 1984). Mahmood (1895) remarked about his friendly manner and mathematical reputation as follows:

> My worthy friend Babu Jadav Chandra Chakravarti, M. A., Professor of Mathematics in the Muhammadan Anglo-Oriental College at Aligarh, to whose mathematical talent and labour I am indebted for the elaborate calculations… and also for the ready assistance which he has kindly given me in connection with other statistics whenever I have had occasion to consult him. (p. vii)

MAO college also recognized his secular attitudes towards the Hindu and Muslim communities. During his professorship of mathematics, he contributed to the development of the mathematics department. Chakravarti, along with support from his former student Dr. Ziauddin Ahmed, organized a group of teachers and researchers, who were interested in astronomy, history of mathematics, and theory of functions, and helped to form the mathematics department. He retired from his job in 1916. Little information is known about how he spent the period after his retirement. He eventually returned to his birthplace, Sirajganj, Bangladesh, and was elected chairman of the local town municipality. In Sirajganj, he also founded a school in 1901. He died in 1920 in his Calcutta residence (Ray, 2023; Pathan, 1984).

Textbook Contents

After the first publication of Chakravarti's textbook, *Arithmetic: For the Use of Schools and Colleges*, in 1890 (in English), it continued to be published for the next century, even after the independence of India (as my own father used this textbook after 1947). In the early part of the twentieth century, at the time of the nationalist movement in India, the textbook was translated from English to many major regional languages including Bengali, Hindi, Assamese, and Nepalese. The book's print was legible, and its language, including Bengali and English, is simple and standard. The local publishing company, Sanyal & Co. at Calcutta, first published the book and its multiple editions. The book does not include any pictures or illustrations, although the author uses diagrams of squares, rectangles, and parallelepipeds. The audience of this textbook is middle school and secondary school students, which the author explicitly mentions in the preface (Chakravarti, 1911).

The topics in the textbook include basic numeration of the decimal systems and operations of addition, subtraction, multiplication, and division; measurements of time, and angle; barter, gain, loss; factors, prime numbers, multiple factors; decimal numbers and fractions; square root and cubic root; proportion, rate, percent; profit, discount, interest, money exchange. The scope of the contents is generally the same in later editions. Other contemporary textbooks, including Smith's *Arithmetic for School* and Mukhopadhaya and Basu's *Patigonith*, have very similar contents (Smith, 1872; Mukhopadhaya & Basu, 1940). In addition, there exists consistency between the mathematics topics covered in the entrance exams of Calcutta University and other provinces (a reference to copies of old exams included in the back of Chakravarti's and Mukhopadhaya and Basu's textbooks) and the topics that the three textbooks covered (Chakravarti, 1911; Smith, 1872; Mukhopadhaya & Basu, 1940). Central and regional governments regulated the syllabus and curriculum for the high-stake entrance exams.

Textbooks prepared students to acquire mathematical knowledge and skills relevant to sociocultural life in colonial India. Chakravarti's textbook starts with tables of measures and units used in that period including the British money table (schillings, pence, pound, guinea), the Indian money table (pie, annas, rupee), English jeweller's weight (grain, pennyweights, ounces), English standard weight (drams, ounces, pounds, quarters), Indian bazar weight (sikis, tola, kancha, chataks, powas, seers, maund), other provincial local weights, English linear measurements (inch, feet, yard), and other Indian measuring systems. There are other topics related to Indian sociocultural life such as money exchange, the barter system, and the measurement of Indian land, among others. Here, there can be mentioned some examples about colonial life embedded in the text. Below are two word-problems, which do not provide any evidence of sociocultural life in colonial India:

> A merchant in New York wishes to remit to London 5110 dollars, a dollar being equal to 4*s*. 6*d*. English: for what sum in English money must he draw his bill when bills on London are at a premium of $9\frac{1}{2}$ per cent.? (Chakravarti, 1911, p. 343)
>
> There is also a physics problem here:
>
> "One pendulum oscillates 6 times in 3.2 seconds, and another pendulum 8 times in 3.6 seconds; if started simultaneously, how often will they tick together in an hour?" (Chakravarti, 1911, p. 297)

However, most of the word problems in the book represent the Indian traditional cultural elements, including the local currency (Rupees), transportation systems, and local measurement systems and units, which engage teachers and students in the teaching and learning of mathematics. For example, the following problem shows the local staple food (rice), and local measuring unit system (*maund*): "A man bought 35 *maunds* of rice on a certain day, and 9 *maunds* on the next day; how many *maunds* did he buy in all?" (Chakravarti,1911, p. 11). The word problems in the book maintain a balance of Indian colonial social life and Indian cultural traditions.

Here is another word problem that demonstrates an example of multi-ethnicities and races of a colonial town: "A certain town contains 87,903 Hindus, 48,093 Mahomedans, 723 Europeans, 1,309 Eurasians and 159 other races; What is the total population of the town?" (Chakravarti, 1911, p. 14). This example also demonstrates inclusivity in the context of race and ethnicity and the author's secular attitude towards different religions including Hindu, Muslim, and other religions, which perhaps increased the book's accessibility to different groups of people.

The following information demonstrates that the textbook had an authoritative role in school mathematics. The scope of the contents of Chakravarti's book and two other contemporary textbooks, including Smith's *Arithmetic for School* and Mukhopadhaya and Basu's

Patigonith, had been maintained almost without change in multiple editions, which indirectly shows the textbooks' central roles about what to teach in classrooms. One edition of Chakravarti's textbook mentions in the preface: "This edition has been specially adapted to meet the requirements of the new syllabus of study drawn up by the Education Department of Bengal. This has necessitated the fragmentary treatment of some of the subjects" (Chakravarti, 1930, p. ii). This provides further evidence of the textbook's authoritative role in mathematics curriculum because the textbook author prioritizes more to include exam topics than to fully develop the mathematics subject topics. The Education Department regulated the school mathematics curriculum, which was implemented through textbooks in classrooms and reflected in the topics of entrance exams. Textbooks play a significant role in preparing students for the entrance exams.

Pedagogy

The author's pedagogical vision is written in the preface section of the first edition of the textbook, where he emphasizes mathematical reasoning and understanding, starting from simple mathematical processes to complex processes:

> I have carefully avoided laying down arbitrary *rules* and have endeavoured to establish the leading propositions of the science of Arithmetic by a process of simple reasoning, being fully convinced that a mere mechanical facility in manipulating figures, sufficient though it may be for the calculations necessary in every-day life, is in no way conducive to a healthy development of the reasoning faculty. I have accordingly explained the processes of Arithmetic by means of specimen examples fully worked out, and in every division of the subject I have begun with simple principles and have tried to proceed by gradual and natural steps to those of a more complex nature. (Chakravarti, 1911, preface)

Here, I will mention some mathematics topics, considering their pedagogical perspectives. The author introduced the topic *addition* operation with definitions, symbols and notations, progressing to mental recitation of *addition* of small numbers and explanation of rules step by step. After the description of the procedures in the worked examples, the section provides exercises of many *addition* problems. For example, the book's *addition* section exercises start with a simple problem at the beginning: "I have purchased a table for 16 rupees and a chair for 7 rupees; how many rupees have I spent in all?" (Chakravarti, 1911, p. 10). Then, a somewhat more complex problem can be mentioned located at the end of the same *addition* exercises: "From a rope are cut off first 27 yards, then 8 yards, and there are 7 yards left; what was the length of the rope?" (Chakravarti, 1911, p. 11).

In the next section, the author also explicitly describes the *addition* rule step by step in simple ways for bigger numbers of 378, 409 and 56:

> We write down the numbers, one under another, thus
>
> $$\begin{array}{r} 378 \\ 409 \\ \underline{+\quad 56} \\ 843 \end{array}$$
>
> placing units under units, tens under tens, hundred under hundreds, and so on; then draw a line under the lowest line of figures. Under this line we place sum which is found in the following way:
>
> We first add the units, thus (8+9+6) units = 23 units = 2 tens + 3 units; we place 3 under the column of units and *carry on* the 2 tens for adding to the column of tens. Next we add the tens, thus (2+7+0+5) tens = 14 tens. … and we place the 8 under the column of hundreds. (Chakravarti, 1911, pp. 11-12)

It is noticeable that the sequence of exercises progresses from simpler to more complex. The example also shows that the author explains the procedures in simple language and in an easy-to-follow fashion. This pedagogy follows throughout the textbook in various mathematical topics.

However, in the above *addition* example, there can be examined another dimension of pedagogy that the textbook reflects throughout the book, which emphasizes rules and procedures to follow rather than inquiry and a constructivist approach. A procedure-based method can be conceptualized as memorizing the rules and procedures and practicing them in the mathematics tasks. On the contrary, in an inquiry-based and constructivist approach, students would be involved in discovering the mathematical formulas and algorithms (rules) as they explore the inquiry-based mathematical activities and problems (Michalowicz & Howard, 2003). In introducing the *addition* topic, the author approaches the conception of *addition* by demonstrating only rules to follow, rather than employing any mathematical tasks for exploring the conception of *addition* and constructing knowledge of mathematics. This procedures- and rules-based method, in general, is applied to other topics too. The book also suggests that sometimes rote memorization learning practice, such as using an algorithm to find the Lowest Common Multiple (LCM), can be beneficial. Chakravarti introduces the LCM algorithm in the Lowest Common Multiple topic without mentioning the reasoning and explanation of how the algorithm functions:

> The following rule gives the most convenient method of finding the L.C.M. of several small numbers:
>
> Places the numbers side by side in a line; divide by any one of the prime numbers 2, 3, 5, 7, 11, ……. which will divide any two at least of the given numbers exactly; set down the quotients thus obtained and the undivided numbers side by side; and proceed in this way until you get a line of numbers which are prime to one another. The continued product of all the divisors and the numbers in the last line will be the L.C.M. required. (Chakravarti, 1911, p. 84)

He then gives an example on how to use this algorithm to find the LCM of some numbers.

The book's different topics also maintain coherence and consistency in which earlier knowledge and skills are applied to current topics. Each topic starts with simple definitions and notations, several worked examples, and many exercises. This style of presentation is consistent throughout the book. In *addition*, Chakravarti's textbook occasionally includes exercises on miscellaneous topics in different locations of the book (review problems) to connect previous topics and concepts with new ones, which makes the textbook more coherent and connected. The book's ample number of exercises, including two *addition*al exercises on miscellaneous topics at the end of the book, give students and teachers wide and flexible opportunities to learn and teach mathematics, both in class and at home. These exercises also reflect students' better preparation for entrance exams, as many of the problems are collected from different sets of the entrance exams.

Chakravarti notes, "And I may add that although the book contains nothing that might strictly be called original, yet it will be found to differ in many ways from any existing text-book on the subject" (Chakravarti 1911, preface). The book's unique styles and pedagogical techniques support this claim. Chakravarti's textbook demonstrates a unique feature in connecting mathematics outside of the classroom to colonial life and Indian traditional culture, and its use of simple and easy-to-follow language in describing the rules and explanations in comparison to two other contemporary textbooks. For example, most of the mathematical problems in Smith's book do not connect to colonial life and Indian traditions, although the book occasionally includes reasoning in describing rules and procedures, and the language of the text is simple and coherent. In contrast, many mathematical problems in Mukhopadhaya and Basu's textbook demonstrate connections to colonial society and Indian culture, although the consistency and coherence of the text in describing the rules and procedures of mathematical conceptions are not consistently maintained. For example, in introducing and describing the rules of *addition*, the authors, to some extent, do not coherently develop the rules of *addition* to follow. Moreover, Chakravarti's book provides many more problems for practice in class and at home in comparison to those of the other two textbooks.

Discussion and Conclusion

The limitation of this study is that there are few surviving documents about the biography of the author, and almost no research or review articles regarding the textbook. From the analysis of the contents and the pedagogical strategies in the text, considering its different editions until 1947, the year of independence of India, and comparing other related mathematics textbooks, this textbook provides a window to how mathematics contents were learned and taught in the contexts and social needs of colonial India in that period. Colonial government regulated school mathematics syllabi, curricula, and entrance exams. In this regard, the textbook plays a dominant role in high-stakes entrance exams preparation, both in terms of contents and rule-based pedagogies in mathematics practice. Even though Chakravarti's textbook is mainly based on rule-method pedagogy, it has simple descriptions of rules and procedures, and a gradual progression from simple to complex in exercises and topics, which was unique at the time. The textbook maintains coherence in its style and presentation of the contents, and engages students and teachers by reflecting social life and cultural traditions in colonial India, which perhaps contributes to a long period of popularity of this book in the Indian subcontinent.

The textbook has its legacy in modern education in the Indian subcontinent. In the subcontinent, textbooks are one of the main sources of mathematics teaching and learning, as well as test preparation for high-stakes exams.

References

Aggarwal, A. (2007). Mathematical books for and in India in the nineteenth century. *BSHM Bulletin: Journal of British Society for the History of Mathematics* 22(1), 11-21.

Chakravarti, J. C. (1911). *Arithmetic: For the Use of Schools and Colleges*. Sanyal & Co. (https://archive.org/details/in.ernet.dli.2015.56600/page/n3/mode/2up).

Chakravarti, J. C. (1930). *Arithmetic: For the Use of Schools and Colleges*. Sanyal & Co. https://archive.org/details/in.ernet.dli.2015.510331/page/n5/mode/2up

Ghosh, S. C. (2015). An intellectual history and present circumstances of curriculum studies in India. In W. F. Pinar (Ed.), *Curriculum Studies in India: Intellectual histories, present circumstances* (pp. 83-110). Palgrave, Macmillan.

Jain, M. (2015). Curriculum studies in India: Colonial roots and postcolonial trajectories. In W. F. Pinar (Ed.), *Curriculum Studies in India: Intellectual histories, present circumstances* (pp. 111-139). Palgrave, Macmillan.

Lelyveld, D. (1975). *Aligarh's first generation: Muslim solidarity and English education in Northern India, 1875-1900.* [Doctoral Dissertation, The University of Chicago]. ProQuest Dissertations Publishing.

Mahmood, S. (1895). *A history of English education in India: Its rise, development, progress, present condition and prospects, being a narrative of the various phases of educational policy and measures adopted under the British rule from its beginning to the present period, (1781 to 1893).* M.A.O. College, Aligarh. (https://babel.hathitrust.org/cgi/pt?id=nnc1.cu56736290&seq=7).

Majumdar, R. C., Raychaudhuri, H. C. & Datta, K. (1946). *An advanced history of India.* Macmillan.

Michalowicz, K. D. & Howard, A. C. (2003). Pedagogy in text: An analysis of mathematics texts from the nineteenth century. In G. M. A. Stanic & J. Kilpatrick (Eds.), *A history of school of mathematics* (pp. 77-109). National Council of Teachers of Mathematics.

Mukhopadhaya, A. & Basu, S. (1940). *Patiganith.* Oxford University Press. https://archive.org/details/in.ernet.dli.2015.338538/page/n1/mode/2up

Nurullah, S. & Naik, J. P. (1943). *History of Education in India: During the British period.* Macmillan & Co. Ltd.

Pathan, M. A. (1984). Babu Jadav Chandra Chakravarty. *Fikro Nazar, Namwaran-e-Aligarh (Urdu)*, 23, 259-262.

Pinar, W. F. (2015). Curriculum studies in India. In W. F. Pinar (Ed.). *Curriculum Studies in India: Intellectual histories, present circumstances* (pp. 206-233). Palgrave, Macmillan.

Raina, D. (2014). Mathematics education in modern India. In A. Karp & G. Schubring (Eds.), *Handbook on the history of mathematics education* (pp. 376-384). Springer.

Ray, P. P. (2023). *The great mathematicians of Bharat.* Rajmangal Prakashan.

Schubring, G. (1987). On the methodology of analysing historical textbooks: Lacroix as textbook author. *For the Learning of Mathematics*, *7*(3), 41-51.

Smith, B. (1872). *Arithmetic for schools.* Macmillan and Co. https://archive.org/details/in.ernet.dli.2015.96156/page/n5/mode/2up

Thapan, M. (2015). Curriculum and its possibilities: Schooling in India. In W. F. Pinar (Ed.), *Curriculum Studies in India: Intellectual histories, present circumstances* (pp. 140-161). Palgrave, Macmillan.

JOURNAL OF MATHEMATICS EDUCATION AT TEACHERS COLLEGE | FALL 2024 | VOLUME 15, ISSUE 2

Analysis of the Integration of Military Themes and Nazi Ideology in the 'Mathematisches Arbeits- Und Lehrbuch' Textbook Series

Christine (Chang) Gao
Teachers College, Columbia University

ABSTRACT This study explores the integration of military themes and Nazi ideology within the "Mathematisches Arbeits- Und Lehrbuch" textbook series, which was a prominent tool in Nazi Germany for embedding National Socialist principles in mathematics education. Authored by Otto Zoll, a Nazi Party member, these textbooks were strategically designed to support the regime's objectives, aligning mathematical instruction with ideological and military indoctrination. The paper focuses on the revisions made to the textbooks after 1937, under directives from the Reich Ministry of Education, which infused them with exercises that directly related to military science, genetics, and Germanic folklore. Notably, students were introduced to complex military applications, such as ballistic calculations and strategic logistics, from as early as grade 10, illustrating the regime's efforts to prepare the youth for future roles in the military and as bearers of Nazi ideology. Through this analysis, the paper highlights the profound impact of these educational materials on shaping the ideologies and values of students, underscoring the potent role of education in propagating state ideologies. By dissecting the content of these textbooks, this study offers insights into how education under authoritarian regimes can be manipulated to achieve specific political ends, contributing to a broader understanding of the intersection between education, ideology, and political control during the Nazi era.

