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Evolving Priorities and Practices in Mathematics Education

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The *Journal of Mathematics Education at Teachers College (JMETC)* is a recreation of an earlier publication by the Program in Mathematics Education at Teachers College, Columbia University. As a peer-reviewed, semiannual journal, it is intended to provide dissemination opportunities for writers of practice-based or research contributions to the general field of mathematics education. Although many of the past issues of *JMETC* focused on a theme, the journal accepts articles related to any current topic in mathematics education, from which pertinent themes for future issues may be developed.

JMETC readers are educators from pre-kindergarten through twelfth-grade teachers, principals, superintendents, professors of education, and other leaders in education. Articles appearing in the *JMETC* include research reports, commentaries on practice, historical analyses, and responses to issues and recommendations of professional interest.

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PREFACE

The Spring 2025 issue of the *Journal of Mathematics Education at Teachers College* features a set of contributions that reflect the evolving priorities and practices in mathematics education as well as the lived experiences of those working within it. The Articles section consists of three research-based pieces, and the Notes from the Field section presents three pieces that offer insights grounded in practice.

In the first research-based article, Osborne & Hibbard use the 2019 Trends in International Mathematics and Science Study data to investigate how eighth-grade students' attitudes towards mathematics relate to achievement. By drawing on a large, nationally representative dataset, Osborne & Hibbard extend prior research on the relationship between students' attitudes and achievement in mathematics. Their findings highlight the role of social and emotional factors in mathematical learning and offer guidance for teachers seeking to foster more positive attitudes about mathematics.

Then, Panthi et al. present a comprehensive conceptual overview of social justice issues in mathematics education, organizing their analysis into a four-part framework: prospects, priorities, processes, and problems. The authors argue that meaningful reform must address curriculum, instruction, and assessment, and they call for greater teacher agency in prioritizing social justice issues in mathematics education despite persistent structural challenges.

Lastly, Sundrani investigates the decision-making processes of secondary mathematics teachers as they select online instructional materials from platforms such as Teachers Pay Teachers. In this study, Sundrani identifies key heuristics that shape teachers' choices and therefore impact classroom instruction.

Molly Stern
Kihoon Lee

Guest Editors

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The Relationship Between Student Attitude Towards Mathematics and Student Mathematics Achievement

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ABSTRACT Using the United States data from the 2019 Trends in International Mathematics and Science Study, which included a total of 9924 randomly selected students in grade eight from 325 randomly selected schools, the present study examined the relationship between the students' attitudes towards mathematics and the mathematics achievement of the students. Due to the hierarchical structure of the educational data, multilevel statistical modeling was used to analyze the data. While including sex and parent education level as student-level control variables and school location and school socioeconomic status as school-level control variables, the students' attitudes towards mathematics demonstrated significant positive effects on the students' mathematics achievement. Furthermore, this relationship was shown to vary significantly across schools; however, school disciplinary climate was not found to be a significant contributor to this variation.

KEYWORDS *Attitude towards mathematics, mathematics achievement, Trends in International Mathematics and Science Study (TIMSS), Hierarchical linear modeling (HLM)*

Introduction

Mathematical literacy is a key that unlocks many doors of opportunity for K-12 students in terms of both educational success and future careers (Wang et al., 2017). Unfortunately, mathematics is also an area in which many students struggle to reach a level of proficiency, and so the doors of opportunity remain locked. Among the various factors that contribute to students' mathematics achievement, research has demonstrated that students' attitudes towards the subject play a key role (Arnold et al., 2019; Yerdelen-Damar et al., 2021).

Review of Literature

Student attitude towards mathematics encompasses such things as the feelings and opinions that a student holds about the subject (Mata et al., 2012). Attitude towards mathematics varies significantly among students and impacts their motivation, engagement, and willingness to learn the subject (Kariadinata et al., 2019). Students with positive attitudes towards mathematics are more likely to persevere when faced with challenges and setbacks in mathematics by seeking out additional resources and support (Clinkenbeard, 2015).

Various factors can influence students' attitudes towards mathematics (Moussa & Saali, 2022). For example, students' previous experiences with, personal beliefs about, and self-efficacy in mathematics can significantly impact their attitudes towards the subject (Chirove et al., 2022). In addition, parents who demonstrate a positive attitude towards mathematics and express belief in their child's ability to succeed in the subject can foster a similar positive attitude in their child. Furthermore, even societal beliefs and stereotypes around mathematics can affect students' attitudes towards the subject (Chirove et al., 2022).

The association between students' attitudes towards mathematics and their performance in the subject is significant. Many studies, such as those by Dowker et al. (2019) and Kiwanuka et al. (2022), have found that students with positive attitudes towards mathematics generally perform better on mathematics assessments than students with negative attitudes. Additional studies have demonstrated that student attitude towards mathematics accounts for a significant portion of the variance in mathematics achievement (Hwang & Son, 2021).

The relationship between student attitude towards mathematics and student mathematics achievement has been demonstrated to hold across grade levels. A study involving the mathematics achievement and attitude towards mathematics of a sample of 240 elementary students between the ages of seven and 11 found that students whose attitudes towards mathematics are positive tend to have higher achievement in mathematics than students whose attitudes towards mathematics are negative (Chen et al., 2018). While controlling for other cognitive and affective factors, the study also revealed that student memory affects the relationship between achievement and attitude.

In another study, Smith et al. (2014) examined data on 7,377 randomly selected middle school students in grade eight who participated in the 2007 Trends in International Mathematics and Science Study (TIMSS). Beyond measuring students' achievement in mathematics, TIMSS also collects information on the background characteristics of students. Some of these characteristics, such as the students' attitudes towards mathematics, can potentially play a role in the students' mathematics achievement. In addition to finding a positiveassociation between achievement and attitude, the study by Smith et al. (2014) also found a positive association between achievement and frequency of group work in the mathematics classroom.

A study by Hemmings and Kay (2010) examined the relationships among prior mathematics achievement, current mathematics achievement, and current attitude towards mathematics, among other variables. For a sample of 78 high school students in grade 10, the data included students' scores from a standardized mathematics test they took in grade seven, scores from a standardized mathematics test they took in grade 10, and scores from a questionnaire they completed in grade 10 that measured their attitudes towards mathematics. The study revealed that a significant positive correlation exists between students' attitudes towards mathematics and their mathematics achievement in both grade seven and grade 10.