KEYWORDS: *Nazi ideology in mathematics education, mathematics curriculum, military mathematics, National Socialist pedagogy, Otto Zoll*

The goal of this paper is to discuss a series of textbooks used in Nazi Germany, which will help us to understand how mathematics teaching was influenced by Nazi ideology. The literature on the teaching of mathematics under Nazism is relatively limited (especially in English). A useful source for the author was the work of Goebel and Schlensag (2008). However, this paper has been purposefully limited to a review of examples, leaving generalizations for the future.

The analysis will focus on Otto Zoll, a textbook author in a period of Nazism, and a member of the Nazi Party. There were several other textbook series from the Nazi era, but this one was selected because most editions from the collection are readily available on the market, and the sellers were willing to ship to the United States. From 1935 to 1945, Zoll served as a member of the National Socialist Teachers League. Zoll received his PhD from the Georg August University of Göttingen in 1901. His dissertation, titled, "Über Flächen mit Scharen von geschlossenen geodätischen Linien" (On surfaces possessing families of closed geodetic lines), was accepted on June 5, 1901. His defense took place on July 29, 1901, for which he was awarded a prize for addressing the problem in an exceptionally satisfying manner and for his clear, accessible writing style.

Figure 1

Introduction of the author

Ueber Flächen
mit Scharen von geschlossenen
geodätischen Linien.

Von

Otto Zoll
aus Düren (Rheinland).

Am 5. Juni 1901
von der philosophischen Fakultät der Universität Göttingen
gekrönte Preisschrift.

Zugleich Inauguraldissertation zur Erlangung der Doctorwürde.

Göttingen 1901.
Druck der Dieterich'schen Universitäts-Buchdruckerei
(W. Fr. Kaestner).

Lebenslauf.

Ich, Carl Otto Zoll, wurde geboren am 2. April 1878 zu Hückeswagen (Rheinland) als Sohn des Fabrikdirektors Fritz Zoll. Ich besuchte die Sexta der Rectoratschule zu Hückeswagen, die Quinta des Progymnasiums zu Wipperfürth, und von da ab das Gymnasium zu Düren, das ich Ostern 1897 mit dem Zeugnisse der Reife verliess. Darauf widmete ich mich dem Studium der Mathematik, und zwar studierte ich von Ostern 1897 bis Mich. 1897 in München, von Mich. 1897 bis Ostern 1899 in Berlin; seit Ostern 1899 bin ich in Göttingen immatrikuliert. Ich besuchte die Vorlesungen der Herren Professoren und Dozenten:

in München: Brunn, Cornelius, v. Lommel (†), Holzhey, Ranke.

in Berlin: v. Bezold, du Bois, Dessoir, Fischer, Frobenius, Fuchs, Glan (†), Hettner, Knoblauch, Lasson, Lehmann-Filhés, Pringsheim, Rawitz, Schwarz, Warburg.

in Göttingen: Baumann, Hilbert, Klein, v. Koenen, Liebisch, G. E. Müller, Rehnisch (†), Roethe, Voigt, Wallach, Wiechert.

Allen diesen meinen verehrten Lehrern spreche ich an dieser Stelle meinen herzlichsten Dank aus, insbesondere Herrn Prof. Dr. Hilbert für die vielseitige Anregung, die er mir während meiner Göttinger Studienzeit hat zu teil werden lassen.

Source: Penn State University Mathematics Library

Figure 2

Portrait of the author

Source: Göttinger Digitalisierungszentrum

At the end of his dissertation (Zoll, 1903), there is a short curriculum vitae (Figure 1). It reveals that his full name was Carl Otto Zoll, born on April 2, 1878, in Hückeswagen (a small town located about 30 miles from Cologne). His father, Fritz Zoll, held the position of managing director at a factory. Zoll went to *Sexta* (today's fifth grade) in Hückeswagen and *Quinta* (today's sixth grade) in Wipperfürth. He then attended Gymnasium in Düren and graduated in 1897. From April 18 to September 29, 1897, Zoll studied mathematics in Munich, then in Berlin. He moved to Göttingen on Easter 1899, and completed his doctorate at the University of Götingen. Zoll's PhD thesis served as his only publication in a scientific journal. Zoll's PhD advisor was the renowned mathematician, David Hilbert. Between 1905 and 1947, Zoll held the position of senior teacher at a Düsseldorf high school, attaining the title of professor in 1916, until his retirement. He passed away on January 20, 1952, in Düsseldorf.

The *Göttinger Digitalisierungszentrum* has a picture (Figure 2) of Otto Zoll, dated 1920-1922, which was part of a photo album with portrait shots of mathematicians presented to David Hilbert in January 1922.

Mathematisches Arbeits- und Lehrbuch

From 1931 to 1943, Zoll contributed as the author and one of the editors to the development of *Mathematisches Arbeits- und Lehrbuch* (Mathematical workbook and textbook), a comprehensive mathematics textbook series for secondary education, published by Vieweg in Braunschweig in Germany.

The series of textbooks consists of three volumes: 1) *Unterstufe* (lower level); 2) *Mittelstufe* (middle level); and 3) *Oberstufe* (upper level). Each volume of the textbook was designed using the framework of the modern curriculum of mathematics education. Each chapter contains five to six sections. In each section, Part A contains tasks that are intended to facilitate the development of the lecture; Part B briefly summarizes what has been developed in Part A; Part C further deepens understanding and develops the relationships of mathematics to other fields; and Part D contains a variety of practice problems from which the teacher can make a selection. Some sections may not contain all of A, B, C, and D (Zoll, 1939).

Differentiation Between the Boys' Version and the Girls' Version

There was another system of counting grades at this time (Goebel & Schlensag, 2008). Accordingly, the Grade 6, 7, and 8 studied in this paper are equivalent to today's Grade 10, 11, and 12 respectively. In 1940, the contemporary military applications already reflected in the text as early as Grade 6 (today's Grade 10). Comparing the *Ausgabe A* (Edition A) for boys with *Ausgabe B* (Edition B) for girls in the volume *Oberstufe: Geometrie und Algebra, Ausgabe A* contains extra content in almost every chapter. For example, in the second chapter, Exponential and Logarithm Functions, the girls study the following topics:

1. Graphical representation of exponential and logarithmic functions. Concept of the logarithm
2. Logarithm systems, especially the decimal logarithms
3. The laws of logarithms
4. The arrangement of logarithmic tables
5. Calculating with logarithms

The boys need to study these five topics, plus *6. Rechenmaschinen*, which is a calculating machine named BRUNSVIGA, and can be considered as a very simple version of Enigma, the encryption device Germany had been using during World War II. The *Ausgabe A* provided a detailed picture of BRUNSVIGA (Figure 3).

In terms of practice problems under this topic, the text asks the students to explain the following mechanism as exercises (Figure 4),

In the 1942 version, another two sections were added in *Ausgabe A* of *Oberstufe*, exclusively for boys: *Geometric Relationships In The Conic Sections and Map Projection and The Trigonometry Of The Spherical Surface*, which include topics such as Map Projection, Nautical Triangle, and Astronomical Position Determination at Sea and in Aircraft. Map Projection and Nautical Triangle topics were relatively challenging; they involved a prerequisite knowledge that was neither mentioned elsewhere in the book, nor in any of the previous volumes and editions.

Figure 3

Brunsviga

§ 12. Rechenmaschinen.

Unter den Aufgaben, die die Arithmetik zu erfüllen hat, bleibt die vom ersten Unterricht an gestellte des reinen Zahlenrechnens auch nach dem Aufbau des mathematischen Lehrgebäudes eine der wichtigsten, die immer wieder geübt und deren Lösung dauernd vervollkommnet werden muß.

Es ist häufig nicht nötig, ganz genau zu rechnen, da Annäherungswerte bis zu einer gewissen Anzahl von Stellen meist genügen. Man kann dann die logarithmische Rechnung, den Rechenstab oder ein zeichnerisches Verfahren verwenden.

Genaue Ergebnisse werden beim Rechnen mit ganzen Zahlen, vor allem bei der Addition und Subtraktion, besonders im Wirtschaftsleben, im Bankbetriebe und bei kaufmännischen Unternehmungen in stets wachsendem Umfange gefordert. Zur Bewältigung dieser ungeheuren Rechenarbeit werden Rechenmaschinen verwendet, die bei guter Bedienung schnell und fehlerlos arbeiten. Sie ermöglichen es, die mechanische Rechenarbeit durch ein Minimum von Aufwand zu erledigen.

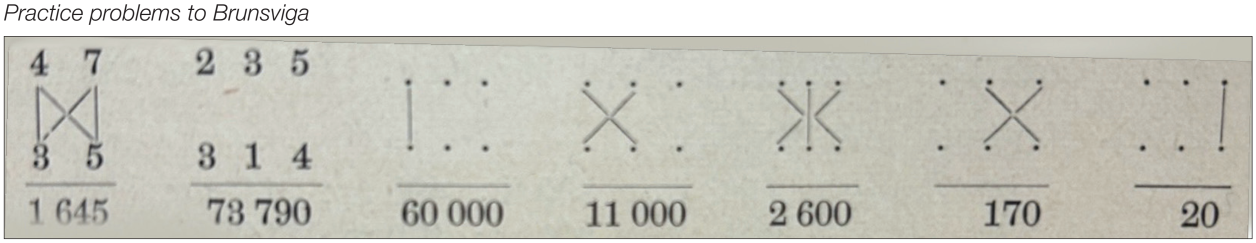
Bei den Additionsmaschinen wird die Multiplikation durch wiederholte Addition bewirkt. An wichtigen Konstruktionsteilen ...

Bild 22. Rechenmaschine „Brunsviga".

Source: *Mathematisches Arbeits- und Lehrbuch, Oberstufe: Geometrie und Algebra, Ausgabe A* (Zoll, 1940, p. 26)

Figure 4

Practice problems to Brunsviga



Source: *Mathematisches Arbeits- und Lehrbuch, Oberstufe: Geometrie und Algebra, Ausgabe A (Zoll, 1940, p. 27)*

Major Revisions with 'National Socialist Thought and the German Spirit'

Under the order of the Reich Ministry of Education, the series' structure and content were periodically revised to reflect changes in the education system. Numerous exercises directly related to National Socialist ideology, and military applications were included in the textbooks with the revisions of 1937 and 1939, initially in the form of supplementary booklets, and later in the body of the textbook (Goebel & Schlensag, 2008). Such ideology and military topics did not exist in the first edition published in 1931 (Zoll, 1931).

The following are quoted directly from the preface of the volume of the 1937 revised version, "Nationalpolitische Anwendungen der Mathematik" of *Mathematisches Arbeits- und Lehrbuch* (Zoll, 1937). All translations of German text from the textbooks analyzed in this study were performed by the author of this article. This edition contains several tasks with the main goal to indoctrinate students in the spirit of Nazi ideology:

> The National Socialist movement has given all German schools a unified goal, namely to educate students to be politically minded German individuals. To achieve this goal, it is necessary to familiarize students with National Socialist thought and imbue them with the German spirit. The present treatises aim to contribute to this task: They discuss a series of life-relevant, significant questions for the German people, in which mathematics plays a role (Zoll, 1937, p. 1).

Ideological issues were presented not just in the preface, but again in the final pages of each edition of the textbooks.

The following was quoted from "Historical Fact" at the end of the *Oberstufe* volume, revised in 1942:

> All people and races of the earth, in their attempt to interact with their environment, have developed some degree of mathematical knowledge and skill, even if sometimes modest. Even the Bushmen and Australian Aborigines possess a numeracy system structured according to their needs; in terms of spatial understanding, they exhibit orientation skills and sometimes an extraordinarily developed capacity for imagination and estimation. However, a vast gulf exists between these mathematical abilities and the mathematical fields discussed in this volume, which are essential to the economic and technical culture developed over the last five centuries. These skills are almost exclusively the result of the intellectual efforts of European Indo-Germanic peoples; among them, the Germanic peoples, and particularly the Germans, have contributed significantly. Compared to these, the mathematical achievements of all other ethnic groups lag considerably behind in the "race of achievement" and remain far behind the European Indo-Europeans, including the most advanced among them, such as the Indians, the inhabitants of the Euphrates and Tigris valleys, the Chinese, the Japanese, and the people of the American continents. (Zoll, 1942, p. 275)

This ideology was reflected not only in the boys' version of the text but also in the girls' version:

> From our Germanic ancestors of more than 2000 years ago, we know practically nothing about their scientific pursuits relevant to our interests here, as well as many other scientific matters. The rise of the sciences has simply brushed aside much of prehistoric Germanic knowledge. However, from the geometric patterns on the pottery of the Neolithic period or the Bronze Age, and from the pleasing geometric shapes of some of these items, we can infer that the Germanic women of that time certainly did not lack an appreciation for the beauty of geometric forms. (Zoll, 1940, p. VII)

Military Applications

With the 1937 revision, the textbook heavily emphasized military applications; the textbook consisted of nine chapters, which were later mixed into the body of different volumes of the textbook series. These chapters are completely independent of one another. The covered fields involved military science, biology (population policy, genetics), and prehistory:

- i From the Theory of Flight
- ii Aircraft Localization
- iii From the Theory of Conclusion (talking about Ballistics)
- iv Perspective (talking about photometry)
- v Image Measurement
- vi Combinatorics along with Applications from the Theory of Inheritance
- vii Fundamentals of Probability Theory along with Problems from the Theory of Inheritance
- viii Fundamentals of Statistics with Consideration of Biological Processes and Population Policy
- ix Problems in Germanic Folklore and Prehistory

Here is an example of the practice problems under the section Aircraft Localization:

> A bomber aircraft B flies at a ground speed of V_b = 360 km/h in an east-west horizontal direction and is attacked by a fighter aircraft K, which also flies in an east-west direction at a speed of V_k = 480 km/h, in the same vertical plane as the bomber but with a 30° inclination to the horizontal. At the moment of the attack, the distance KB = 400 m and the inclination of KB to the horizontal a = 50°. The projectile trajectory can be assumed to be straight and aligned with the sighting direction for this short distance; furthermore, the average projectile speed on this path is c = 800 m/s, and air resistance is not considered. What is the lead angle in the assumed position? How must the bomber's defensive fire be directed against the fighter? How does the solution change if the bomber flies in a west-east direction, but all other numerical data remains the same? (Zoll, 1937, p. 28)

Under the section on Ballistics, the text introduced External Ballistics by presenting an image of a mortar (Figure 5), which is a type of artillery weapon that fires explosive shells in a high-arching trajectory, making it effective for targeting entrenched positions or enemies behind cover.

Then, as a practice problem, the text asks students to calculate a projectile's flight from the moment it leaves the barrel until it reaches the target.

Here is an example:

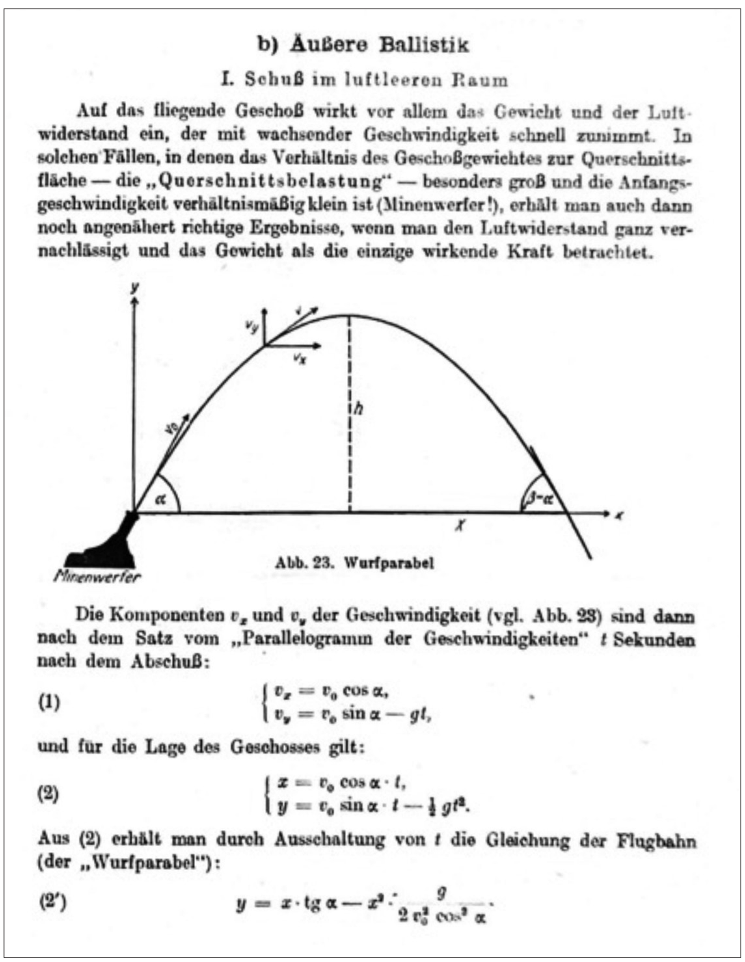
> A hand grenade was thrown from a trench with an initial speed of 26 m/s at an angle of 28°. What is the maximum height it reaches, where, and when does it land? How large must the elevation angle be if one (again with $v_0 = 26$) wants to hit an enemy trench 62 meters away? (Zoll, 1937, p. 41)

In another problem under the Ballistic section, technical parameters of a heavy mortar are given and students are asked to calculate which area of the terrain can be covered. By what value must a certain technical parameter be increased (or decreased) so that the next area seamlessly connects? (Zoll, 1937, p. 41)

Under the section of Image Measurement, military applications such as the determination of the location of a shell's burst by photographic capture, altitude measurement, evaluation of aerial, and sequential aerial photographs are included (Zoll, 1937, p. 78)

Figure 5

Ballistics



b) Äußere Ballistik

I. Schuß im luftleeren Raum

Auf das fliegende Geschoß wirkt vor allem das Gewicht und der Luftwiderstand ein, der mit wachsender Geschwindigkeit schnell zunimmt. In solchen Fällen, in denen das Verhältnis des Geschoßgewichtes zur Querschnittsfläche — die „Querschnittsbelastung" — besonders groß und die Anfangsgeschwindigkeit verhältnismäßig klein ist (Minenwerfer!), erhält man auch dann noch angenähert richtige Ergebnisse, wenn man den Luftwiderstand ganz vernachlässigt und das Gewicht als die einzige wirkende Kraft betrachtet.

Abb. 23. Wurfparabel

Die Komponenten v_x und v_y der Geschwindigkeit (vgl. Abb. 23) sind dann nach dem Satz vom „Parallelogramm der Geschwindigkeiten" t Sekunden nach dem Abschuß:

$$(1) \quad \begin{cases} v_x = v_0 \cos \alpha, \\ v_y = v_0 \sin \alpha - gt, \end{cases}$$

und für die Lage des Geschosses gilt:

$$(2) \quad \begin{cases} x = v_0 \cos \alpha \cdot t, \\ y = v_0 \sin \alpha \cdot t - \frac{1}{2} gt^2. \end{cases}$$

Aus (2) erhält man durch Ausschaltung von t die Gleichung der Flugbahn (der „Wurfparabel"):

$$(2') \quad y = x \cdot \operatorname{tg} \alpha - x^2 \cdot \frac{g}{2 v_0^2 \cos^2 \alpha}.$$

Source: Nationalpolitische Anwendungen der Mathematik, p. 39

Post WWII

Zoll attempted to resume his career as a textbook author after 1945 (Krömer & Nickel, 2023). His affiliations with the Nazi party caused him troubles in the Allied-occupied Germany. The Vieweg publisher tried to revise his textbook, along with authors and editors, to remove all disagreeable content. Due to objections from the education division of the British military government, the publisher eventually replaced Zoll in the fall of 1947 as editor with his former co-author, Hans Brandes (Goebel & Schlensag, 2008).

In a letter dated October 2, 1945 (archived at the TU Braunschweig library) to the publisher, Vieweg, Zoll raised the question of whether it was even possible for him to remain as an editor, since he had been a member of the National Socialist German Workers' Party (NSDAP):

> […] with the new book, it might be questionable whether an education counselor, who was a Parteignosse (although no longer active), can be an editor. Should you have any concerns, I suggest assigning a colleague who was not Parteignosse as the editor. (Zoll, 1945)

In a letter dated May 5, 1947, the education branch of the British Military Government responded to Vieweg: "In reply to your letter of 27th March, would it not be preferable to concentrate on books by Bauer-Hanxleden than on the work of a man known to everyone to contain pernicious material from the Nazi era? We suggest a new edition under a different editor's name" (Goebel & Schlensag, 2008). Without further publications, Zoll died on January 20, 1952, in Düsseldorf, Germany.

References:

Goebel, M., & Schlensag, C. (2008). Hans Brandes (1883 – 1965). Promotion in Halle – Lehrer in Braunschweig. *Reports on History of Mathematics, 58.* Martin-Luther-Universität, Halle-Wittenberg.

Krömer, R. & Nickel, G. (2023). Band 17. *SieB - Siegener Beiträge zur Geschichte und Philosophie der Mathematik.* Universität Siegen.

Zoll, O. (1903). Über Flächen mit Scharen geschlossener geodätischer Linien [Doctoral dissertation, University of Göttingen]. *Mathematische Annalen, 57*. pp 108-133.

Zoll, O. (1931). Mathematisches Arbeits- und Lehrbuch. Mittelstufe. Braunschweig: Vieweg.

Zoll, O. (1937): Nationalpolitische Anwendungen der Mathematik. *Für die Oberstufe höherer Lehranstalten*, pp 28. Vieweg. Braunschweig.

Zoll, O. (1939). Mathematisches Arbeits- und Lehrbuch für höhere Lehranstalten. Unterstufe: Rechnen und Geometrie (1. und 2. Klasse). Vieweg. Braunschweig. pp 14

Zoll, O. (1939). Mathematisches Arbeits- und Lehrbuch. Mittelstufe: Geometrie und Algebra (3., 4. und 5. Klasse). Vieweg. Braunschweig. *Ausgabe A (für Jungenschulen)*, Vorwort (Preface).

Zoll, O. (1940). Mathematisches Arbeits- und Lehrbuch. Oberstufe: Geometrie und Algebra (6., 7. und 8. Klasse). Vieweg. Braunschweig. *Ausgabe A (für Jungenschulen)*, pp 26-27.

Zoll, O. (1940). Mathematisches Arbeits- und Lehrbuch für höher Lehranstalten. Oberstufe: Geometrie und Algebra (6., 7. und 8. Klasse). Vieweg. Braunschweig. *Ausgabe B (für Mädchenschulen).* Geschichtlicher Einleitung (Historical Introduction).

Zoll, O. (1942). Mathematisches Arbeits- und Lehrbuch. Oberstufe: Geometrie und Algebra (6., 7. und 8. Klasse). Vieweg. Braunschweig. *Ausgabe A (für Jungenschulen)*, pp 199-254.

Zoll, O. (1943). Mathematisches Arbeits- und Lehrbuch. Oberstufe: Geometrie und Algebra (6., 7. und 8. Klasse). Vieweg. Braunschweig. *Ausgabe B (für Mädchenschulen)*, pp 11-20.

Vieweg-Archive, Library at Technische Universität Braunschweig, Letter Exchange between Vieweg Verlag - Otto Zoll: Akte VIZ:25 Zoll, Otto

JOURNAL OF MATHEMATICS EDUCATION AT TEACHERS COLLEGE | FALL 2024 | VOLUME 15, ISSUE 2

Higher Arithmetic: A Textbook Analysis

Samantha Moroney
Teachers College, Columbia University

ABSTRACT This paper investigates the historical context, internal content, and educational significance of Higher Arithmetic, a 1919 mathematics text by David Eugene Smith and George Wentworth, which has been insufficiently studied. Through historical and content analysis, this study reveals the authors' intention to provide a review and extension of knowledge in a modern context of post-World War I industrial United States, intended for both students and pre-service teachers who need thorough teacher preparedness. Although Higher Arithmetic may not be well-recognized today, its progressive intentions for teacher training and the modernization of material remain goals that are as important today as in Smith and Wentworth's time.

KEYWORDS *Mathematics textbooks, Historical analysis, David Eugene Smith*

David Eugene Smith (1860-1944) is one of the most notable names in the history of mathematics education. During his life, he produced many works, including a significant number of textbooks, both independently, and in collaboration with his contemporaries and other peers. In his works, Smith always focused on historical perspective, modernization, and teacher preparedness, solidifying him as a leader in mathematics education. While some of his many textbooks have been studied in papers and student dissertations (Goodwin, 2012; Murray, 2012; "Notes and News," 1921), others have been less explored. In 1919, David Eugene Smith collaborated with fellow mathematics educator, George Wentworth (1868-1921), on one such underexplored textbook, *Higher Arithmetic*, published by Ginn and Company.

At the time of the textbook's publishing, the world (and the United States) was experiencing great change due to significant events in world politics and the United States economy, advancements in technology and manufacturing, and education. This paper aims to assess the historical context in which *Higher Arithmetic* was published, and to examine the contents of the text itself, to better understand its place in the progression of mathematics education. The current political climate, both nationally and internationally, consists of troubling economic realities, rising immigration numbers, and wars, which could affect education in similar ways to the climate of the early 1900s. Through a historical analysis of archival texts and an analysis of how *Higher Arithmetic* is structured for both thorough teacher preparedness and modern contextualization, we may gain a clearer picture of how to write educationally relevant textbooks today.

Historical Context

The 1890s to 1920s saw a progressive era of reforms in the United States, coinciding with a transition to industrialization and urbanization. Approaching the 1900s, the second largest economic depression in US history, the Panic of 1893, preceded the final stage of the Industrial Revolution. This brought a period of immigration, leading to an influx of children entering the public school system, and advances in industry and technology that affected the educational system. Schools needed to teach labor specialization so workers and technicians could adapt to the shift from factories to machine-run, mass-production facilities. Early 1900s surveys on the arithmetic and practices needed in business occupations appeared (Goodwin, 2012). Smith himself emphasized omitting "obsolete" topics for modern education (Smith, 1903).

Numerous scientific innovations marked this era as well. The Wright brothers achieved powered flight in 1903, Einstein proposed his theory of Relativity in 1905, Henry Ford opened his first automobile assembly line in 1913, and the telephone (which had been patented in the late 1800s) became a more common household device. Amidst such innovation, however, the world fell into turmoil from 1914-1918 with World War I. The Child Labor Act of 1916 established the minimum working age of 14 years, extending the average time spent in school for children and teenagers when the number of students attending four-year high schools had already nearly doubled from 1890 to 1900 (Donoghue, 2006). The end of the war in 1918 saw a damaged postwar economy of disrupted trade, debt, and reparations in Europe, which would ultimately contribute to the Great Depression and economic downturn of the 1930s.

During all this time, David Eugene Smith continuously produced textbooks, motivated by his goal of modernization and his belief that effective mathematical education required better teacher preparation. Smith believed that excellent teachers needed not only rigorous content knowledge, but also pedagogical expertise, including historical, philosophical, and psychological perspectives, as well as experience teaching under supervision. Smith's texts, therefore, emphasized teacher preparation and historical context, and were often published in collaboration with other noted mathematics educators (both posthumously and with his contemporaries) such as Jekuthiel Ginsburg, Wooster Woodruff Beman, George A. Plimpton, Karl Fink, G. St. L. Carson, and George A. Wentworth (Ockerbloom, n.d.). *Higher Arithmetic* from 1919, authored by Smith and Wentworth, focuses on these areas Smith deemed so important.

Higher Arithmetic

Wentworth and Smith produced 32 unique publications (not including later editions) related to arithmetic, algebra, geometry, trigonometry, and more, written for primary, junior-high, and high-school levels, or specified for beginner, intermediate, and advanced mathematics. (Catalogs can be found through UPenn, "The online books page," and viewed at HathiTrust.) Wentworth and Smith's most recognizable textbooks include the first of the Wentworth-Smith mathematical series: *Arithmetic*—a three-book collection for an extensive elementary school arithmetic course. Considered "one of the most important efforts of recent times to produce a notably successful school arithmetical course that shall be scholarly, progressive, practical, and usable by all teachers" (Review, 1911a, p. 328), *Arithmetic* provided ample exercises so that teachers could be flexible with their selection, but would never lack practice work for students who required it. Wentworth and Smith notably omitted obsolete material and centered problems in the real-life of school children, such as farm work. The "Review" (1911a) stated, "David Eugene Smith has undoubtedly the most scholarly arithmetical equipment of anyone in America, and the Wentworth arithmetics have long been in the center of the stage, so that this combination brings into action the most complete scholarship and highly successful schoolroom methods" (p. 328). This review provides significant evidence for the importance both men held in the educational world, each revered for their work. An earlier "Review" (1909) on the first book concludes that it is the first arithmetic textbook to incorporate the best of both the old and new ideas, methods, and scientific principles. In a "Review" (1911) for their *Plane and Solid Geometry* textbook, a comparison is made to the trusted and widely-used texts of George A. Wentworth, citing a "simplicity of treatment, clearness of expression, symmetry of page" (p. 440) as the best of the old, which Wentworth and Smith combine with the new and modern.

On *Higher Arithmetic* itself, little review or analysis was found by this author. This lack of findings does not, however, preclude its popularity. While this author was unable to determine the success or amounts of use, it was recognized as part of the Wentworth-Smith Series, identified in journals of the time to be, "A series well known throughout the country for its sane pedagogy and sound service. Thousands are now using the series." (Font Matter, 1919, p. 2). *Higher Arithmetic* was mentioned one year later for its practicality and significance as a review of the fundamental principles and practices in arithmetic (Front Matter, 1920).

Smith and Wentworth

David Eugene Smith started a promising law career in 1884 when he took on a temporary mathematics teaching position. Having found a love for mathematics and teaching, he took the role full-time, while still working in law and pursuing other educational degrees. In 1891, he became chair of mathematics at the Michigan State Normal School (i.e. teachers' college), where he first began writing textbooks with Wooster Beman, a noted mathematics educator, and innumerable papers in both mathematics education and its history. He also began building his extensive mathematics library. He took the position of chair in mathematics at Teachers College, Columbia University in 1901 before retiring in 1926 and donating his library of texts to Columbia University in 1931 (O'Connor & Robertson, 2015).

George Wentworth, born in Exeter, New Hampshire, took after his father, George A. Wentworth (1835-1906), by attending Phillips Exeter Academy as a boy and then Harvard University. G.A. Wentworth became a mathematics educator and worked at Phillips Exeter Academy as the head of the mathematics department, even serving as principal on two separate occasions, before resigning his position in 1891. G.A. Wentworth then dedicated himself full-time to the writing of approximately 50 math textbooks, many of which became the standard of the day, utilized in a vast number of schools and courses (American Council of Learned Societies [ACLS], 1936). His son, George Wentworth, left Harvard in his final year to do commercial work out west, but later returned to New Hampshire and began writing textbooks, like his father. He became the owner of his father's works upon his father's death, which would prove useful in the coming years. Wentworth and Smith officially entered a partnership, resulting in the *Wentworth-Smith Mathematical Series* and numerous other collaborations in the early 1900's. Although the official year of their partnership is not agreed upon--news sources of the time reported 1913 (Notes and News, 1921) while the *Journal of Education* wrote about the Wentworth-Smith Mathematical Series in 1911 (Review, 1911a)—their work was consistent throughout the 1910s.

Confusion in distinguishing between the two Wentworths does exist in the academic community. This is likely due to several factors: 1) George Wentworth's death in 1921 (Notes and News, 1921) is often absent from online and university websites, including the University of Pennsylvania and Teachers College libraries; 2) the distinction between father and son has been made using only a middle initial in the father's name; 3) both were notable in the mathematics education community and recognized to have collaborated with Smith (Ockerbloom, n.d.-a, Ockerbloom, n.d.-b); and 4) several examples of mistaken Wentworth attributions can be found through library searches for their publications and misinformed dissertations. Finally, although David Eugene Smith never collaborated with either Wentworth until *after* the death of G. A. Wentworth in 1906 (Ockerbloom, n.d.-a), owning his father's textbook rights made it possible for George Wentworth to give access and use of his father's work to Smith, resulting in publications (largely accessible through HathiTrust) (2024) from Smith and George Wentworth, posthumous publications from Smith and George A. Wentworth, and collaborations from all three.

The influence of all three men is evident in their collaborative works, and it is clear that a close relationship developed between Smith and Wentworth, fueled by their collaboration and mutual appreciation and respect for Wentworth's father's work. Smith became closely acquainted with Wentworth's entire family. Correspondences from both George Wentworth and his sister, Ellen Lang Wentworth, to Smith have survived through the Smith personal papers in the Columbia Rare Books Collection. Through George's correspondences, we gain a sense of his growing desire to collaborate with Smith over other authors with whom they had both collaborated previously, particularly G. St. L. Carson, a teacher of mathematics in England (Carson & Smith, 1913). Indeed, George Wentworth indicated in a letter to Smith in 1915: "To tell the truth I have much less interest in any of the books which we write with Carson than in any American book." The correspondence shows how Wentworth and Smith successfully worked together with detailed updates, frequently sent pages, 'casts' from the press, and thoughts on book structure and order. They write "send me the 1R of the Essentials and I will incorporate my changes with yours on the 1R. We ought to cast from the 2R at the latest" (Wentworth, 1915) or, when reflecting on the inclusion of a table of Logarithm Reciprocals, stating "The only reason I can see is that this table should be used for the logarithms of numbers in the denominator of a fraction–that is used in place of our American 'cologarithm.' If this was the reason for writing pages 340 and 341, why on earth do we not use this table in solving examples" (Wentworth, 1915). The subject of Carson's involvement in some of their projects becomes a point of humor on both men's parts, furthering this author's belief in their working relationship *and* in their personal, trusting friendship. Smith is quoted, "I sympathize with your desire to see Carson's [manuscripts] for the key to Algebra II. It will be here about three weeks ago and I will let you know when and that it came, whenever this happened." (Smith, 2015) In response, Wentworth supplied, "I should also like to make a bet large or small and am willing to offer you long odds. If the Germans succeed in sinking any English mail steamers Westward bound I am willing to bet that the hold will turn out to be full of manuscripts from Carson" (Wentworth, 1915).