A study by Hemmings et al. (2011) extended the study by Hemmings and Kay (2010) by considering the effect of sex on the relationships among prior mathematics achievement, current mathematics achievement, and attitude towards mathematics. The sample included 53 female and 47 male high school students in grade 10. Overall, the data revealed that females are more likely than males to have a positive attitude towards mathematics. Further, the researchers found that prior mathematics achievement and attitude towards mathematics, when taken together, are good predictors for current mathematics achievement.

While the relationship between student attitude towards mathematics and student mathematics achievement is well-established (Arthur, 2019; Ma & Kishor, 1997), it is also complex and multifaceted. Therefore, the present study sought to join the ongoing body of research seeking to explain this relationship by addressing the following three research questions:

1. What is the nature of the relationship between student attitude towards mathematics and student mathematics achievement, with control over sex and parent education level at the student level and city population size and school socioeconomic status (SES) at the school level?
2. How does the relationship between student attitude towards mathematics and student mathematics achievement vary across schools?
3. How does school disciplinary climate contribute to the school-level variation?

Method

Data

TIMSS is an international assessment of fourth- and eighth-grade students conducted every four years by the International Association for the Evaluation of Educational Achievement (IEA). The aim of TIMMS is to measure students' mathematics and science achievement, with the goal being to enable "countries around the world to make evidence-based decisions to improve educational policies related to mathematics and science teaching and learning" (IEA, n.d.).

TIMSS 2019 made use of a multi-stage, random sampling procedure involving both stratified and cluster methods for obtaining its sample. First, for each participating country, a number of schools with students in grades four or eight was randomly selected based on the number of students enrolled. Next, for each selected school, a number of classes was randomly selected from the subpopulation of all classes meeting TIMSS eligibility criteria. Finally, every student in the selected classes was invited to participate in the study (Martin et al., 2020). In addition to the standardized achievement tests, students, as well as teachers and principals, filled out questionnaires that included information about beliefs and background characteristics of the students and schools. In our study, we considered only the U.S. sample of eighth-grade students, utilizing the nationally representative sample of 9924 students from 325 schools (Egan et al., 2022).

Table 1

Items Used to Construct Composite Variable of Student Attitude Towards Mathematics

Items	Possible Responses
How much do you agree with these statements about learning mathematics? • I enjoy learning mathematics. • I wish I did not have to study mathematics.* • Mathematics is boring.[1] • I learn many interesting things in mathematics. • I like mathematics. • I like any schoolwork that involves numbers. • I like to solve mathematics problems. • I look forward to mathematics lessons. • Mathematics is one of my favorite subjects.	0 = disagree a lot 1 = disagree a little 2 = agree a little 3 = agree a lot

Note. Students chose one of the possible responses for each statement.
[1] Responses for this item were reverse coded.

Outcome Variable

TIMSS 2019 included over 200 items in its assessment of mathematics (Martin et al., 2020). These items measured student achievement across four mathematical content domains: number, algebra, geometry, and data and probability. The number domain included topics such as integers, fractions, decimals, ratios, and proportions; the algebra domain included topics such as expressions, operations, equations, relationships, and functions; the geometry domain included topics such as geometric shapes, transformations, area, volume, and the Pythagorean theorem; the data and probability domain included topics such as data collection, graphs, numerical summaries, interpretation of data, and basic probability (Mullis & Martin, 2017).

To measure student mathematics achievement for our study, we used as our outcome variable a student's overall score (combining the four content domain scores) on the TIMSS mathematics assessment. For reference, scores on the TIMSS assessment are distributed according to a bell-shape with a mean (referred to as the scale centerpoint) of 500 points and a standard deviation (SD) of 100 points (Mullis et al., 2020).

Predictor Variables

For the present study, our predictor variables were derived using information gathered from the student, teacher, and principal questionnaires (see Martin et al., 2020). The primary predictor variable for our study was student attitude towards mathematics, measured at the student level.

Student Attitude Towards Mathematics.

TIMSS 2019 measured the type of attitude students have towards mathematics by using information gathered from some student questionnaire items (see Table 1). Based on student responses, TIMSS created (and standardized) a composite variable of student attitude towards mathematics, where each student was assigned one score (Cronbach's $\alpha = .94$; Martin et al., 2020).

School Disciplinary Climate.

Based on information gathered from the school questionnaire, TIMSS 2019 determined whether each school's disciplinary climate is considered to be problematic. The variable of school disciplinary climate (1 = problematic, 0 = not problematic) was included as a school-level variable.

Student-Level Control Variables.

There are some variables that often differ from student to student that are related to the students' personal and family backgrounds. These variables, sometimes called student background characteristics, often help explain variation in students' performance in academic subject areas in general, and in mathematics in particular (Ma et al., 2008). Two such variables include a student's sex and the education level of the student's parent(s), measured by whether the student has at least one parent with a bachelor's degree or higher. The present study included sex (1 = male, 0 = female) and parent education level (1 = at least bachelor's degree, 0 = less than bachelor's degree) as student-level control variables.

School-Level Control Variables.

As with background characteristics of students, there are variables that often differ from school to school that may also help explain the variation in students' mathematics achievement (Ma et al., 2008). Two such variables include the population size of the city where a school is located and the school's SES. As a result, the present study included city population size (1 = at least 50,000 people; 0 = less than 50,000 people) and school SES (0 = high SES, 1 = middle SES, 2 = low SES) as school-level control variables.

Statistical Analysis

Due to the hierarchical structure of the educational data, multilevel statistical modeling was used to analyze the data. Specifically, we employed a hierarchical linear model (HLM) with two levels (student and school). The analysis included three steps. Step one incorporated the null model, meaning that neither the student nor school level included any predictor variables. The purpose of the null model was to estimate the mean student mathematics achievement and to determine how the variance in student mathematics achievement is split between the student and school levels. Here is the null model:

$$MATHij = \beta_{0j} + r_{ij}$$
$$\beta_{0j} = \gamma_{00} + u_{0j}$$

where $MATH_{ij}$ is the mathematics achievement for student i in school j, β_{0j} is the mean mathematics achievement for school j, r_{ij} is the error term representing the unique effect associated with student i in school j, γ_{00} is the grand (overall) mean mathematics achievement, and u_{0j} is the error term representing the unique effect associated with school j.

For step two of the analysis, student attitude towards mathematics, as well as all other control variables at both the student and school levels, were included in the model. The purpose of this model was to determine whether student attitude towards mathematics has a statistically significant relationship with student mathematics achievement. Further, in this model, student attitude towards mathematics was allowed to vary randomly at the school level to determine any school-to-school variance in its relationship with student mathematics achievement. The first two research questions were addressed using the results from this second step. Here is the model at this step:

$$MATH_{ij} = \beta_{0j} + \beta_{1j} (ATT)_{ij} + \beta_{2j} (SEX)_{ij} + \beta_{3j} (PED)_{ij} + r_{ij}$$
$$\beta_{0j} = \gamma_{00} + \gamma_{01} (SES)_j + \gamma_{02} (POP)_j + u_{0j}$$
$$\beta_{1j} = \gamma_{10} + u_{1j}$$
$$\beta_{2j} = \gamma_{20}$$
$$\beta_{3j} = \gamma_{30}$$

where $MATH_{ij}$ is the mathematics achievement for student i in school j, β_{0j} is the mean mathematics achievement for school j, $(ATT)_{ij}$ is the attitude towards mathematics score for student i in school j, β_{1j} is the slope associated with $(ATT)_{ij}$, $(SEX)_{ij}$ is the sex for student i in school j, β_{2j} is the slope associated with $(SEX)_{ij}$, $(PED)_{ij}$ is the parent education level for student i in school j, β_{3j} is the slope associated with $(PED)_{ij}$, r_{ij} is the error term unique to student i in school j, γ_{00} is the grand (overall) mean mathematics achievement, $(SES)_j$ is the SES for school j, γ_{01} is the slope associated with (SES)j, (POP) j is the city population size for school j, γ_{02} is the slope associated with (POP)j, u_{0j} is the error term of school j unique to the intercept, γ_{10} is the effect of $(ATT)_{ij}$, u_{1j} is the error term of school j unique to the slope of $(ATT)_{ij}$, γ_{20} is the effect of $(SEX)_{ij}$, and γ_{30} is the effect of $(PED)_{ij}$.

Finally, in step three of the analysis, school disciplinary climate was added to the model at the school level to examine whether it contributes to the between-school variance in the relationship between student attitude towards mathematics and student mathematics achievement. The third research question was addressed using the results from this third step. Here is the model at this step:

$$MATH_{ij} = \beta_{0j} + \beta_{1j} (ATT)_{ij} + \beta_{2j} (SEX)_{ij} + \beta_{3j} (PED)_{ij} + r_{ij}$$
$$\beta_{0j} = \gamma_{00} + \gamma_{01} (SES)_j + \gamma_{02} (POP)_j + u_{0j}$$
$$\beta_{1j} = \gamma_{10} + \gamma_{11} (DIS)_j + u_{1j}$$
$$\beta_{2j} = \gamma_{20}$$
$$\beta_{3j} = \gamma_{30}$$

where MATHij is the mathematics achievement for student i in school j, β_{0j} is the mean mathematics achievement for school j, $(ATT)_{ij}$ is the attitude towards mathematics score for student i in school j, β_{1j} is the slope associated with $(ATT)_{ij}$, (SEX)ij is the sex for student i in school j, β_{2j} is the slope associated with $(SEX)_{ij}$, $(PED)_{ij}$ is the parent education level for student i in school j, β_{3j} is the slope associated with $(PED)_{ij}$, r_{ij} is the error term unique to student i in school j, γ_{00} is the grand (overall) mean mathematics achievement, $(SES)_j$ is the SES for school j, γ_{01} is the slope associated with $(SES)_j$, $(POP)_j$ is the city population size for school j, γ_{02} is the slope associated with $(POP)_j$, u_{0j} is the error term of school j unique to the intercept, γ_{10} is the effect of $(ATT)_{ij}$, (DIS)j is the disciplinary climate for school j, γ_{11} is the slope associated with $(DIS)_j$, u_{1j} is the error term of school j unique to the slope of $(ATT)_{ij}$, γ_{20} is the effect of $(SEX)_{ij}$, and γ_{30} is the effect of $(PED)_{ij}$.

Results

Descriptive Statistics

We calculated descriptive statistics for each of the predictor variables in order to explore their primary features (see Table 2).

Table 2

Descriptive Statistics for Predictor Variables

	M	SD
Student-level variables		
Student attitude towards mathematics[1]	0.00	1.00
Sex (1 = male, 0 = female)	0.50	0.50
Parent education level (1 = at least bachelor's degree, 0 = less than bachelor's degree)	0.41	0.49
School-level variables		
School disciplinary climate (1 = problematic, 0 = not problematic)	0.61	0.49
City population size (1 = at least 50,000 people; 0 = less than 50,000 people)	0.48	0.50
School SES (0 = high SES, 1 = middle SES, 2 = low SES)	1.33	0.81

[1] This variable was standardized.

Student-Level Variables.

The TIMSS 2019 index used to measure student attitude towards mathematics was standardized to have a mean of 0 with an SD of 1. Regarding the sex of the students, 50.3% were male, and 49.7% were female. In terms of parent education level, about 41% of the students had at least one parent with a bachelor's degree (or higher).

School-Level Variables. About 48% of the schools were in cities whose population size was 50,000 or greater. In terms of SES, 22% of the schools were high SES, 23% of the schools were middle SES, and 55% of the schools were low SES. The disciplinary climate was considered to be problematic in 61% of the schools.

Estimating the Mean and Partition of Variance for Mathematics Achievement

The outcome variable for the present study was student mathematics achievement. However, since TIMSS uses a plausible-value approach to assign scores to students, descriptive statistics could not be calculated. Rather, we used the null model from an HLM to produce estimates for both the mean and the variance for student mathematics achievement. The estimated mean mathematics achievement for the students was 519.49 (t = 117.84, p < .001), which is about one-fifth of an SD higher than the overall TIMMS mean of 500.

After using the HLM to partition the variance in student mathematics achievement, we found that about 54% can be attributed to student-level differences, while about 46% can be attributed to school-level differences (variance at student level: $\sigma^2 = 5241.79$, variance at school level: $\tau = 4458.44$; $\sigma^2/(\sigma^2+\tau)=0.54$, $\tau/(\sigma^2+\tau)=0.46$). As the school-level variance was statistically significant ($\chi^2 = 6576.05$, p < .001), we affirmed that the mathematics achievement of students does indeed differ from school to school.

Relationship With Student Attitude Towards Mathematics

Student attitude towards mathematics was shown to have a statistically significant relationship with student mathematics achievement (t = 18.82, p < .001), even when controlling for the student-level variables of sex and parent education level and the school-level variables of city population size and school SES. In particular, each 1-point increase in student attitude towards mathematics was associated with an increase of 22.42 points in student mathematics achievement. As such, we found that student mathematics achievement is positively related to student attitude towards mathematics.

With the TIMSS mathematics achievement having a mean of 500 points and an *SD* of 100 points (Mullis et al., 2020), the effect size calculation was straightforward, yielding an effect size of 22.42% of an *SD* (Cohen's d=22.42/100=0.2242) for student attitude towards mathematics.