In Ellen's correspondence with Smith, we see evidence of Smith's appreciation for her father's work and the long-standing connection he held with the Wentworth family. Ellen Wentworth wrote to Smith in 1931, "I am very much pleased that you have recommended the name of my father in the Dictionary of American Biography," and, "I am glad that you saw Emily and her dear baby recently. Emily makes a lovely mother and we enjoy seeing them both wherever we can," referring to her niece, Emily Wentworth, the daughter of George Wentworth. The long-lived relationship with the entire family speaks to the closeness and respect Wentworth and Smith undoubtedly held for each other throughout their collaboration. As one of the final texts written by the two before Wentworth died, *Higher Arithmetic* is an excellent piece to analyze for evidence of Wentworth and Smith's relationship.

The Preface

The authors' intent for *Higher Arithmetic* is offered in the preface, written as a direct letter to the readers, in which Smith and Wentworth discuss their intended audience, prerequisite recommendations, the book's purpose, the approach for instructors of mathematics, the sequence of topics, and the overall focus of the content.

The textbook is directed toward students and teachers in "normal school" and high school mathematics and is intended for both review and extension of prior knowledge. This reflects Smith's educational desires, as normal schools, popularized during this era, were largely developed with his ideas in mind: focus on content knowledge and pedagogical understanding for future teachers (Wentworth & Smith, 1919, p. iii-iv). The authors further emphasize modernized arithmetic with some applied work relevant to day-to-day life, focusing on a thorough understanding of principles with commercial applications and only some formal definitions, rather than on memorization, drill, and technicalities of specific vocational trades. They further encourage teacher flexibility, to adapt the materials to match the needs and personal preferences of their students. No particular rules are provided for how to follow the text as the sequence of topics is considered conventional but intentionally non-binding.

The tone is informative and instructive, intent on preparing preservice teachers with knowledge beyond the elementary level and providing high school commercial students with the broad and thorough knowledge needed to enter the workforce. This includes attention to logarithms and the slide rule for the "growing demand" for arithmetic teachers and students planning to enter the business world to know these instruments (Wentworth & Smith, 1919, p. iii-iv).

The Contents

The structure of the Contents is in line with their previous works and the text's preface. The book includes twelve distinct chapters on different topics: Reading and Writing Numbers, Addition, Subtraction, Multiplication, Division, Percentage, Thrift and Investment, Mercantile Arithmetic, Corporation Arithmetic, Industrial Arithmetic, Arithmetic of the Bank, and Arithmetic of Civic Life. Each of these twelve chapters is written to house two pages of initial explanatory information on one subtopic followed by one page of exercises, then alternating single pages of written explanation with exercises for other subtopics, and finally a single page of exercises for cumulative chapter review. Consider, for example, Chapter IX: Corporate Arithmetic. This chapter holds subsections on Stocks (two pages of content, one page of exercises); Bonds, Wages, Graphs, and Circular Pictograms (each one page of content, one page of exercises); and Review of Chapters I-IX (one page of exercises). After these first twelve chapters is Chapter XIII: Advanced Problems. This chapter houses a brief explanation before it delves into seven and a half pages of advanced problem exercises. A section for Supplementary Work follows, focusing on square roots, logarithms, and the slide rule, and including written pages, tables, and exercises. The textbook concludes with a Tables for Reference section (measurement conversions) and an index. Specific topics included in each chapter can be seen in Figure 1.

This topic-related content is as reported: a review and extension of elementary work with a focus on the practical world applications, without being too technically specific to vocations. Starting with elementary topics of numbers, addition, subtraction, multiplication, division, and percentages, the concepts begin as a review, presented in the same order one might structure the elementary math curriculum to get progressively more difficult. However, within each section, the concepts move from the simple to more extended use that is average for an elementary school student.

Figure 1

Chapter subtopic content of "Higher Arithmetic," 1919

Chapter/Section	Subtopics
I. Reading and Writing Numbers	Notation, Number Names, Notation, Index Notation, RomanNumerals, Fractions, Per Cents, Ratio, Abstract and Concrete Numbers, Algebraic Notation, Review of Chapter 1
II. Addition	Integers and Decimals, Checks in Addition, Reduction of Fractions, Common Fractions, Compound Numbers, Short Methods, Algebraic Addition, Review of Chapters I and II
III. Subtraction	Methods of Subtraction, Complementary Method, Checks and Short Methods, Review of Chapters I-III
IV. Multiplication	Nature of Multiplication, Multiplication of Integers, Multiplication of Fractions, Multiplication of Decimals, Approximate Results, Finding Per Cents, Checks in Multiplication, Check of Nines, Short Methods, Compound Algebraic Numbers, Review of Chapters I-IV
V. Division	Nature of Division, Integers and Decimals, Checks in Division, Division of Fractions, Approximate Results, Short Methods in Division, Division of Compound Numbers, Algebraic Division, Review of Chapters I-V
VI. Percentage	Formulas, Formulas in Commission, Formulas in Discount, Formulas in Simple Interest, Review of Chapters I-VI
VII. Thrift and Investment	Personal Cash Accounts, Budgets, Household Accounts, Living within the Budget, Economy and Discount, Investing Money, Compound Interest, Simple and Compound Interest, Stocks and Bonds, Stocks, Other Kinds of Investment, Life Insurance, Review of Chapters I-VII
VIII. Mercantile Arithmetic	Cash Checks, Bills, Trade Discount, Invoices and Statements, Profit and Loss on Cost, Profit or Loss on Selling Price, Borrowing Money, Simple Interest, Property Interest, Review of Chapters I-VIII
IX. Corporation Arithmetic	Stock, Bonds, Wages, Graphs, Circular Pictograms, Review of Chapters I-IX
X. Industrial Arithmetic	Common Measures, Accuracy of Measurement, Plane Rectilinear Figures, Polyhedrons, Curvilinear Figures, Round Bodies, Measurement of Land, Metric System, Metric Length, Metric Area and Volume, Metric Capacity and Weight, Specific Gravity, Review of Chapters I-X
XI. Arithmetic of the Bank	Depositing Money, Drawing Money, Bank Books, Transmitting Money, Collecting by Draft, Borrowing from a Bank, Foreign Money, Review of Chapter I-XI
XII. Arithmetic of Civic Life	Tariff, State and Local Taxes, Internal Revenue, Review of Chapter I-XII
XIII. Advanced Problems	Micellaneous Problems
Supplementary Work	Advanced Theory and Practice: Square Roots, Logarithms and Logarithmic Properties, Slide Rule
Tables for Reference	Measurements
Index	

To this end, there are, as reported in other works of Wentworth and Smith, a vast number of exercises, intentionally ranging from simple to advanced. This not only expands the topics, but allows for flexible teaching, allowing a teacher to use what they need to fit the demands of their students and those students' personal needs, whether that be advanced materials only, or a lot of practice on every aspect, starting at a more beginner level. After percentages, the topics then increase in both difficulty and practicality by focusing on thrift, investment, mercantile, corporation, industry and banking arithmetics, and arithmetic for civic life. None of these go into depth on any particular vocation, rather, they are all aimed at real-world use and include expansion from simple to difficult problems.

In introducing content, the authors try to do so in a clear, concise manner. They identify important terms and principles, providing definitions (only sometimes formally), and occasionally light historical background. They do this before providing any examples or exercises to help in building the solid foundation and understanding desired. In addition, the natural progression of the book builds on notational awareness. Notation is introduced topic by topic, utilizing previously discussed notations.

Figure 2

Chapter I- Reading and Writing Numbers, Exercises 17-24.

Write in words, as on a check, the following numbers:

17. $225. **19. $1200.** **21. $1950.** **23. $10,500.**
18. $250. **20. $1505.** **22. $2775.** **24. $15,708.**

Figure 3

Chapter IV- Multiplication, Exercise 1 in Multiplication of Fractions.

1. Draw a line $\frac{3}{4}$ in. long, divide it into 3 equal parts, and then find the combined length of two of these parts. In other words, show that $\frac{2}{3}$ of $\frac{3}{4}$ is $\frac{1}{2}$, thus justifying the definition of the multiplication of fractions.

Figure 4

Chapter VIII- Mercantile Arithmetic, Bill Example.

JAMES McCREERY & CO.
Fifth Avenue Thirty-Fourth Street
New York

May 1, 1922

Mrs. David Dunham
501 West 120th St., City

	Account rendered		Charges		Credits			
Apr.								
7	3 doz. towels	7.50	22	50				
8	1 suit case		19	00				
	1 doz. towels				7	50		
12	1 pc. ribbon	.60						
	1 " "	.85						
	6 yd. linen 30	1.80	3	25				
			44	75	7	50	37	25

If this bill is paid by check and no further receipt is required, please detach this coupon and mail with check.
Folio 534 **James McCreery & Co.**

Name, Mrs. David Dunham **Date, May 1, 1922**
Address, 501 West 120th St. **Amount, $37.25**

Figure 5

Chapter VIII- Mercantile Arithmetic, Invoice Example.

THE BRONSON IMPORTING COMPANY
2487 Boston Avenue
Chicago

May 17, 1924

Sold to **J. M. Bryan Co.,**
Terms: 2/10,N/30 **La Salle, Ill.**

May	17	7 bx. Castile soap	4	20	29	40		
		750# Java coffee		30	225	00		
		16 hf.cht. Oolong tea	56	25	900	00		
							1154	40

The exercises include a focus on real-life, practical application, financial literacy, and problem-solving skills through a variety of topics, avoiding the abstract mathematical concept in favor of a practical, application-centered understanding. Some of these problems appear in a limited drill-like practice, such as Figure 2 (Wentworth & Smith, 1919, p. 5), while others are designed to build a deeper understanding of a given principle, definition, or process through their problem-solving, such as Figure 3 (Wentworth & Smith, 1919, p. 53), Figure 3 comes right after seeing $\frac{2}{3} \times \frac{3}{4}$ as an example of how to take the product of two fractions, using cancellation.

As discussed, much of this book is intended to help those who would work in the commercial and industrial world. The problems thereby reflect this in industry and sales-related financial needs, such as bills, invoices, and statements. For example, Wentworth and Smith included what the documents looked like and how they would be used before giving exercises related to the understanding of such documents. See Figure 4 (Wentworth & Smith, 1919, p. 126), Figure 5 (Wentworth & Smith, 1919, p. 130), and Figure 6 (Wentworth & Smith, 1919, p. 131).

One significant contextual event affecting this textbook is World War I, referred to in its pages as the European War. Although WWI is incorporated somewhat subtly, there are several references throughout the text to the post-war economy and the needs of the common man. For example, the large focus on financial literacy is driven by these personal needs as well as the economic and industry needs. Figure 7 is an excerpt from the beginning of Chapter VII on Thrift and Investment (Wentworth & Smith, 1919, p. 98). The topic is approached as the first of many problems that the authors explain are guaranteed to be faced by everyone and therefore must be learned and understood for their security.

In other areas, references to the war, and life during it, are embedded into problems, relating them to very real experiences for American people, as seen in Figure 8 (Wentworth & Smith, 1919, p. 111), or otherwise used as

Figure 6

Chapter VIII- Mercantile Arithmetic, Exercises 1 and 3 in Invoices and Statements.

1. If the invoice on page 130 is paid within 10 da., what is the net price paid by the J. M. Bryan Co.?

Find the net amounts of the following bills:

3. 600 bags coffee @ $32.50 less 33⅓, 10; 200 bags coffee @ $42 less 25, 10, 5; 400 bags coffee @ $48.50 less 40, 15, 5; 300 bags coffee @ $53.80 less 30, 20, 10.

Figure 7

Chapter VII– Thrift and Investment, discussion on personal cash accounts.

Personal Cash Account. One of the first uses that an adult has for arithmetic is the keeping of his personal cash account. The recent extension of the income tax and the increased necessity for thrift and economy imposed by war conditions has made it important that everyone, whatever his financial standing, should keep a record of his receipts and expenditures. The following is a model personal cash account:

1924					1924				
May	1	Cash on hand	276	85	May	2	Rent	35	
	2	Wages	30			3	Groceries	8	75
	5	R. J. Jones	10			4	Meat bill	5	40
						8	Balance	267	70
			316	85				316	85
May	8	Balance	267	70					

Figure 8

Chapter VII- Thrift and Investment, Exercise 11 in Compound Interest

11. A war savings stamp was bought on Jan. 1, 1918, for $4.12. On Jan. 1, 1923, it was redeemed by the government by the payment of $5. Show that this was approximately equivalent to allowing 4% interest, compounded quarterly.

driving forces behind specific subject matter, including the increased need for understanding the metric system, given a vast increase in post-war foreign trade.

Another important aspect to consider is how concepts are presented in ways that help prepare teachers. Consider Chapter III: Subtraction. The purpose of the chapter is to think about subtraction with different kinds of numbers, particularly as may be seen in the business world. Pedagogical and methodological considerations are often built into the written portions of the chapter to help guide pre-service teachers to recognize good student practices and their implications for teacher knowledge. For example, at the start of this chapter, before any content information, the authors explain that students with knowledge of any of five popular methods of subtraction in America can use that method to move forward with their elementary arithmetic, but that a teacher or *Higher Arithmetic* student will need working knowledge of all five. (Wentworth & Smith, 1919, p. 39) They then move on to language and etymology, including minuend, subtrahend, remainder, difference, rest, balance, and minus. For the average student, this may not be the most interesting, but for the teacher-in-training, it refines their language so that they speak about topics with intent and knowledgeable direction.

Before moving on to detail the five methods, Smith and Wentworth first offer a relationship, both numerically and algebraically, to explain how difference remains the same when numbers are increased by the same values (7-2=17-12=27-22=…) (Wentworth & Smith, 1919, p. 40) They also offer the method of partitioning numbers. These supply a deeper understanding of principles between numbers in subtraction which leads to deeper conceptual understanding. They are relationships that a teacher should be able to see and readily explain, and that a *Higher Arithmetic* student should be able to understand and recognize algebraically. Four of the five methods are then displayed through examples that remain simple, not broaching how to handle values of ten or higher.

The page of exercises that follows this introduction first offers five practice problems related to the methods (four practice and one pedagogical defense on a method), before offering four more problems that use the four methods but expand their use to include other numbers and units such as fractions, length metrics, money, time, subtraction of numbers with different amounts of digits, and numbers which are generally larger. This is completed with a single question on the algebraic variable notation justification/understanding (Wentworth & Smith, 1919, p. 41).

The chapter continues by introducing the fifth method, the "complementary method" which is more involved and requires new vocabulary and a full, detailed example. The subsequent page of exercises focuses on identifying a "complement" before including practice using the complementary method. These again remain simple initially but expand to include one question with numbers without the same number of digits, and one question using decimals (Wentworth & Smith, 1919, p. 42-43).

The third section of the chapter briefly introduces three concepts: checks in subtraction, a special case in which a short method can be used and what it looks like, and the first look at solving truly algebraic subtraction problems (expressions with a's and b's). The majority of the 23 practice problems are drill exercises for checks and short methods, eventually expanded to include units of length and degree, decimals, and two problems to practice algebraic subtraction (Wentworth & Smith, 1919, p. 44-45).

The chapter concludes with a page of exercises that use these concepts and expand upon them by including problems with more than two subtracted values, or by applying the topic to subtracting two Roman numeral values (which is included to show how the principles of subtraction can be applied to this particular case, and therefore likely to others as well) (Wentworth & Smith, 1919, p.46).

In general, none of the topics included in the short chapter are exceedingly difficult, and most can be used at the elementary level. However, they are presented in a way that highlights what is simple and what complicates the concepts for students. In this case, different numbers of digits, different numbers of numbers, remainders over 10 that must carry a digit, units, the sudden appearance of variables (used sparingly so as not to overwhelm), and the actual algebraic relationships that govern subtraction. While students need to learn to navigate all these things, the take-aways for teachers are the importance of the order in which you may introduce the various concepts, the language that you use, the small additions that complicate things for children and how you might slowly or limitedly introduce them, and deeper teacher understanding, so that they know how to use all the various pieces at the same time to reach all of their students.

Conclusion

Higher Arithmetic, written by Wentworth and Smith in their textbook collaborations would be one of the last textbooks written by the two authors while Wentworth was still alive. In Smith's ongoing aim for better-prepared secondary mathematics teachers, he and Wentworth set about after World War I to write a textbook that would apply to the real-world needs of the common man in a fast-developing time of industry and machinery. As more school-aged children found their future working-skills needed developing, Smith and Wentworth endeavored to create a secondary system of arithmetic that would provide the review and advancing tools they would need, while simultaneously helping their teachers to be better educators with more options and choices at hand. By all accounts, they were successful in this. They created a textbook that was flexible and adaptable for students, while informative for future teachers, rooting the content in the post-war needs of the time, with hands-on practice and a natural progression of student challenges and mathematical understanding.