Between-School Variance in Relationship With Student Attitude Towards Mathematics

In addition to examining the relationship between student attitude towards mathematics and student mathematics achievement, we also examined whether this relationship varies from school to school. Indeed, the between-school variance in the relationship between student attitude towards mathematics and student mathematics achievement was shown to be statistically significant (χ_2 = 282.53, p = .023). In other words, the extent to which a student's mathematics achievement is related to the student's attitude towards mathematics depends on the school in which the student is enrolled.

Contribution of School Disciplinary Climate to Relationship With Student Attitude Towards Mathematics

Because a statistically significant between-school variance exists in the extent to which student mathematics achievement is related to student attitude towards mathematics, disciplinary climate was included in the model as a school-level variable to determine if it helps explain the between-school variance; however, it was not found to do so (t = -0.23, p = .817). In other words, there is insufficient evidence to conclude that differences in schools' disciplinary climates help explain why the relationship between student attitude towards mathematics and student mathematics achievement is greater at some schools than at others.

HLM Model Performance

To determine how well the HLM model performed, we calculated the proportion of variance explained. Specifically, the full model explained roughly 14% of the student-level variance in student mathematics achievement and roughly 41% of the total school-level variance in student mathematics achievement. As a whole, the full model explained roughly 27% of the total variance in student mathematics achievement. Based on the standard benchmarks used in educational research (see Gaur & Gaur, 2006), these results are considered acceptable, demonstrating that the HLM model performed well.

Discussion

Revisit of Research Literature

Although there is a substantial amount of literature on the positive relationship between student attitude towards mathematics and student mathematics achievement, most studies have involved relatively small, nonrandom samples from relatively small geographical regions (e.g., one school district). In addition, most existing studies have not taken into account the hierarchical structure of educational data. The use of large-scale, randomly selected data for the present study lends further evidence of a positive relationship between student attitude towards mathematics and student mathematics achievement. Further, the present study made use of multilevel modeling techniques to account for students being nested within schools. Taken together, these constitute a marked improvement in research methodology. As a result, our study contributes to the research literature by yielding findings that are more generalizable.

Educational Implications

While we recognize that variables being correlated does not necessarily mean there is a causal relationship between them, there is a clear connection between attitude and achievement; thus, it follows that using strategies that foster a positive attitude towards mathematics is crucial in promoting student academic achievement in the subject. One such strategy is to address any negative beliefs or stereotypes that students may hold about mathematics (Awoniyi & Butakor, 2021). This can be done by challenging misconceptions and providing examples of successful individuals who have excelled in mathematics. By debunking the idea that math is only for "geniuses" or "mathematical prodigies," students can develop a more positive view of their own potential in mathematics and become more motivated to succeed in the subject (Hwang & Son, 2021).

Since the classroom environment plays a crucial role in shaping students' attitudes towards mathematics (Awoniyi & Butakor, 2021), another strategy for fostering positive attitudes is for teachers to create a supportive and inclusive learning environment. This can be achieved by valuing and respecting every student's contribution, providing opportunities for collaborative learning, and promoting a growth mindset. Furthermore, integrating real-world applications and problem-solving activities into the mathematics curriculum can also help students develop a positive attitude towards the subject. By demonstrating the relevance and practicality of mathematics in everyday life, students can see the value in learning the subject and become more engaged (Elçi, 2017).

Teachers can also influence students' attitudes towards mathematics through their own attitudes towards the subject. This relationship between teacher attitudes and student attitudes towards mathematics is well-established and supported by research (Elçi, 2017; Hwang & Son, 2021). Teachers who demonstrate enthusiasm for the subject, provide clear explanations, and create a supportive learning environment can help students develop a positive attitude towards mathematics, which, in turn, can also increase the students' academic achievement in the subject (Awoniyi & Butakor, 2021). Because of this connection, it is imperative that educational administrators aim to help teachers improve (when needed) their own attitudes towards mathematics in order to establish a supportive and engaging classroom learning environment that fosters students' interest in and enthusiasm for mathematics (Ayob & Yasin, 2017).

Moreover, it has been observed that a decline in student enrollment in advanced mathematics classes may be attributable to poor attitudes towards mathematics (Arnold et al., 2019). This decrease in enrollment limits the number of students who can pursue careers in science, technology, engineering, and mathematics (STEM) fields, demonstrating that fostering positive attitudes towards mathematics among students is not only important for their academic success but also for their future career prospects in fields that rely on mathematical proficiency. This connection raises concerns among mathematics educators and educational policymakers, as well as society as a whole, about the continued STEM workforce shortage (U.S. Chamber of Commerce Foundation, 2022).

Limitations and Suggestions

While this study found that student mathematics achievement is positively related to student attitude towards mathematics, only certain student and school background characteristics were included in the study. Future studies could examine the ways in which student attitude towards mathematics interacts with other such variables, including student race/ethnicity (Parks & Schmeichel, 2012). Including other key background variables could provide further insight into the relationship between attitude and achievement. In addition, student attitude towards mathematics could be considered alongside other outcome variables in mathematics. For example, one could analyze whether improving students' attitude towards mathematics helps decrease students' mathematics anxiety.

Another key finding from the study is that the relationship between student attitude towards mathematics and student mathematics achievement varies across schools, meaning there are school-level variables that affect the relationship. Since school disciplinary climate was not found to contribute to the relationship, future studies could consider other school-level variables that might help explain the variation. An obvious limitation of this study is that it only included the U.S. data from TIMSS 2019. It is possible that the relationship between student attitude towards mathematics and student mathematics achievement might also vary across countries. With that in mind, future research could examine the larger set of TIMSS data that includes other countries and country-level variables.

Finally, this study measured student attitude towards mathematics in a particular way, specifically by making use of TIMSS' composite variable of student attitude towards mathematics developed using information obtained through the TIMSS 2019 student questionnaire. While the TIMSS attitude variable had a high level of internal consistency, for future studies, researchers could seek to measure student attitude towards mathematics in different ways by using other well-established attitude surveys or even designing their own.

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Navigating through Social Justice in Mathematics Education: Prospects, Priorities, Processes, and Problems

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ABSTRACT Social justice in mathematics education remains a critical concern. Despite growing awareness, progress has been insufficient. Mathematics education plays a pivotal role in fostering social justice through creativity, critical thinking, innovation, and collaborative problem-solving. However, existing curriculum, teaching practices, and assessments reveal significant shortcomings. This review-based argument highlights four critical areas—prospects, priorities, processes, and problems—aimed at creating a socially just environment with equitable access to high-quality mathematics for all students. The authors emphasize the interrelatedness of social justice within these criteria to promote equity, access, and empowerment in mathematics education.

KEYWORDS *Social justice, prospect of social justice, priority of social justice, process of social justice, problems of social justice*

Introduction

Justice in mathematics education is a critical concern. It encompasses fairness, equity, equality, and equal access to learning opportunities inside and outside mathematics classrooms (Esmonde & Caswell, 2010). Mawarti and Nurlaelah (2020) describe that mathematics education holds immense potential to enhance social justice through student participation in learning and development. According to them, this potential can be materialized through membership based on equality while using resources in and for education and through cognitive and affective impact on the development of humanity, fostering a just and peaceful world. To achieve this, we must integrate mathematics and social justice by aligning contents with the contexts (Nolan, 2009). Transforming mathematics education into participatory democracy is essential for sustainable futures with social justice (Nolan, 2009) in a way that ensures justice, equity, and fairness for all students in the schools and classrooms (Gutstein, 2006). For this transformation, teachers and students should critically challenge the social and political structures to combat injustice and unfairness (Cotton & Hardy, 2004). Socially just educational pedagogy such as dramatic, interactive, and experiential teaching and learning can develop personal and group participation, exemplify reflection, and engage teachers and students in critical issues through dialogue (Boylan, 2009). Disciplinary boundaries no longer exist in socially just pedagogy to ensure equitable learning opportunities (Moje, 2007).

Paulo Freire's (1970) critical pedagogy acknowledges the existence of oppressed and oppressors in the social, cultural, historical, and political context of educational practices, and it further links opportunities and justice in education (AydÕn et al., 2010). From this view, socially just pedagogy embodies a vision of equality across race, class, ability, and gender in any classroom (Lynch & Baker, 2000). Consequently, teachers and institutions may shape what to teach, how to teach, and why to teach a subject matter or content in the mathematics curriculum (Buell & Shulman, 2019). Despite mathematics education being a powerful and essential tool for maintaining social justice (Gutstein, 2003), most teachers may not teach for social justice due to the pressure to prepare students for mandated assessments. They may have difficulty planning lessons integrating social justice with mathematics subject matter (Register et al., 2022). If considered and applied correctly, social justice pedagogy in mathematics can make students confident and creative with a broader public mission to prepare them to be responsible citizens (Colby & Erlich, 2000). Social justice pedagogy in mathematics classrooms may engage, empower, and motivate students to solve social problems through mathematics as a tool (Bond & Chernoff, 2015). Integrating social justice issues into mathematics classes may help teachers seek a broader social, cultural, and political connection between classroom practices and the community through lesson modeling and reflection (Garii & Rule, 2009).

Social justice in mathematics education is complex and multifaceted. Achieving clarity in its planning and implementation is essential, and this goes beyond transforming beliefs, dispositions, and knowledge with self-awareness and self-reflexivity in mathematics teaching and learning (Boylan & Woolsey, 2015). The issue is pervasive across various educational contexts (Panthi & Belbase, 2017; Panthi et al., 2021) and affects social dynamics in teaching mathematics (Wright, 2016). Mathematics teachers often find themselves in the role of advocates, needing to counsel and persuade not just students but also other stakeholders of the value of social justice in the classroom (Panthi et al., 2018a, 2018b). Yet, they face obstacles, such as cultural diversity, disconnected curricula, traditional teaching methods, seating arrangements, and large class sizes (Panthi & Belbase, 2017; Panthi et al., 2021), all of which can impede the integration of social justice into mathematics education.

This paper seeks to establish a consistent approach to social justice within mathematics education, recognizing that mere advocacy may not suffice to affect change in classroom practices. It examines four critical dimensions: the potential benefits (prospects), the areas of greatest need (priorities), the method of implementation (processes), and the challenges faced (problems) in integrating social justice into mathematics education. The central question is: “What are the prospects, priorities, processes, and problems associated with social justice in mathematics education?” This inquiry aims to contribute to the evolving understanding of how social justice can be effectively woven into the fabric of mathematics teaching and learning.

Method of Study

This study conducts a comprehensive literature review to explore the four key dimensions of social justice in mathematics education: prospects, priorities, processes, and problems. The methodology of this study is document collection, review, and analysis to understand and address these critical aspects effectively.

Document Collection

Our literature review encompassed a diverse range of sources, including peer-reviewed articles, books, book chapters, theses, dissertations, and website content. Initiated in January 2021, the collection process involved gathering relevant documents from various online platforms and libraries, focusing on four primary types of materials to inform our study on social justice in mathematics education.

A comprehensive search was conducted using Google and the Education Resources and Information Center (ERIC), utilizing a variety of keywords including ‘justice’, ‘social justice’, 'socially just pedagogy', 'culturally relevant pedagogy', 'culturally responsive assessment', 'mathematics and social justice', 'reconstruction of mathematics', 'transformative learning', 'critical pedagogy', 'social justice in mathematics teaching', 'social justice in mathematics', equity in mathematics education', 'critical mathematics education', 'prospects of social justice', 'priorities of social justice', 'processes of social justice', and 'problems of social justice'. This search yielded a multitude of websites and online resources pertaining to social justice in mathematics education.

Our search synthesized a wealth of literature on social justice in mathematics education, drawing from a variety of sources: Primary information came from educational websites, while journal articles and conference papers provided in-depth analyses on topics like equity pedagogy and critical mathematics education. Influential texts such as D'Ambrosio's (2012) exploration of social justice, Koestler's (2012) insights on equitable teaching, and Freire's (1970/1999) seminal work on oppressed pedagogy formed the backbone of our theoretical framework. Other works, such as critical pedagogy (Darder et al., 2003), pedagogy for social justice (Gutstein, 2006), and curriculum and evaluation standards (National Council of Teachers of Mathematics, 1989 & 2000), were additional resources used to conceptualize social justice in mathematics education.

Additionally, theses and dissertations (e.g., Bialick, 2021; Colquitt, 2014; Kari, 2017; Seegmiller, 2020; Wonnacott, 2011; & Wright, 2015) offered fresh perspectives on the subject, and institutional content further enriched our understanding of the practical applications of social justice in educational settings.