So, the question remains as to why, although it was likely very successful given the popularity of all *Wentworth-Smith Series* texts, little attention seems to have been given to *Higher Arithmetic*. One possible answer lies in the prolific nature of their textbook creation. In the same year as *Higher Arithmetic*, Wentworth and Smith published School Arithmetic, which would have a secondary publishing the following year in 1920. It is possible that this publication, the last in Wentworth's lifetime, overshadowed *Higher Arithmetic*. Another possible factor is that only ten years after Wentworth's death, the United States would experience a massive economic decline, leading to the Great Depression, and calling for very different needs of the populace for some time. It is possible that *Higher Arithmetic* simply became lost in transition.

Despite this, it remains clear that Wentworth and Smith were, together, a powerful and collaborative team with many surviving texts that are, to this day, highly thought of and remembered for their progressive intentions for teacher training and modernization of material. These goals are as important today as they were in Smith and Wentworth's time. Considering the turbulent climate we find ourselves in today, with ongoing wars and possible economic decline, a text that was well-placed for similar events is quite relevant. *Higher Arithmetic* suggests that teacher preparedness and modern relevance are critically important to both educational success and effective textbook creation.

References

Archival sources

Smith, David Eugene. Professional papers. Rare book and manuscript library. Columbia University, New York:
Letters to and from Ellen Lang Wentworth. Box 54.
Letters to and from George Wentworth. Box 54.

Other sources

American Council of Learned Societies. (1936). *Dictionary of American Biography* (Vol. 19). Charles Scribner's Sons.

Beman, W. W., & Smith, D. E. (1897). *Higher Arithmetic*. Ginn & company.

Carson, G. St. L., & Smith, D. E. (1913). *Essays on mathematical education.* Ginn and Company.

Donoghue, E. F. (2006). The education of mathematics teachers in the United States: David Eugene Smith, early twentieth-century pioneer. *Paedagogica Historica, 42*(4/5), 559–573. https://doi.org/10.1080/00309230600806831

Front Matter. (1919). *The High School Journal, 2*(8). http://www.jstor.org/stable/40361007

Front Matter. (1920). *The High School Journal, 3*(4). http://www.jstor.org/stable/26290427

Goodwin, D. (2012). Historiography for educational leadership in mathematics: Content analysis of a 1904 K-8 mathematics series and the early 20th century context of its use Available from ERIC. (1651840402; ED549869). Retrieved from https://tc.idm.oclc.org/login?url=https://www.proquest.com/dissertations-theses/historiography-educational-leadership-mathematics/docview/1651840402/se-2

HathiTrust. (2024) *HathiTrust Digital Library.* HathiTrust Digital Library. https://www.hathitrust.org/

Murray, D. R. (2012). *A Cabinet of Mathematical Curiosities at Teachers College: David Eugene Smith's Collection* (Order No. 3504178). Available from ProQuest Dissertations & Theses Global. (1010783420). https://tc.idm.oclc.org/login?url=https://www.proquest.com/dissertations-theses/cabinet-mathematical-curiosities-at-teachers/docview/1010783420/se-2

Notes and News. (1921). *The American Mathematical Monthly, 28*(10), 402–406. http://www.jstor.org/stable/2972172

O'Connor, J. J., & Robertson, E. F. (2015, June 2). *David Eugene Smith.* MacTutor. https://mathshistory.st-andrews.ac.uk/Biographies/Smith_David/

Ockerbloom, J. M. (Ed.). (n.d.-a). *David Eugene Smith (Smith, David Eugene, 1860-1944).* The Online Books Page. https://onlinebooks.library.upenn.edu/webbin/book/

Ockerbloom, J. M. (Ed.). (n.d.-b). *George Wentworth (Wentworth, George, 1868-).* The Online Books Page. https://onlinebooks.library.upenn.edu/webbin/book/lookupname?key=Wentworth%2C%2BGeorge%2C%2B1868-lookupname?key=Wentworth%2C%2BGeorge%2C%2B1868-

Professor George A Wentworth. (1886, April 29). The *Boston Globe*, p. 1.

[Review of *WENTWORTH-SMITH MATHEMATICAL SERIES*, by G. Wentworth & D. E. Smith]. (1909). *The Journal of Education, 70*(8 (1743)), 218–218. http://www.jstor.org/stable/42812176

[Review of *WENTWORTH'S PLANE AND SOLID GEOMETRY*, by G. Wentworth & D. E. Smith]. (1911). *The Journal of Education, 74*(16 (1851)), 440–440. http://www.jstor.org/stable/42819110

[Review of *THE WENTWORTH-SMITH MATHEMATICAL SERIES*. *Threebook Series: Book I. (intended for grades III and IV.), Book II. (for grades V. and VI.), Book III. (for grades VII. and VIII.)*, by G. Wentworth & D. E. Smith]. (1911a). *The Journal of Education, 74*(12 (1847)), 328–328. *http://www.jstor.org/stable/42818739*

Smith, D.E. (1903). The teaching of elementary mathematics. New York: The Macmillan Company.

Wentworth, G., & Smith, D. E. (1919). Higher arithmetic. Ginn and company.

JOURNAL OF MATHEMATICS EDUCATION AT TEACHERS COLLEGE | FALL 2024 | VOLUME 15, ISSUE 2

Gazeta Matematică: A Historical Perspective on Its Role in Romanian Mathematics Education

Elizabeth Wilson
Teachers College, Columbia University

ABSTRACT This study examines Gazeta Matematică's (GM) influence on mathematics education in Romania from 1895-1945. Analyzing 13 articles from GM's archives, this research explores discussions on mathematics methodology and didactic issues and highlights Ion Ionescu's contributions. Findings reveal tensions between idealized rigorous pedagogy and practical classroom challenges. The study highlights GM's dual role as a mirror reflecting the complexities of mathematics education in a modernizing Romania and as a catalyst for educational reform. It demonstrates GM's significance in fostering mathematical discourse and provides insights into evolving priorities and attitudes toward mathematics instruction during a period of significant social and political transformation.

KEYWORDS: *Mathematics education history, Romanian education reform, Gazeta Matematică, educational periodicals, mathematical pedagogy*

Introduction

The transformative epoch spanning from the 1848 revolution through World War II proved instrumental in shaping contemporary Romania. This half-century witnessed an unprecedented acceleration in the evolution of Romania's public sphere. This can be seen particularly in its geographic and political configurations through the unification of the Danubian Principalities in 1859, the achievement of independence from Ottoman suzerainty in 1877, and the subsequent territorial consolidation of the Romanian state. This period of profound transformation culminated in the establishment of the Kingdom of Romania in 1881, marking a decisive shift in both the country's geopolitical positioning and its internal administrative structures. The territorial configuration continued to evolve dramatically, with Romania reaching its maximum extent after World War I through the acquisition of Transylvania, Bukovina, and Bessarabia (1918-1920), only to face significant losses in 1940 to Hungary, the USSR, and Bulgaria, before settling into its modern borders by 1945 (Hitchins, 2007).

Within this context of rapid transformation, the educational system experienced considerable volatility. During this period of institutional flux, the periodical *Gazeta Matematică* (GM) emerged as a pivotal journal in Romanian mathematics education, simultaneously articulating foundational theoretical perspectives while fostering the development of mathematical pedagogy throughout the nation.

The publications of *Gazeta Matematică* impacted mathematics education despite social and political changes. The aim of this study is two-fold. The first aim of the study is to understand what was written regarding mathematics methodology and didactic issues. The second aim of the study is to provide insight on how Ion Ionescu, a prominent author, potentially influenced broader societal attitudes and perceptions toward mathematics. A central insight emerging from this study is that this journal served as a crucial channel through which international ideas were disseminated, ultimately enhancing the discourse on national methodologies for teaching mathematics.

This study follows methods used in Furinghetti (2003) and Karp & Schubring (2014). It analyzes GM within its historical context and examines the interplay between the journal and the broader socio-political landscape shaping mathematics education in Romania. The articles were retrieved from *Gazeta Matematică - Electronica Edition* created by the Romanian Society of Mathematical Sciences, in collaboration with SOFTWIN company and the SIGMA Publishing House. This software holds the entire collection of *Gazeta Matematică* series B from 1895 to 2010, encompassing over 55,000 mathematical problems and articles. In 2010, an initiative was launched to digitize the publication's available physical copies, and a dedicated team of over 20 mathematicians undertook the meticulous task of categorizing the Gazette's contents into distinct mathematical fields, thereby creating a comprehensive and easily navigable digital archive of this extensive collection.

The electronic database revealed 31 publications in the journal under the category 'mathematics methodology and didactic issues' between 1895 and 1945. This pedagogically oriented category, rather than the pure mathematics category, was particularly relevant to this research, as it was the only one focused on teaching methods and educational approaches. Out of the 31 items, only 13 articles were available through the electronic database. The researcher was unable to locate the physical copies of the articles not included in the digital repository. The following study is based on the 13 articles that were available to the researcher at the time of this writing. The articles, originally in Romanian, were carefully reviewed and translated by the researcher for this study.

Historical Background

The decades encompassing 1895-1945 experienced many social, political, and ideological shifts reverberating across Romanian society. Major events, from World War I and II to the rise of communist and fascist movements, fundamentally reshaped the nation's economic, cultural, and educational landscapes. However, the extent to which these turbulent shifts impacted the realm of mathematics education merits deeper investigation. Andonie (1965) highlights the importance and impact of *Gazeta Matematică* (GM) in his three-book volume of the history of mathematics in Romania. The journal *Gazeta Matematică*, published continuously throughout this period, offers a unique window into the priorities, pedagogical approaches, and human perspectives that shaped how mathematics was taught and perceived in Romania during one of its most volatile eras.

Politically, Romania began this period (1895-1945), as a constitutional monarchy under King Carol I. However, public dissatisfaction with the corrupt and autocratic rule of the monarchy eventually led to the outbreak of the Romanian Peasants' Revolt in 1907, one of the earliest modern revolutions in Europe. Although the revolt was violently suppressed, it revealed deep societal divides and the strength of the peasant movement. After Carol I's death in 1914, his nephew Ferdinand ascended to the throne. Ferdinand made the pivotal decision to enter World War I on the side of the Allied Powers in 1916, seeking to acquire territory from the Austro-Hungarian Empire. However, this resulted in much of Romania being occupied by 1917, before eventually being liberated with help from Russian forces (Drace-Francis, 2006; Hitchins, 2014).

In the aftermath of the war, the territories of Bessarabia, Bukovina, and Transylvania were united with the Kingdom of Romania through the 1918 Great Union. This doubled Romania's territory and population. However, integrating the new territories and their ethnic minorities proved challenging. The interwar period also saw the long-awaited transition from monarchy to democracy. Though this transition was marred by political instability, antisemitism, and the rise of fascist movements like the Iron Guard. In 1938, King Carol II initiated a personal dictatorship, governing by decree until being forced into exile in 1940 by the military and territorial losses to Hungary and the Soviet Union (Hitchins, 2014).

Socially, Romania experienced the stirrings of modernization during this period, although it lagged behind Western European nations. The peasantry remained the dominant social class, comprising over 80% of the population and living in extreme poverty. Land reforms in 1921 and 1923 aimed to uplift the peasantry by breaking up large estates but were only modestly successful. Industrialization gathered pace, with the growth of the oil industry, manufacturing, and infrastructure. Railroads connecting urban centers fueled the expansion of the urban middle and working classes. However, progress was hindered by political instability, underdevelopment, and most of the economic benefits accruing to foreign business interests rather than Romanian interest (Drace-Francis, 2006).

Mathematics School Education in Romanian 1895-1945

Within this context of rapid transformation, the educational system experienced considerable volatility. Different regions inherited diverse educational traditions. In Transylvania, the Austro-Hungarian administration established a robust educational framework that prioritized technical training and embraced multilingual instruction in Hungarian, German, and Romanian. The Old Kingdom (Wallachia and Moldavia) adopted a distinctly French-inspired approach, emphasizing classical studies, literature, and philosophy in line with Western European traditions. Meanwhile, Bessarabia's schools operated under the Russian imperial model, which restricted Romanian-language education and primarily used Russian as the language of instruction (Andonie, 1965; Bachman,1991). Mathematics education in elementary and secondary schools went through several reforms and changes, reflecting the broader educational and political developments in Romania during 1885-1945 (Andonie, 1965).

In the late 19th century, the education system in Romania was heavily influenced by the French model. Mathematics education at the elementary level focused primarily on basic arithmetic operations, weights and measures, and some practical geometry concepts. At the secondary level, which included lower secondary (gimnaziu) and upper secondary (liceu), the mathematics curriculum included algebra, geometry (plane and solid), trigonometry, and elements of calculus. The teaching methods were largely traditional, emphasizing theoretical knowledge, memorization, and exercises (Andonie, 1965).

In the early 20th century, there were efforts to modernize the mathematics curriculum and teaching methods. The 1904 education reform introduced some changes, such as incorporating more practical applications and problem-solving skills in mathematics education. After World War I and the unification of Romanian territories in 1918, further reforms were implemented to standardize the education system across the country. The 1925 reform emphasized the importance of practical mathematics and introduced new textbooks and teaching guidelines (Andonie, 1965; Szakács, 2018).

It is interesting to note that Romania entered the twentieth century with extremely low literacy rates of around 20-30%, which limited student accessibility to mathematics. The 1896 Haret reforms introduced compulsory elementary education and established teacher training programs. In 1904, secondary education opportunities for girls were improved, although poor families still prioritized their sons' schooling. The 1920s and 1930s saw great progress in improving literacy and school enrollment, with the literate population tripling by 1930. Investments were made in new school construction and teacher training. However, a shortage of qualified teachers, inadequate funding, and rural/urban divides persisted. By the end of this period, literacy rates still lagged at around 60-70% (Hitchins, 2014; Hoivik, 1974; Drace-Francis, 2006).

However, the quality of mathematics education varied significantly between urban and rural areas, as well as between different regions of the country. Rural schools often faced challenges such as a lack of qualified teachers, inadequate resources, and large class sizes.

Throughout 1895-1945, geometry and arithmetic remained the core components of mathematics education at the elementary level, while algebra, geometry, trigonometry, and elements of calculus were taught at the secondary level. The emphasis was on developing theoretical knowledge and problem-solving skills, although there were efforts to incorporate more practical applications and modernize teaching methods (Szakács, 2018).

It is important to note that the availability of educational resources, teacher training, and overall educational quality varied significantly depending on the specific region, urban or rural setting, and socioeconomic factors.

Gazeta Matematică

Gazeta Matematică was founded in 1895 by engineers who were passionate about elevating mathematics instruction, and it remains in publication today. During the construction of the Anghel Saligny Bridge, a few of the engineers who were schooled abroad were astonished at the lack of mathematical rigor among other Romanian engineers and were galvanized into creating *Gazeta Matematică*. During its initial 50-year span, the journal established itself as an influential voice shaping discourses around mathematics within Romania's educational sphere. The journal included problems, articles, solutions, biographies, and announcements on competitions (Andonie, 1965).

Analysis of Content in Articles

The following narrative synthesizes the core discussions within the 13 selected articles in *Gazeta Matematică*, offering a succinct yet comprehensive summary of their contents. Among the 13 articles, there are five editorial articles, three articles written by the committee, and five articles by Ion Ionescu, who was a prominent Romanian mathematics educator and editor of the journal.

In the early 20th century, there was a growing recognition among Romanian mathematics educators of the need to reform the teaching of geometry and arithmetic in secondary schools. Romania became a member of the International Congress of Mathematicians (ICM) in 1911, with Romanian mathematicians actively participating in the congresses, contributing to the development of mathematics, and representing Romania on the global stage. The ICM, which was established in 1893, aims to promote and coordinate the international cooperation of mathematicians. Romania's involvement in ICM reflects its growing interest and involvement in the international mathematical community during the early 20th century (Andonie, 1965.

Prominent figures like G. Ţiţeica, a prominent mathematics educator, advocated adopting the rigorous, axiomatic approach to geometry promoted by the Italian mathematician, Veronese. This involved grounding geometric instruction in precise definitions, postulates, and logical deductions, while still utilizing intuition and models (Ţiţeica,1903). Ţiţeica provided an in-depth analysis of Veronese's approach in articles published in 1903. He outlined how Veronese redefined foundational geometric concepts like equality and parallelism.

Another issue GM addressed was the lack of standardized mathematical terminology across Romania. An effort was made in 1903 by a commission to establish consistent vocabulary for concepts like equations, geometric objects, and operations.