Document Analysis

We employed document analysis as a method for synthesizing key codes, concepts, and categories from the different sources outlined above (Bowen, 2009; Morgan, 2021). Through meticulous review and analysis, we extracted key concepts, which were then coded to unearth potential categories and themes following Morgan's (2021) and Bowen's (2009) detailed approach to document analysis. In this process, we reviewed and identified key concepts related to prospects, priorities, processes, and problems of social justice in mathematics education and noted them down in a table forming a matrix of codes, concepts, and relevant sources (see Tables 1-4). This open coding and conceptualizing of key ideas from different sources in an open exploration continued until the conceptual matrix was saturated enough to cover key ideas related to the four critical dimensions of social justice in mathematics education. This iterative process of reading and coding key concepts from the documents allowed us to distill the essence of social justice in mathematics education into distinct codes. Once we accumulated enough codes, we grouped them into categories that formed the subthemes of our conceptual matrix (refer to Tables 1-4). Our analytical framework was further refined by adopting categorical thinking, as suggested by Freeman (2017), and grounded theory coding, suggested by Bryant (2017), to systematically organize these insights into four overarching themes of social justice in mathematics education.

There were three sub-themes under the prospect of social justice in mathematics education: social justice movement, sustainability and democracy, and critical pedagogy (Figure 1). In the second theme, priorities, we generated two sub-themes: curriculum reform and pedagogical reform. We generated three sub-themes for the third theme, process: a curriculum process, a pedagogical process, and an assessment process. The last theme was problems associated with social justice in mathematics education, which included the two sub-themes: the difficulty of social justice in mathematics education and the challenges of social justice in mathematics education. Figure 1 shows the schematic of these four dimensions in our study.

Thematic Results and Discussion

We organized the codes into conceptual groups, which are grouped under prospects, priorities, processes, and problems. Figure 1 presents each theme and its categories, and each theme and its categories are discussed separately.

Prospects for Social Justice in Mathematics Education

The prospect of social justice in mathematics education is related to the possibility of integrating social justice with mathematics. The prospects are summarized in the following codes and categories in Table 1 with three distinct sub-themes—social justice movement, sustainability and democracy, and critical pedagogy. Each subtheme has been explored under separate subheadings.

The Social Justice Movement In Mathematics Education

There has been a call to restructure mathematics education that includes reform in teaching and learning mathematics for social justice (Grazino, 2017), acknowledging students' background, family values, race, class, language, or culture (Acharya et al., 2021). Social justice

Figure 1

Conceptual framework of prospects, priorities, and problems of social justice in mathematics education

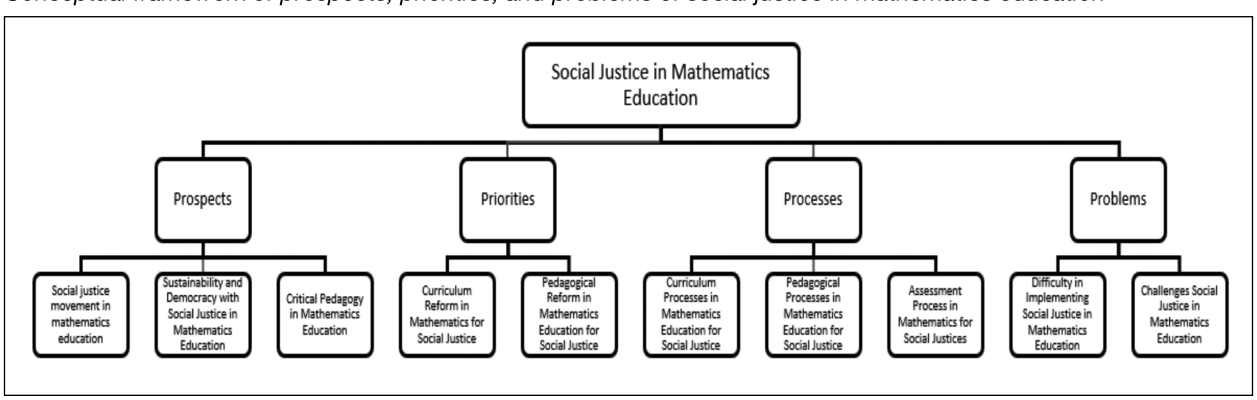

Table 1

Prospects of social justice in mathematics education

Primary Codes	Categories	Relevant Literature
M1. Reform teaching and learning. M2. Realistic mathematics M3. Developing equitable pedagogy M4. Differentiation in teaching M5. Equity principle M6. Landless movement pedagogy M7. Habitus in classroom M8. Critical mathematics M9. Re-imagining social justice in mathematics. M10. Democracy in mathematics classroom	Movement	Grazino (2017), Mawarti & Nurlaelah (2020), Bartell (2013), Wager (2008), NCTM (2000), Knijnik & Wanderer (2015), Fletcher (2012), Skovsmose (2011), Le Roux (2016), Ball & Bass (2008)
SD1. Sustainable development SD2. Mathematics for sustainability SD3. Participation in decisions SD4. Social justice and sustainability SD5. Numeracy and sustainability SD6. Quantitative and qualitative interpretation SD7. A fair share of resources SD8. Taking a stand for justice SD9. Discrimination and unfair treatment SD10. Sustainable development goals SD11. Mathematics for sustainability	Sustainability and Democracy	Campbell (2013), Fraser (2013), Mawarti & Nurlaelah (2020), Miller (2002), Norton (2003), Dryzek (2010), NCTM (2014),Mawarti & Nurlaelah (2020), OECD (2015), Roe et al. (2018)
CP1. Empowering students CP2. Functional math CP3. Democratic community CP4. Students' critical eye CP5. Dialogical pedagogy CP6. Banking model of education CP7. Professional development for critical thinking CP8. Inclusive and responsive teaching CP9. Connection to real-world issues CP10. Culture and context-bound	Critical pedagogy	Ernest (2016), Jorgensen (2016), Andersson & Valero (2016), Tutak et al. (2011), Uenrostro (2016), Freire (1970), Shapira-Lishchinsky (2016), Mumu et al. (2017), Selvianiresa & Prabawanto (2017), Skovsmose (2011)

in mathematics education has two related components: mathematics literacy and realistic mathematics (Mawarti & Nurlaelah, 2020). The first component emphasizes mathematics literacy as a civil rights or social justice issue that incorporates lessons and activities to apply knowledge in a socially responsible way. The second component includes understanding social justice issues through a mathematical framework to address social problems (Mawarti & Nurlaelah, 2020). In this sense, social justice in mathematics education has been realized as a new movement for developing equitable mathematics pedagogy considering cultural and social justice (Bartell, 2013). There is a renewed interest in teaching for and about social justice by differentiating between teaching about, with, and for social justice (Wager, 2008). The National Council of Teachers of Mathematics (NCTM, 1989) initiated social justice as a framework for inclusive mathematics education through the equity principle that further continued in NCTM (2000) its commitment to social justice with a strong emphasis on helping all students learn mathematics despite their background and ability groups. The social justice movement in mathematics education has been growing in different parts of the world. For example, projects such as active school programs and landless movement pedagogy in Brazil (Knijnik & Wanderer, 2015), habitus (an internalized pattern of behavior, action, and disposition) created by mathematics teachers and students in the classrooms in Ghana (Fletcher, 2012), critical mathematics education in Denmark (Skovsmose, 2011 & 2012), and re-imagining the possibility of equity and social justice through mathematics education in a South African university (Le Roux, 2016), are a few initiatives in social justice movements in different places.

The restructuring of mathematics education has been proposed to include a focus on teaching and learning that promotes social justice. This reform, as suggested by Graziano (2017), involves recognizing and valuing students' diverse backgrounds, family values, races, classes, languages, and cultures. Social justice within the context of mathematics education encompasses two interconnected components: mathematics literacy and realistic mathematics education, as identified by Mawarti and Nurlaelah (2020). The former views mathematical literacy as an issue of civil rights and social justice, integrating lessons and activities that encourage the application of knowledge for societal benefit. The latter seeks to comprehend social justice concerns using mathematical concepts to tackle societal issues.

This approach to social justice has sparked a new movement aimed at developing an equitable mathematics pedagogy that takes into account cultural and social justice considerations, as discussed by Bartell (2013). Moreover, a growing interest is in differentiating the teaching methods for, with, and about social justice, as outlined by Wagner (2008). The National Council of Teachers of Mathematics (NCTM) laid the groundwork for this approach in 1989 by introducing social justice as a framework for inclusive mathematics education, which was reinforced in 2000 with a strong commitment to ensuring all students have access to mathematics education, regardless of their background or abilities.

Sustainability and Democracy with Social Justice in Mathematics Education

The interplay between equity, justice, and sustainability forms a crucial axis in mathematics education. Campbell (2013) posits that these elements are integral to sustainable development, yet social justice alone cannot fully realize this goal. When social justice and equity principles are woven into the fabric of mathematics education, they can profoundly shape students' attitudes, values, and competencies, such as numeracy. This integration enables students to connect mathematical concepts with societal challenges, fostering sustainable development.

From the vantage point of social justice, mathematics education catalyzes heightened awareness of societal values, encourages active citizen participation in decision-making processes, and advocates for equitable resource allocation and distribution (Fraser, 2013). Mawarti and Nurlaelah (2020) and Roe et al. (2018) further affirm that embedding social justice within mathematics education is a step toward achieving sustainable development.

Sustainability encompasses not only environmental stewardship but also economic and educational equity, fairness, and the democratization of resources and the opportunity to access these resources with a fair share and utilization (Roe et al., 2018). Renert (2011) emphasizes that natural capital, a shared inheritance of humanity, should be equitably distributed across current and future generations, irrespective of background. This equitable distribution is facilitated by linking educational pursuits, particularly mathematics, with ecological sustainability.

The ideal learning environment marries mathematical thinking and quantitative reasoning with real-world applications, fostering a culture of informed decision-making and risk-taking (Roe et al., 2018). Such an environment is predicated on citizens' numeracy and quantitative skills (Volchok, 2019), complemented by qualitative insights into their social, cultural, historical, and political identities (Miller, 2002; Norton, 2002).

Sustainability is also deeply entwined with democratic ideals, where equitable resource sharing, opportunity access, and balanced power dynamics are not merely the privileges of voters but the rights of all critically thinking, informed, and mathematically literate individuals (Dryzek, 2010). The National Council of Teachers of Mathematics (NCTM, 2014) advocates for mathematics education that champions equity, justice, and democratic values, challenging discrimination and promoting fairness.

Various global initiatives exemplify the commitment to teaching mathematics through the lens of sustainability and democracy. In the United States, equity pedagogy (Cabana et al., 2014; Jackson, 2013), democratic practices in South African mathematics classrooms (Christiansen, 2007; Vithal, 1999 & 2012), and educational reforms in Denmark (Valero, 2002) all contribute to this movement. The OECD's Sustainable Development Goal Four underscores the importance of equitable, inclusive, and quality education, including mathematics, as a foundation for sustainable development (OECD, 2015). In conclusion, mathematics education is poised to play a pivotal role in instilling democratic principles and values within classroom teaching and extending these concepts into daily life, thereby shaping a sustainable present and future (Ball & Bass, 2008).

Critical Pedagogy in Mathematics Education

Critical mathematics education is designed to endow students with the functional knowledge and skills of mathematics and the critical consciousness necessary for fostering democratic communities committed to social justice, both within and beyond the school environment. This educational approach, championed by Ernest (2016), Jorgensen (2016), and Andersson and Valero (2016), encourages educators to adopt democratic principles such as justice, equity, and culturally responsive pedagogy, nurturing students to engage with social justice issues in their surroundings critically.

The practice of mathematics education involves a reflective critique of conventional norms and the application of theories that promote a socially just pedagogy characterized by equity, access, and fairness (Tutak et al., 2011). Teachers are encouraged to engage in meaningful dialogue with students, embracing inclusive practices that reflect thoughtfulness and consideration for all learners (Jorgensen, 2016).

Critical pedagogy, as advocated by Uenrostro (2016), supports dialogic teaching methods that facilitate social agency and societal transformation by questioning established societal norms. This pedagogical stance, as outlined by Freire (1999), rejects the traditional 'banking' model of education, where teachers passively deposit information into students, who are seen as mere receptacles. Instead, it calls for an educational paradigm shift towards a more interactive and caring approach to teaching that emphasizes social justice.

Therefore, professional development programs for teachers should focus on fostering a pedagogy of social justice that encompasses equity, access, fairness, and care, as Shapira-Lishchinsky (2016) suggested. Critical pedagogical practices can address and rectify students' misconceptions about mathematics by promoting an inclusive and responsive teaching environment.

Moreover, critical pedagogy enables students to connect mathematical concepts to real-world situations and their life experiences, empowering them to challenge the traditional notions of objectivity and truth (Mumu et al., 2017). This approach reveals the culturally and contextually bound nature of mathematical knowledge, allowing for subjective interpretations and a broader understanding of mathematics as a discipline intertwined with the real world (Skovsmose, 2011; Selvianiresa & Prabawanto, 2017).

The prospects for mathematics education, as discussed above, extend across various disciplines, social and cultural contexts, and educational systems globally. These prospects have paved the way for social justice to become a priority in mathematics education, promoting values such as peace, harmony, tolerance, and resilience, transcending the confines of strict disciplinary knowledge and skills.