Concurrently in England, committees were formed to rethink the conventional reliance on Euclid's Elements for geometry education (Ionescu, 1903; 1913). Ionescu (1903) reported on their recommended modifications in GM. The English reform approach displayed some alignment with Veronese's axiomatization by separating constructions from deductive theory. Despite these reform efforts aimed at injecting more logical rigor, assessments in the following decades revealed alarming deficiencies in students' arithmetic and trigonometry knowledge, both in secondary schools and for those attempting to enter technical universities. Ionescu (1903) repeatedly criticized the "indulgence" and "guilty leniency" of mathematics teachers who passed unqualified students lacking foundational knowledge (Ionescu, 1913). The searing critiques illustrated how idealized visions for revamped mathematics pedagogy were running up against harsh classroom realities.

In 1945, the educator Simionescu gave a speech titled *Mathematics and Secondary Education* at a conference outlining his perspectives of mathematics education in Romania. He began by detailing the various branches of mathematics taught at the time, including arithmetic, geometry, algebra, trigonometry, analytical geometry, mechanics, and astronomy. Simionescu stated that the main purpose of teaching mathematics was for intellectual training and spiritual development of students, following the vision of the renowned educator, Spiru Haret, the founding father of mathematics education in Romania. However, he lamented that the Romanian school system had not always fulfilled this purpose. He praised the achievements of the early 1900s led by Spiru Haret. While Romanian compulsory education fluctuated between 7 to 10 years in the early 20th century, Simionescu criticized the reduction of required school years in compulsory education because it led to a concerning degradation of the curriculum. Notably marked by the removal of advanced topics such as higher algebra and analytical geometry (Simionescu,1945). Simionescu cited examples of students being unfamiliar with basic geometric constructions as evidence of this decline. Simionescu identified several key factors hindering effective math instruction in Romania: large class sizes, student indiscipline, poorly trained teachers due to inadequate salaries, and an inexplicable indulgence from teachers who passed underperforming students (Simionescu, 1945). He chastised the lack of official recognition and rewards for excellent educators.

Simionescu credited private initiatives like the *Gazeta Matematică* journal and student mathematics clubs for fostering interest and higher achievement in the subject. He advocated expanding the curriculum to include more advanced math concepts like calculus and geometric transformations. Ultimately, Simionescu stressed the need for collaboration between university faculty and secondary teachers to continuously uplift mathematics education standards. He expressed optimism that prioritizing quality mathematics education would propel Romania's cultural renaissance and economic development as a modern nation.

In summary, influential authors like G. Ţiţeica advocated for adopting rigorous, axiomatized approaches to geometry inspired by figures like Veronese, but the realities of implementation presented significant obstacles. Recurring critiques from educators like Ion Ionescu and Simionescu highlighted alarming deficiencies in student knowledge, pointing to issues like unqualified students being promoted due to "indulgent" grading, large class sizes, and inadequate teacher training. A key tension emerged between the idealistic vision of rigorous, logic-based mathematics teaching and the practical realities of classroom instruction. To improve mathematics education standards, Ţiţeica (1903), Ionescu (1913), and Simionescu (1945) called for an ongoing collaboration between university professors and secondary school teachers.

Ion Ionescu

Of the 13 articles examined, five bore the signature of Ion Ionescu, showcasing his prolific contribution and profound influence within the field. Ion Ionescu (1870-1946) was a pioneering figure in the field of mathematics in Romania. Born in Bucharest, he displayed an early aptitude and passion for mathematics during his school years. After completing his studies in engineering, Ionescu embarked on a lifelong career dedicated to teaching mathematics and nurturing young minds (Andonie, 1965).

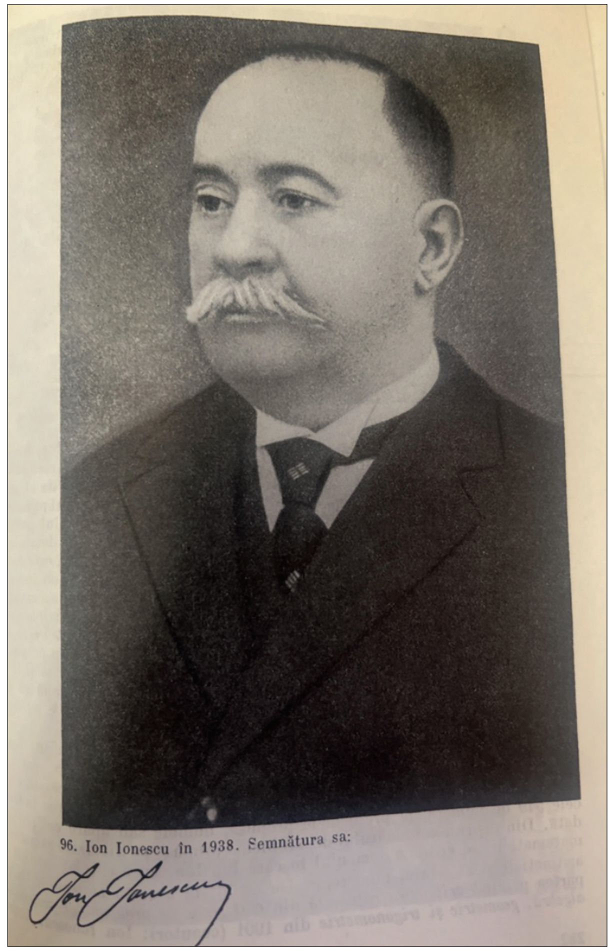
In 1895, at the age of 25, Ionescu founded the *Gazeta Matematică* journal, which would become his legacy. This monthly publication was designed to cater to students and mathematicians alike, offering a unique platform for exploring mathematical problems and solutions. Ionescu served as the editor-in-chief of *Gazeta Matematică* for 51 years, until his passing in 1946 (Andonie, 1965).

Under Ionescu's stewardship, *Gazeta Matematică* flourished, attracting contributors from across Romania and beyond. The journal's pages were filled with intricate problems, solutions, and thought-provoking articles that challenged readers to think critically and to develop their problem solving abilities. Ionescu himself was a prolific contributor, authoring a great number of articles, problems and solutions. According to Andonie (1965), Ionescu contributed over 210 publications. Beyond his editorial duties, Ionescu was a gifted teacher. He taught mathematics at the Polytechnical school in Bucharest, inspiring generations of students with his passion for the subject. As this study highlights, Ionescu had an unwavering commitment to academic rigor. He believed that Romania economics would grow only if the mathematical curriculum and teaching in elementary and secondary schools remained rigorous. He claimed that teachers who expected bribes would be detrimental to Romanian society (Andonie,1965).

Ionescu's impact extended far beyond the confines of the classroom or the pages of *Gazeta Matematică*. He

Figure 1

Photo of Ion Ionescu in 1938 (Andonie, 1965)



played a pivotal role in promoting the teaching of mathematics in Romania, fostering both practical understanding and intellectual curiosity in mathematics. Moreover, Ionescu's contributions transcended national borders. *Gazeta Matematică* gained international recognition, with contributors and readers from around the world participating in the intellectual discourse it fostered. Ionescu also participated in committees in London (Andonie,1965).

In recognition of his achievements, Ionescu received numerous accolades and honors. He was elected a member of the Romanian Academy, and his name became synonymous with excellence in mathematics education (Andonie, 1965). Even after his passing, his legacy lived on through the continued publication of *Gazeta Matematică* and the enduring appreciation for his contributions.

Conclusion

These thirteen articles published in *Gazeta Matematică* were devoted to critiquing and describing the mathematical education landscape of that time. The articles give insights into the evolving priorities, pedagogical approaches, and underlying attitudes toward mathematics instruction. GM served as a platform for debating educational reforms, standardizing mathematical terminology, and addressing the challenges of implementing new teaching methods.

It must be noted that during the period of political instability from 1915 to 1944, *Gazeta Matematică* (GM) saw a decline in publishing articles containing opinions and suggestions on mathematics pedagogy. Instead, the journal maintained its focus on mathematical problem sets and solutions. This shift reflected the broader climate of academic repression, where many scholars faced imprisonment for expressing views contrary to the existing political power. Nevertheless, the very existence of GM and the consistency of monthly publications was seen as a championed endeavor to the public. Ionescu's tireless dedication as editor over five decades, had the goal of shaping discourses and inspiring students through its pages. Despite the challenges of the time, GM remained a beacon for mathematical education and enthusiasm in Romania. This underscores the journal's significance in fostering mathematical discourse and its enduring impact on Romanian mathematics education.

Ultimately, *Gazeta Matematică* served as both a messenger documenting the complex realities of mathematics education in a nation grappling with modernization, and as a catalyst fueling aspirations for a revitalized curriculum that could propel Romania's cultural and socioeconomic development. The journal's historical significance extends beyond its immediate temporal context. Analysis of *Gazeta Matematică*'s role from 1895-1945 demonstrates its fundamental contribution to Romanian mathematical education during a period of substantial socio-political transformation. *Gazeta Matematică* established and maintained high academic standards while successfully bridging traditional and modern teaching methods. This approach led to the development of comprehensive curriculum models that would shape Romanian mathematics education for generations to come. The cultural and social impact of *Gazeta Matematică* extended far beyond mathematics education. The journal played a significant role in Romania's national identity formation, effectively responding to the increased demand for technical education during a period of rapid social change. It successfully balanced Romanian intellectual traditions with Western European educational models, contributing to a broader national cultural revival. During a time of urbanization and industrialization, the journal adapted its content and approach to support the growing needs of middle-class education. The journal's success has also brought international recognition to Romanian mathematical achievements, enabled the export of educational methodologies, and facilitated valuable cross-cultural educational exchange.

In the modern era, *Gazeta Matematică* has successfully transformed to meet contemporary challenges. The journal's digitization represents a significant evolution, enabling the preservation of its rich historical content while enhancing accessibility. This digital transformation has facilitated integration with online learning platforms and provided global access to Romania's mathematical heritage, extending its influence beyond national borders.

References

Andonie, G. S. (1965). *Istoria matematicii in Romania* (Vols. 1-3). Editura Stiintifica.

Bachman, R. D., Library of Congress Federal Research Division, & Keefe, E. K. (1991). *Romania: A country study.* U.S. Government Printing Office.

Drace-Francis, A. (2006). *The making of modern Romanian culture: Literacy and the development of national identity.* I.B. Tauris.

Editorial Board. (1903). Mathematical vocabulary. *Gazeta Matematică, (2)*.

Editorial Board. (1937a). The analytical syllabus decided for the "Gazetei Matematice" contest

of 1938. *Gazeta Matematică, (3)*.

Editorial Board. (1937b). Report for the Vasile Conta award for the year 1937. *Gazeta Matematică, (3).*

Editorial Board. (1940). The analytical program decided for the "Gazetei Matematice" contest from 1941. *Gazeta Matematică, (4).*

Furinghetti, F. (2003). Mathematical instruction in an international perspective: The contribution of the journal L'enseignement Mathématique. In D. Coray & F. Furinghetti (Eds.), *One hundred years of L'Enseignement mathématique: Moments of Mathematics Education in the Twentieth Century* (pp. 19–46). L'Enseignement Mathématique.

Gazeta Matematică. (1895-1945). Volumes 1-50, Issues 1-12. Romanian Mathematical Society.

Hitchins, K. (2007). *Rumania, 1866-1947.* Oxford University Press.

Hitchins, K. (2014). *A concise history of Romania.* Cambridge University Press.

Hoivik, T. (1974). The development of Romania: A cohort study. *Journal of Peace Research*, 11(4), 281-296.

Ionescu, I. (1903a). The reform of geometry education in England. *Gazeta Matematică, (6).*

Ionescu, I. (1903b). The reform of geometry education in England (continued). *Gazeta Matematică, (7).*

Ionescu, I. (1913). Teaching trigonometry in real high schools. *Gazeta Matematică, (4).*

Ionescu, I. (1921). Parabola. *Revista Matematică Din Timișoara,* 1(1), 7–8.

Ionescu, I. (1932a). Arithmetic education in our high schools. *Gazeta Matematică, (10).*

Ionescu, I. (1932b). Arithmetic education in our high schools (part II). *Gazeta Matematică, (12)*.

Simionescu, G. D. (1945). Mathematics and secondary education. *Gazeta Matematică, (6).*

Societatea de Științe Matematice din România, SOFTWIN, & Editura SIGMA. (2010). *Gazeta Matematică - Electronica Edition.* Societatea de Științe Matematice din România.

Szakács, S. (2018). The Romanian education system: Historical perspectives. *Europe in the Classroom*, 39–86.

Ţiţeica, G. (1903a). Geometry in secondary education. *Gazeta Matematică*, (1).

Ţiţeica, G. (1903b). Geometry in secondary education. *Gazeta Matematică, (3).*

NOTES FROM THE FIELD

The Fall 2024 issue features three notes from the field that each resulted from a NYC STEM study tour that took place in the spring of 2024, as explained in the introductory piece by Dr. Rochy Flint. Edwin Geng discusses instructional practices that support the development of a growth mindset, as observed during a visit to a mathematics class at the Borough of Manhattan Community College. Baldwin Mei describes opportunities for mathematical learning at the American Museum of Natural History and at the Museum of Mathematics, in New York City. Lastly, Eugene Ho and Cleha Kodama describe the innovative opportunities available to students at the Brooklyn STEAM Center.

NOTES FROM THE FIELD: NYC STEM STUDY TOUR

Dr. Rochy Flint,
Teachers College, Columbia University

Teachers College has a rich history of conducting study tours across the globe, with some participants sharing their findings here at *JMETC*. In 2019, we, in the Program of Mathematics at Teachers College, began looking closer to home. Situated in the heart of NYC, a city with a rich tapestry of diverse cultures and educational institutions, we decided to explore the state of STEM education in NYC. In the spring of 2024, we conducted a second NYC Study Tour, eager to see what is happening in STEM education in a post-COVID-19 world.

In 2019, we organized the NYC STEM study tour with a focus on specialized high schools. While these schools were accomplishing remarkable things, we observed inequities in the types of opportunities available to certain demographic groups, institutionalizing a lack of access for Black and Latinx communities. In 2024, we expanded beyond specialized high schools. Where in 2019 we were distraught by inequities, this year's tour not only reframed the narrative but also revealed something profoundly inspiring– a celebration of excellence and culture. We encountered the lighthouses of NYC, where young scholars of all backgrounds are leading the way in technological advancements, with each project driven by an urgency to serve the community.

Rather than trying to push students into the specialized framework—though we did learn about the relatively new Discovery Programs, which are addressing needs within the specialized high school sector– our recent tour focused on institutions that are built with a culturally relevant pedagogy framework, providing comprehensive educational experiences that showcase innovation and success while catering to their surrounding communities.

We explored questions such as: How does the pedagogy cater to the school's demographic? What programs are offered to help ameliorate the vast segregation issues in NYC public schools? Are classes interdisciplinary? Do they implement student-centered models that foster collaboration over traditional lecture-based instruction? How do students feel about mathematics inside and outside the classroom? Although these questions may not be directly addressed in the following articles, they were at the forefront of our observations during the tour.

Throughout the tour, we met with leading STEM educators, observed classes, and engaged with teachers, administrators, and students. We also visited renowned museums with a focus on STEM initiatives. A true highlight was the opportunity to connect with TC alumni who are at the forefront of STEM education, doing incredible things. The institutions we visited were diverse: an all-girls school, a magnet institution, a bilingual school supporting a migrant community, inner-city schools, a community college, and two renowned museums.

There is a Hebrew adage, *Chinuch L'naar Al Pi Darko*, which translates to "educate a child according to their way." This principle emphasizes that the child should be central to the framework of the education system. We must create a system that nurtures the needs of the child rather than forcing the child to acclimate to the needs of the pre-constructed educational system.

One instance on the tour that electrified our participants was sitting in TC alum Andrés Rodrigues-Aponte's classroom. We observed Aponte guide young scholars as they defined triangles in their own terms. He passionately led his students down a path of deep learning and discovery, ending the lesson with the following affirmations in both English and Spanish: "I am great. I will do great things in life. I will respect and help my classmates, because we are a great family; Yo soy grande. Yo lograré grandes cosas en la vida. Yo respetaré y ayudaré a mis compañeros y compañeras de clase, porque somos una gran familia." The students brought their whole selves into the classroom, where their culture and mindset were valued.

We are grateful to all of the institutions for generously, enthusiastically, and warmly opening their doors, classrooms, and labs, giving us a unique opportunity to observe them in action. We enjoyed eye-opening conversations with students, teachers, supervisors, and administrators from all walks of life. The levels of excellence we observed left us optimistic—there were plenty of "wow" moments! STEM education is thriving!

This section of the journal offers a glimpse into some of the observations TC participants made and analyzed during the tour, showcasing the diverse work happening in real time in NYC. While some institutions focus on siloed subjects in a traditional approach, others adopt an interdisciplinary method. For us, as active observers, it became clear that excellence does not come in one form; however, a common thread was that an immersive interdisciplinary approach fosters the deepest engagement.

JOURNAL OF MATHEMATICS EDUCATION AT TEACHERS COLLEGE | FALL 2024 | VOLUME 15, ISSUE2

NYC STEM STUDY TOUR

Fostering Growth Mindset and Grit within Students in the Mathematics Classroom

Edwin Geng
Teachers College, Columbia University

KEYWORDS *Growth mindset, fixed mindset, grit, perseverance, effort, mathematics education, problem solving, learning, achievement*

The capacity for learning and achievement in mathematics is neither static nor predetermined. Intelligence is adaptive and can be developed through structured support and pedagogical methods within the classroom. Alfred Binet, the creator of the IQ test, posited that individuals could enhance their intellectual abilities, including focus, memory, and judgment, through deliberate practice, training, and the correct methodology (Binet, 1909). Despite this, many students seem to hold a fixed belief that their intelligence is determined by genetics or innate ability, leading to self-defeating behavior in the face of mathematical challenges (Merseth, 1993). As educators, it is crucial to address and counteract this mindset to foster a more resilient and growth-oriented approach in students.

As children progress through different stages of cognitive development, they begin to compare their abilities with those of their peers and to evaluate their own performance. External feedback from parents, teachers, and peers often reinforces the notion that success is praiseworthy, while failure is undesirable. This dynamic can breed fear of challenges and lead students to believe they are inherently less capable than others. Consequently, students may avoid meaningful opportunities for learning, internalizing failure as a reflection of their personal worth rather than viewing it as a valuable learning opportunity (Dweck, 2006). In contrast, students who adopt a growth mindset view intelligence as something that can be developed and instead turn challenges into learning opportunities. They cultivate their abilities through mindful effort and commitment as well as support from others. This persistence yields mastery, and studies consistently provide evidence that students who have a growth mindset tend to perform better academically (Blackwell, Trzesniewski, & Dweck, 2007) and socially (Yeager et al., 2014) than those who have a fixed mindset.

In addition to fostering a growth mindset, the concept of grit, defined as combination of passion and perseverance towards a goal by Angela Duckworth in 2016, is elemental to academic success. Angela Duckworth's research highlights that students who exhibit grit are more likely to succeed academically because they demonstrate resilience and adapt their strategies when faced with challenges (Duckworth, 2016). These students do not repeat ineffective approaches; instead, they seek new methods and continuously assess how to achieve their objectives. Through grit, students persist in the face of difficulty, exhausting all available resources until they find a solution.

Anyone can develop a growth mindset and practice grit with the proper support and methodology (Jaffe, 2020). Elisabeth Jaffe's article "Mindset in the Classroom: Changing the Way Students See Themselves in Mathematics and Beyond" details various methods to promote growth mindsets and grit in students in the mathematics classroom. During the 2024 Spring STEM Study Tour, I observed Professor Jaffe in her Intermediate Algebra and Trigonometry course at the Borough of Manhattan Community College (BMCC). Her teaching methods illustrate effective ways to foster these traits in students, and several key strategies are outlined below.

One approach Professor Jaffe employs at the start of each semester is to read and assign passages on grit, prompting students to reflect on personal experiences where they demonstrated perseverance. This can include everyday challenges such as commuting to school or learning a new language (Jaffe, 2020). Throughout the semester, students are regularly reminded that they possess both grit and the ability to solve math problems with patience and practice. By encouraging students to recognize their own resilience, Professor Jaffe helps them realize that their intelligence is not fixed and that they have the capacity to develop through persistence. She believes that new self-awareness nurtures a growth mindset, empowering students to face challenges with confidence rather than fear.

Another significant aspect of Professor Jaffe's teaching is her emphasis on presuming confidence in and holding high expectations of all students. Regardless of a student's prior academic performance, she provides challenging material that requires critical thinking and application of mathematical concepts. This creates a mandate for grit; students must adapt to the degree of difficulty. Simultaneously, her use of scaffolding techniques, such as posing a difficult problem and guiding students through the process of breaking it down into smaller, simpler steps, help maintain high academic standards while make achieving and applying conceptual understanding for students much more manageable.

An illustrative example of this approach occurred when Professor Jaffe posed the question: "How many times do you need to fold an 8.5" by 11" sheet of paper in half for it to be taller than the Empire State Building?" Initially, students made intuitive guesses, ranging from several hundred to several thousand folds. Professor Jaffe then provided key information, including the height of the Empire State Building (1,250 feet) and details about a ream of paper such as the height per ream (2 inches) and how many sheets of paper per ream (500 sheets). Students were tasked with determining the height of a single sheet of paper, then using logarithmic equations to solve the problem. This challenge pushed students to draw upon their prior knowledge, collaborate, and persevere until they arrived at the surprising answer of 22 folds. This exercise not only illustrated the power of exponential growth but also reinforced the importance of grit and persistence in problem-solving.

Another example of Professor Jaffe's approach to fostering perseverance occurred when a student left an exam question blank and attempted to submit it early. Instead of accepting the incomplete exam, Professor Jaffe returned it to the student, challenging them to reconsider the problem using the knowledge they had acquired throughout the semester. Although initially discouraged, the student eventually solved the problem, demonstrating to them that they could push through initial struggles by persevering. While this doesn't mean that all students are able to solve difficult problems through perseverance alone, the idea is that by holding students to high standards and encouraging them to push through difficulties, Professor Jaffe cultivates an environment where students learn to embrace challenges and see failure as an opportunity for growth, which in turn may increase the chances of success.

If grit and growth mindset can be cultivated in adult students ages 18 and older at the community college level, then it might be possible to develop them in adolescent students during secondary education. Math teachers can implement the various approaches demonstrated by Professor Jaffe in their classroom. For instance, they can have students keep journal entries of when they have demonstrated grit throughout the school year. Teachers can further nurture this grit by offering high-level, challenging math content and problems to all students in conjunction with proper scaffolding, guidance, and encouragement. By having students apply grit in persevering through difficult tasks, a message can be sent to students that achievement and learning depends more on grit and effort than on genetics or innate talent. This sends a message to students that intelligence is not fixed – that their own intelligence is improvable through grit and effort. This in turn promotes growth mindset in students and prepares them to persist through challenges and problems later in life without shying away from obstacles at the first sign of difficulty.

Teachers must help students come to the realization that effort and grit play more of a role in long-term success than genetic predisposition and innate talent. However, grit and growth mindset by themselves cannot guarantee success. They are not substitutes nor solutions to problems and inequities in education such as overcrowded classrooms, inadequate curriculum, and unqualified teachers. Neither do they replace nor supersede all other support systems that schools must provide for their students (Jaffe, 2020). Learning and achieving in the mathematics classroom are shared responsibilities between the teacher and student as well as the overall educational community. Grit and growth mindset must be taught with proper support systems in place. At the same time, issues in equity and other problems in the educational system still need to be addressed. Just as students should be held responsible for their own learning, we as educators hold an equal responsibility to ensure high quality education for all students. After all, learning and achieving in the mathematics classroom should be a universal right of all students and not a privilege reserved just for the few.

References

Binet, A. (1909). Modern ideas about children. (S. Heisler, Trans. 1975). Suzanne Heisler, Publisher.

Blackwell, L. S., Trzesniewski, K. H., & Dweck, C. S. (2007). Implicit theories of intelligence predict achievement across an adolescent transition: A longitudinal study and an intervention. Child Development, 78, 246-263.

Duckworth, A. (2016). Grit: The power of passion and perseverance. Scribner/Simon & Schuster.

Dweck, C. S. (2006). Mindset: The new psychology of success. Random House.

Jaffe, E. (2020). Mindset in the classroom: Changing the way students see themselves in mathematics and beyond. The Clearing House: A Journal of Educational Strategies, Issues and Ideas, 93(5), 255–263.

Merseth, K.K. (1993). How old Is the shepherd? An essay about mathematics Education. Phi Delta Kappan, 74, 548.

Yeager, D. S., Johnson, R., Spitzer, B. J., Trzesniewski, K. H., Powers, J., & Dweck, C. S. (2014). The far-reaching effects of believing people can change: Implicit theories of personality shape stress, health, and achievement during adolescence. Journal of Personality & Social Psychology, 106, 867-884.

JOURNAL OF MATHEMATICS EDUCATION AT TEACHERS COLLEGE | FALL 2024 | VOLUME 15, ISSUE 2

NYC STEM STUDY TOUR

Creating Opportunities Through Career and Technical Education: A Case Study of Brooklyn STEAM Center

Eugene Ho
Teachers College, Columbia University

Cleha Kodama
Teachers College, Columbia University

KEYWORDS *STEM Education, Career and Technical Education, Community School, Career Pathways*

The Brooklyn STEAM Center, located in Building 77 of Brooklyn Navy Yard, exemplifies how strategic partnerships between public education institutions and local industries can enrich and empower young adults. While it may be somewhat difficult to place within the traditional educational landscape, as its model simultaneously falls under traditional high school, extracurricular enrichment, career and technical education (CTE), and college preparation, the Brooklyn STEAM Center is without a doubt an innovative institution that serves its community's needs. We briefly describe here, based on our visit to Brooklyn STEAM Center during the Spring 2024 study tour, some of the key components that make Brooklyn STEAM Center work, and how their students benefit from its unique programming.

To apply to Brooklyn STEAM Center, a student must be a rising eleventh-grade student at one of the eight nearby partner public schools. Furthermore, they must be in good academic standing and have completed accelerated coursework in their ninth- and tenth-grade years. In their application to Brooklyn STEAM Center, students rank their preference for all six programs and receive admission offers based on the strength of their application and program availability. Once accepted into a program, during eleventh and twelfth grade, students attend their home institutions for half the day for traditional academic coursework and attend Brooklyn STEAM Center for the other half for CTE coursework. Many local public high schools were initially considered and evaluated under various criteria such as demographics and socioeconomic status. Ultimately, eight high schools with large, underserved populations that highlight the diverse, multicultural Brooklyn community were chosen. As Brooklyn STEAM Center continues to grow (during our visit, they were renovating more floors in its building to accommodate continued expansion), it continues to seek new partnerships with nearby institutions.

The Center offers six CTE pathways in culinary arts, construction, cybersecurity, engineering, media, and full-stack (website) development. Each demonstrates the interdisciplinary nature of industry professions and gives students early exposure to their potential careers. What makes the Brooklyn STEAM Center unique is that it is not just a CTE institution that graduates students directly into the workforce. At the Brooklyn STEAM Center, they believe that academic rigor and vocational skills are not separate education tracks that students must choose from; in fact, close to 99% of the center's graduates move on to attend post-secondary education institutions, in both two-year and four-year programs, or even beyond. As repeatedly emphasized during our visit, there simply does not need to be a distinction between academic and vocational training – both can be obtained concurrently at Brooklyn STEAM Center. The programming at the Center provides students with certifications and vocational training. However, the true focus is on providing students with engaging, project-based learning that develops problem-solving skills in a real-life context. That is, the program helps students obtain as many useful skills and experiences as possible to prepare them for the future.

During our tour, we had the opportunity to visit and talk to students in the cybersecurity and full-stack development course. The cybersecurity program provides students with the skills in both (ethical) hacking and defending against hackers. As cybercrime becomes ever prevalent in the increasingly online world we live in, with a record 12.5 billion dollars of losses to Internet crime in 2023 (Federal Bureau of Investigation, 2023 p. 7), it was no wonder that the cybersecurity programs were in high demand and had the most competitive admissions out of the six offered programs. Students begin this program by building their own computers and assembling all the hardware components such as the CPU, RAM, and hard drive into their appropriate positions. This provides the opportunity for students to gain hands-on experience in handling computer hardware and perhaps increases the amount of personal investment and attachment to the program. As students continue to work through the Cybersecurity program, students develop both theoretical and practical skills in hacking, understanding where system vulnerabilities can occur and how to defend them against exploits. On the industry level, there is considerable demand for expertise in what is typically referred to as "white-hat" hacking (as opposed to "black-hat"), a type of ethical hacking where (under the owner's consent) the hacker attempts to pre-emptively hack systems and find their vulnerabilities, so that defenses may be developed against actual malicious attacks down the line. In the Cybersecurity program at Brooklyn STEAM Center, tasks and assignments are initially scaffolded. However, as students progress, the difficulty increases, and solutions become increasingly sophisticated, just as they would in the real world. In their final assessment, students are tasked with a long-term project that requires them to hack a series of successively difficult problems, while fully documenting their thinking processes and techniques used. Students work in pairs, but are otherwise completely on their own, with only their experience and knowledge to guide them.

In the full-stack development class, students were working on websites they devised, designed, and created from scratch. During class time, the instructor walked around and provided assistance as students worked, but the main learning activities we observed were very much student-driven. Students shared ideas and worked through problems together and on boards on walls, which provided us with artifacts of their brainstorming and designing process. Even when learning practical and technical skills, their education felt personalized. Some students' websites sought solutions to inconveniences and problems they noticed in their everyday lives. Other students created programs based on their interests and hobbies. An example of a final project a student team shared with us was a food-reviewing mobile app that assigned scores to reviewers based on whether their views agreed with overall consensus. That is, if your reviews strongly reflect what most people thought, your "rank" would increase, and your ratings would carry more weight on the app. They even showed us a prototype installed on their phones with functional user interfaces. It was clear that students felt a great sense of ownership and investment in their work and ideas. This was a common thread throughout our conversations with students.

While speaking with the students at Brooklyn STEAM Center, what also stood out to us was how concrete the students' goals were. As students happily shared their plans with us, they were particularly candid about what requirements they needed, such as coursework in majors they hoped to study, potential difficulties they might face, and what anxieties they have for the future. In general, it seemed that the students appreciated the job-preparedness, the training, and certifications brought. Nevertheless, many wanted to continue their studies to learn more about computer science or data science, and to eventually obtain a bachelor's degree. To this end, Brooklyn STEAM Center also works actively with the City University of New York (CUNY) to align its curriculum and ensure that students who wish to attend college after high school have a smooth transition. CUNY representatives also participate in events that allow students to learn more about the college and build connections between the institutions.

When discussing future colleges they would like to attend, many mentioned schools in the CUNY or State University of New York (SUNY) systems, citing the affordable cost of attendance, location, and, most importantly, the availability of programs that they wanted to pursue. According to a report by the Brookings Institute, "six of CUNY's senior colleges and six community colleges [rank] among the top 10 four-year and two-year colleges nationwide with the greatest success in lifting low-income students into the middle class. Three other CUNY senior colleges were ranked in the top 25" (City University of New York, 2020). At Brooklyn STEAM Center, it was clear that students also understood the great value of these nearby higher learning institutions and the socioeconomic potential they represented. Perhaps these types of understanding were what attracted them to Brooklyn STEAM Center, and students seemed to be appreciative of the opportunities made available to them through this program.

As an innovative CTE provider, Brooklyn STEAM Center also aims to provide robust industry opportunities. Work-based learning is a key component of the Center's programming, and students are heavily incentivized to find work placements relevant to their program (although this is not required). The strategic location of Brooklyn STEAM Center and its partnerships with local industries allow the center to offer its students many opportunities for paid internships and work experience. Currently, Brooklyn STEAM Center partners with over 40 industries and has provided students with over 35,000 hours of work-based learning experience, with more than $500,000 paid out in student earnings. Furthermore, the center hosts events to help students showcase their projects and network with industry professionals. Students are also encouraged to personally reach out to potential employers to practice their networking and negotiating skills. On multiple occasions, students have been able to secure paid positions without any assistance. The Brooklyn STEAM Center then provides the necessary legal documentation, contract drafting, as well as general oversight to ensure that the student employment process occurs without issue. Through these various initiatives, students can increase their employable skills beyond what is learned in the classroom, improving their social and economic potential before even graduating high school.

The Brooklyn STEAM Center helps underrepresented and underserved communities access opportunities that help them gain valuable experience and expertise. Through practical and rewarding work-based learning opportunities, they challenge stigmas associated with CTE programs and showcase the impacts of high-quality experience-driven learning. Brooklyn STEAM Center also supports young adults in college preparation. It reimagines education as preparation for the future, where academic learning and career preparation do not have to be mutually exclusive. While the Brooklyn STEAM Center is relatively new to the STEM education landscape, it is an inspirational model that provides access to invaluable experiences and aims to support the diverse needs of the New York City community.

References

The City University of New York. (2020, June 17). New Study Confirms CUNY's Power as National Engine of Economic Mobility. The City University of New York. https://www1.cuny.edu/mu/forum/2020/06/17/new-study-confirms-cunys-power-as-national-engine-of-economic-mobility/

Federal Bureau of Investigation. (n.d.). 2023 Internet Crime Report. Internet Crime Complaint Center. https://www.ic3.gov/AnnualReport/Reports/2023_IC3Report.pdf

JOURNAL OF MATHEMATICS EDUCATION AT TEACHERS COLLEGE | FALL 2024 | VOLUME 15, ISSUE 2

NYC STEM STUDY TOUR

Learning and Inspiring Outside the Classroom: Museums of STEM in NYC

Baldwin Mei
Teachers College, Columbia University

KEYWORDS *museum, informal learning, STEM*

When we think about learning, perhaps the first word that comes to mind is "school": a place where children congregate, spending hours in classrooms learning about a set of prescribed topics from an adult expert, the teacher. A ubiquitous experience, but not necessarily one filled with amazement and awe, leading many to have school memories that are less than inspiring. However, places of knowledge and learning exist beyond the neatly polygonal confines of the classroom and can serve as important, additional places of student learning (Malone, 2008). In New York City, a great resource that students, parents, and educators can leverage is the wide array of world-class museums dotted around the city. Two of these institutions are the American Museum of Natural History and the Museum of Mathematics (MoMath). The STEM Study Tour conducted by Teachers College, Columbia University, a tour designed to bring current and future mathematics educators to places of mathematics teaching and learning around NYC, visited these museums as they both of had rich, innovative, and inspirational learning opportunities for mathematics.

A day (or *Night at*) at the American Museum of Natural History is a staple NYC attraction. The museum offers a space to explore a dizzying array of fossils, artifacts, specimens, and samples from Earth's past and present, meaning there's a little something there for everyone. There are taxidermied animals and preserved insects for those looking for a biological adventure, plant samples and environmental displays for museumgoers curious about the ecology of the planet, dinosaur and extinct animal fossils for explorers looking to delve into mysteries of the planet's ancient fauna, and rock and mineral samples for the aspiring geologist, with many further undiscussed exhibits still. In terms of where these exhibits lie within the STEM space, the museum heavily leans into the S—Science—part of things, many of which have strong connections to core K-12 sciences such as biology, earth science, and ecology. However, the M—Mathematics—in STEM isn't excluded from prominence in the museum, something that can be seen in the Hall of Planet Earth and the Rose Center for Earth and Space.

In the Hall of Planet Earth, mathematics is a central cog to the interactive *Causes of Climate and Climate Change*. Guests can freely explore a multimodal wall of climate change data presented through maps, text descriptions, and a multitude of graph types. Mathematical concepts such as interpreting trends, rates of change, positive and negative graph values, and probabilities are woven in throughout, tacit requirements to understanding the messages being presented in the exhibit's data. To facilitate this communication of information, the museum also implemented accessibility features such as audio cues that correspond to graph values, colors to indicate positive and negative values, and tactile buttons for easier exhibit navigation. As shown in Figure 1, the whole package is tied together in a visually alluring display that encourages curious investigators to congregate around key parts of the exhibit.

Encapsulating how this all comes together for teaching was a jigsaw activity (i.e., an activity where students are divided into groups to investigate different phenomenon and then recombine into smaller groups to share their results) used by a museum guide during the STEM Study Tour as an example of a typical field trip learning experience at the museum. In the activity, a "class" of students was divided into three groups, each group investigating a specific section of the *Causes of Climate and Climate Change* exhibit. This exploration phase involved many skills previously mentioned. Students engaged with the visual representations of climate data and noted that temperatures generally increased over time, with some latitudes increasing faster than others. More than the skills used, the process of exploration generated interest and excitement: Unexpected quirks of the exhibit caused bystanders to quickly surround each curiosity, one person's epiphanous moments started a chain reaction of realizations, and this all contributed to lively, engaged conversations when jigsaw pieces (the students) reconvened into small groups to share their findings of each part they were assigned of the Climate and Climate Change exhibit.

Another mathematically rich exhibit lying directly adjacent to the Hall of Planet Earth is the *Cosmic Pathway* in the Rose Center for Earth and Space. As its name suggests, this is a path that museum visitors walk through that physically represents a cosmic phenomenon, specifically the timeline from the birth of the universe to today. Mathematical topics such as scientific notation, ratios, and proportions are critical to understanding this exhibit. While the tie-in for scientific notation is straightforward due to the large numbers associated with the universe, ratios and proportions are, quite literally, embedded in the path. The path, 360 feet in total, is exactly proportional to the 13 billion years of the universe's history as can be seen in Figure 2, with significant events placed in the appropriate spots along the path. This includes all of human history, which participants of the STEM Study Tour (and likely any future learners who walk the *Cosmic Pathway*) were shocked to learn was the width of one strand of hair right at the end of the path, the exhibit using the subversion of expectations and established prior distances to create a sense of surprise and awe.

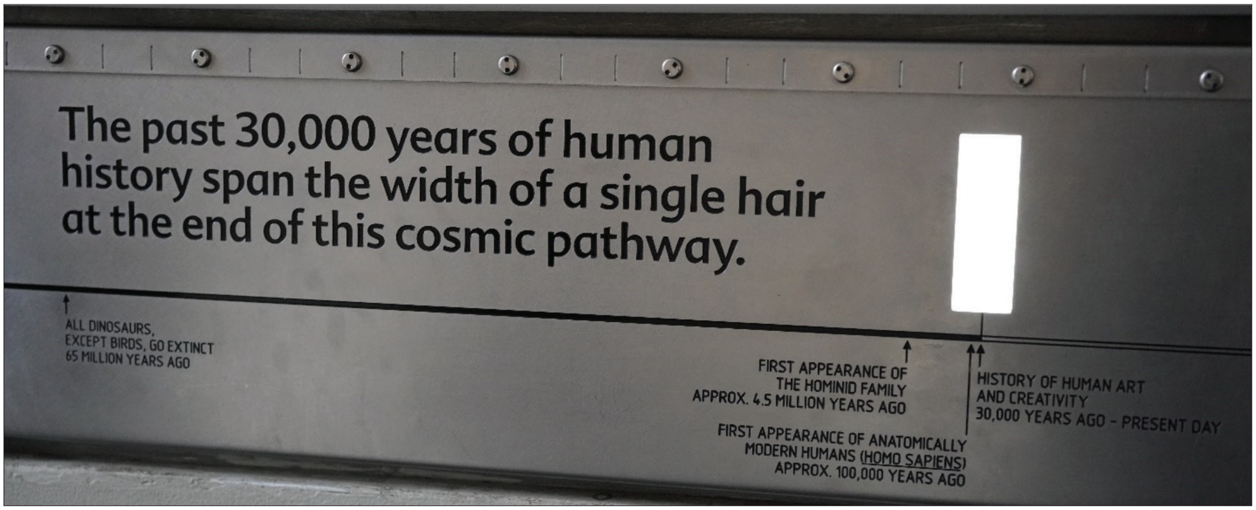
MoMath covers "only" one area of STEM: mathematics, but don't let that seemingly simple word deceive you. The exhibits show how mathematics goes beyond what is presented in a school textbook. Exhibits are inspired by a wide range of mathematical subtopics, some of which are familiar to students, some of which are far beyond the standard curriculum in typical K-12

Figure 1

Causes of Climate and Climate Change Exhibit in the Hall of Planet Earth

Figure 2

Cosmic Pathway Walkway Length to Time Scale



education. All of them are interactive, meaning students can come to grips with a subject that is predominantly thought of through pencil and paper abstractions. While every exhibit in MoMath is replete with rich mathematical connections that inspire mathematical exploration, I will highlight two that have close ties to curriculum taught in New York State's K-12 mathematics curriculum: *Done in a Million* and *PolyScope*. *Done in a Million* is an exhibit where museum goers use a wheel to turn a series of interconnected gears 1,000,000 times to break a drinking glass. The size of the machine and the way the gears are laid out entranced the entrance to the museum, causing many visitors to see how they could get the wheels to spin. As the wheel turned and turned though, machine operators quickly realized that while some of the wheels with lower powers of 10 turned swiftly, the wheels with larger powers of 10 required significantly more effort and to get moving. The physical, tactile method of representing the differences in size between powers of 10 is a non-traditional approach to the topic, a kinesthetic method to develop a person's number sense that importantly exposes students to different representations of the same topic (Kilpatrick et al., 2001). As it happens, realizing the size disparity between 10, 100, 1,000, and higher powers of 10 is a realization that one of the museum's staff noted is a common epiphany experienced by those who operated the *Done in a Million* machine.

PolyScope, on the other hand, is a geometric exhibit where small wooden objects are placed into a mirrored crevice to create three-dimensional shapes through reflections. Reflections are a topic often covered in K-12 geometry but almost exclusively with two-dimensional figures. This exhibit acts as an extension opportunity, posing the question to learners of what reflections look like with 3D shapes and what happens when multiple

Figure 3

Done in a Million Exhibit

reflections are applied to the same figure. Museum goers and members of the STEM Study Tour were drawn to this, trying to figure out how many and what figures could be created, with a handy list of 3D solids as scaffolding available to all who want it.

Throughout the discussion of these museums, the emphasis has been on both the learning opportunities and the spectacle and inspiration they present to visitors. The latter, in particular, is critical to the museums' success as, in education lingo, museum exhibits and experiences serve as powerful ways to motivate people who find it difficult to get excited in a traditional classroom setting. At the time of writing, for example, *Mathemalchemy*, an exhibit with various mathematical art pieces created by artists and mathematicians, is hosted at MoMath. This offers a rare glimpse into the potential for math and art to fuse, something which may serve to motivate those unconvinced by the logical skew mathematics is usually given in schools.

In conclusion, learning doesn't just happen in the classroom! The STEM Study Tour's museum stops, the American Museum of Natural History and the Museum of Mathematics, offered a small glimpse into some informal locales for STEM learning with both institutions providing opportunities for multimodal exposure to STEM, deeper exploration of K12 topics, and STEM spectacles that inspire and motivate learning.

References

Malone, K. (2008) *Every Experience Matters: An evidence based research report on the role of learning outside the classroom for children's whole development from birth to eighteen years.* Farming and Countryside Education for UK Department Children, School and Families.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics.* National Academy Press.

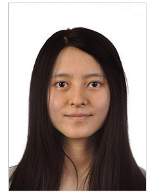
ABOUT THE AUTHORS

Articles

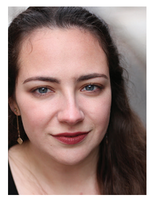
Alexander Karp is a professor of mathematics education at Teachers College, Columbia University. He received his PhD in mathematics education from Herzen Pedagogical University in St. Petersburg, Russia and also holds a degree from the same university in history and education. Currently, his scholarly interests span several areas, including the history of mathematics education, gifted education, mathematics teacher education, and mathematical problem solving. He served as the managing editor of the International Journal for the History of Mathematics Education and is a co-editor of the Springer series on the history of mathematics and its teaching. He is the author or editor of over one hundred twenty publications, including over forty books.

Daria Chudnovsky is currently a second-year Ph.D. student in Mathematics Education at Teachers College, Columbia University. She has been a private math enrichment tutor for several years now and currently holds an adjunct lecturer position in the math department at Baruch College. Her research interests include problem posing, student self-efficacy, and the history of mathematics education in Ukraine.

Omar Faruque is a doctoral student in the Mathematics Education program in the Department of Mathematics, Science and Technology at Teachers College, Columbia University. He is a high school mathematics teacher at the Department of Education, New York City. His research interests include the history of mathematics education and colonial mathematics education, teacher education, international mathematics education, and ethnomathematics.



Christine (Chang) Gao is a doctoral student at Teachers College, Columbia University, specializing in the history of mathematics education. Her research examines mathematics education during the Nazi era, focusing on the integration of military and ideological content into the curriculum. Through her studies, she aims to uncover how authoritarian regimes manipulate mathematics education and its impact on shaping societal values and political systems.



Samantha Moroney is currently a doctoral student, pursuing her EdD in Mathematics Education at Teachers College. Samantha has a passion for education, and has worked for the past four years as an adjunct lecturer for both Rutgers University and Baruch College. She will continue her adjunct work at both Baruch College and New York University in Spring 2025, while pursuing her research interests in mathematics anxiety and the intersection of music and math.

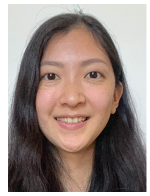
Elizabeth Wilson is a doctoral candidate in Mathematics Education at Teachers College, Columbia University. Her research explores the historical development of mathematics education in Romania, examining pedagogical traditions and educational practices that have shaped Romanian mathematical pedagogy.

Notes

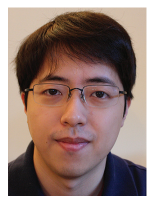
Rochy Flint is a mathematician with research interests in three-dimensional geometry and topology, the intersection of women and mathematics, and student-centered learning models. She has been a mathematics educator for 20+ years, and is a lecturer at Teachers College, Columbia University. She earned a B.S. in mathematics from Columbia University, and a Ph.D. in mathematics from The Graduate Center in the City University of New York. Rochy is passionate about mathematics outreach, multicultural MathSpaces, and enjoys large-scale collaborative projects that combine mathematics, mathematics education, and art.

Edwin Geng is a Master's in Mathematics Education student at Teachers College, Columbia University who has experienced mathematics education as a student and educator in both Taiwan and the United States. He aspires to utilize his educational experience from both countries to develop a good mathematics curriculum and establish a classroom norm that fosters growth and grit mindset in students. He enjoys constructing DIY projects such as origami in his spare time and lives with two cats at home.

Eugene Ho is a Master of Arts student in Mathematics Education at Teachers College, Columbia University. He is most interested in using cultural approaches to build passionate learning environments for math.



Cleha Kodama is a middle school math teacher at the Berkeley Carroll School in Brooklyn, New York. Since starting her career as a teacher, she has worked with students in grades 6-12 in the US, Japan, and Malaysia. She received an MA in Mathematics Education at Teachers College, Columbia University. Her interests lie in creating collaborative math classrooms that empower students and encourage them to see mathematics as a flexible and creative tool.



Baldwin Mei is a Mathematics Education doctoral candidate at Teachers College, Columbia University. His research interests include preservice and in-service teacher education, technology, and multicultural education. He currently works to support the development of preservice secondary mathematics teachers in the Initial Certification program at Teachers College.

ACKNOWLEDGEMENT OF REVIEWERS

The Editorial Board would like to acknowledge the following reviewers for their effort and support in reviewing articles for this issue of the *Journal of Mathematics Education at Teachers College.* Without the help of these professionals, it would be impossible to maintain the high standards expected of our peer-reviewed journal.

Leo Ayarig
Teachers College, Columbia University

Mine Cekin
Teachers College, Columbia University

Yuxi Chen
Teachers College, Columbia University

Art Walden A. Dejoras
University of Science and Technology of Southern Philippines

Omar Faruque
Teachers College, Columbia University

Jimmy Giff
Teachers College, Columbia University

Kihoon Lee
Teachers College, Columbia University

Yiran Li
Teachers College, Columbia University

Michelle Longhitano
Teachers College, Columbia University

Jonathan Maitre
Teachers College, Columbia University

Mara Markinson
Queens College, City University of New York

Anthony Miele
Teachers College, Columbia University

Nasriah Morrison
Teachers College, Columbia University

Sarah Nelson
Teachers College, Columbia University

Marina Nouel Perez
Teachers College, Columbia University

Maya Schwartz
Teachers College, Columbia University

Yuang Song
Teachers College, Columbia University

Micah Swartz
Texas State University

Sunita Verma
Teachers College, Columbia University

Chris Vicevich
Teachers College, Columbia University

Kaori Yamamoto
Teachers College, Columbia University

JOURNAL OF MATHEMATICS EDUCATION AT TEACHERS COLLEGE

CALL FOR PAPERS

This call for papers is an invitation to mathematics education professionals, especially Teachers College students, alumni, and associates, to submit articles describing research, experiments, projects, innovations, or practices in mathematics education. The journal features full reports (approximately 3500 to 4500 words) and short reports (approximately 500 to 1500 words). Full reports describe findings from specific research, experiments, projects, innovations, or practices that contribute to advancing scholarly knowledge in mathematics education. Short reports ("Notes from the Field") provide examples, commentary, and/or dialogue about practices out in the field of mathematics education or mathematics teacher education; examples from classroom experience are encouraged. Although many past issues of *JMETC* focused around a theme, authors are encouraged to submit articles related to any current topic in mathematics education, from which pertinent themes for future issues may be developed. Articles must not have been submitted to or accepted for publication elsewhere. All manuscripts must include an abstract (approximately 150 words in length) and keywords. Manuscripts should be composed in Microsoft Word and follow APA format. Guest editors will send submitted articles to the review panel and facilitate the blind peer-review process. Articles for consideration should be submitted online at jmetc.columbia.edu, and are reviewed on a rolling basis; however, to be considered for the Spring issue, articles should be received by **January 31, 2025**.

CALL FOR REVIEWERS

This call for reviewers is an invitation to mathematics educators with experience in reading or writing professional papers to join the review panel for future issues of *JMETC.* Reviewers are expected to complete assigned reviews within three weeks of receipt of the manuscript in order to expedite the publication process. Reviewers are responsible for editorial suggestions, fact and citations review, and identification of similar works that may be helpful to contributors whose submissions appear appropriate for publication. Neither authors' nor reviewers' names and affiliations will be shared with one another; however, reviewers' comments may be sent to contributors of manuscripts to guide revision of manuscripts (without identifying the reviewer). If you wish to be considered for review assignments, please register and indicate your willingness to serve as a reviewer on the journal's website: jmetc.columbia.edu.

CALL FOR EDITOR NOMINATIONS

Do you know someone who would be a good candidate to serve as a guest editor of a future issue of *JMETC?* Students in the Program in Mathematics Education at Teachers College are invited to nominate (self-nominations accepted) current doctoral students for this position. Being asked to serve as a guest editor is a testament to the high quality and standards of the student's work and research. In particular, anyone nominated as a guest editor should be a current doctoral student whose scholarship is of the highest quality, whose writing skills are appropriate for editorial oversight, and whose dedication and responsibility will ensure timely publication of the journal issues. All nominations should be submitted to Ms. Juliana Fullon at jmf2213@tc.columbia.edu .