Priorities for Social Justice in Mathematics Education

Mathematics, as a universal body of knowledge, belongs to all of humanity and should be imparted through a pedagogy steeped in social justice and equity. Such as pedagogy would transcend the barriers of race, class, and gender within any educational setting, as advocated by Buell and Shulman (2019). In light of this, mathematics teachers and educators must place social justice at the forefront of mathematics education. This initiative entails a commitment to reforms in curriculum, teaching methods, and assessment practices, all aimed at fostering an environment where learning and teaching mathematics is a socially just experience. The following discussion elaborates on these priorities, further delineating with specific codes and categories in Table 2.

Curriculum Reform in Mathematics Education for Social Justice

Mathematics curriculum reform is a multifaceted process with deep historical, social, and political roots. It encompasses various movements and initiatives across the globe, such as the ‘new math’, ‘back to basics’, and ‘standards movement’ in the US (NCTM, 2000), ‘realistic mathematics education’ (RME) in the Netherlands (van Zanten & de Heuvel-Panhuizen, 2021), and a series of reform efforts in the UK, including the National Numeracy Strategy (1999), Primary National Strategy (2003), Every Child Matters (2004), and Every Child Counts (2004) (Vollaard et al., 2008). These ongoing reforms raise the question of how social justice has been integrated into the mathematics curriculum.

In the US, curriculum reforms have aimed to address equity, access, and fairness in mathematics education (NCTM, 2000 & 2014). The RME approach has shown potential for promoting social justice within mathematics classrooms, as evidenced by various studies (Gutstein, 2006; Kathotia et al., 2021; Solomon et al., 2021; Stephan et al., 2014). However, these reforms have often been entangled in discourses of inclusion that, paradoxically, may lead to marginalization or require assimilation into the prevailing mathematics education culture, thus maintaining its core characteristics (Sojoyner, 2017). Notably, past reforms, particularly in the US, have not always been inclusive of all groups, especially black learners who have historically faced racism, white supremacy, and legalized segregation (Berry et al., 2014). Despite efforts towards equity and inclusion, many black learners in the US still encounter oppressive educational practices (Martin, 2019).

The Ontario Secondary Mathematics Curriculum in Canada emphasizes the importance of students actively constructing new knowledge based on their prior experiences, preparing them for societal roles (Ontario Ministry of Education, 2005). This approach aligns with the transformative nature of the curriculum that

Table 2

Priorities of social justice in mathematics education

Primary Codes	Categories	Relevant Literature
CR1. Standards movement CR2. Realist mathematics education (RME) CR3. National numeracy strategy CR4. Every child matters. CR5. Discourse on inclusion. CR6. Legalized segregation CR7. Enclosure and containment CR8. Transformative curriculum CR9. In and out of classroom math CR10. Critical stance CR11. Inquiry-based curriculum	Curriculum reform	NCTM (2000), van Zanten & de Heuvel-Panhuizen (2021), Vollaard et al. (2008), Martin et al. (2010), Berry et al. (2014), Sojoyner (2017), Lam (2012), Mitescu et al. (2011), Chapman et al. (2016), McHugh (2015)
PR1. Daily life and social context PR2. Constructing and reconstructing math PR3. Scrutinizing student and teacher roles PR4. Pedagogical reform PR5. Reformative motion PR6. Personal empowerment PR7. Transformative learning PR8. Critical reflection PR9. Understanding social issues PR10. Engaging in community issues in math PR11. Day-to-day life math PR12. Specific circumstances	Pedagogical reform	Skovsmose (2011), Wedege (2016), Alderton (2020), Boaler (2008), Colquitt (2014), Jackson (2013), Hassi & Laursen (2015), Mezirow (1991), Evans & Davies (1993), Hendrickson (2015), Bartell (2013), Cochran-Smith, (1999)

refines learners' experiences (Lam, 2012). Reform-oriented teaching practices that are grounded in social justice principles advocate for higher-level thinking, deep content knowledge, and a cooperative learning environment where students respect and learn from one another, thereby creating rich and meaningful learning opportunities both inside and outside the classroom (Mitescu et al., 2011). Furthermore, these practices promote a critical examination of mathematical content within a democratic context that addresses broader structural and societal issues (Chapman et al., 2016). An inquiry-based curriculum that explores societal inequities through mathematics enables students to engage with and interpret the world mathematically (Gutstein, 2006; McHugh, 2015).

Pedagogical Reform in Mathematics Education for Social Justice

The integration of mathematics education with daily life and societal contexts is a critical aspect of learning and teaching, as it involves the construction and reconstruction of mathematical knowledge within the context of students' backgrounds and aspirations (Skovsmose, 2011, 2013; Wedge, 2016)"). This process also entails a critical examination of the roles of students and teachers, focusing on their actions and potential growth (Alderton, 2020). Central to this is the concept of social justice in mathematics education, which necessitates pedagogical reforms that transform the identities of students and teachers through dynamic relationships and context-sensitive discourse (Alderton, 2020; Boaler, 2008).

Such transformation provides opportunities for student empowerment, enabling them to engage with mathematics through inquiry, collaboration, and communication (Colquitt, 2014). Equity pedagogy plays a vital role in facilitating these transformative learning experiences, which require teachers to have a profound understanding of their students' learning processes (Hassi & Laursen, 2015; Jackson, 2013).

Transformative learning is characterized by self-reflection and agency, shifting the interpretation of experiences through critical awareness and reflection (Mezirow, 1991). It promotes autonomy and self-directed learning, often achieved through collaborative group work (Hassi & Laursen, 2015). In a socially just educational environment, transformative learning leads to significant shifts in how students derive meaning from their experiences, a hallmark of this educational approach (Mezirow, 1991, 1997). Empowerment in this context is not just about the learning process but also the outcomes, particularly in college mathematics (Hassi & Laursen, 2015).

By recognizing the value of mathematics as a tool for understanding and addressing social issues, students can balance social justice with mathematical learning, engaging in community-centered activities within the classroom (Evans & Davies, 1993; Hendrickson, 2015). This approach enhances students' awareness of social realities by connecting mathematical concepts to everyday life and adapting to their unique circumstances (Bartell, 2013; Cochran-Smith, 1999; Skovsmose, 1994).

To advance socially just mathematics education, curriculum and pedagogical reforms must be prioritized, requiring ongoing efforts to uphold social justice through clear visions, plans, and actions within classrooms, schools, and educational policies. These priorities should extend to pedagogical and assessment practices that emphasize equity, access, and a high-quality learning environment, thereby fostering a more equitable and just educational landscape.

The Processes of Social Justice in Mathematics Education

The pursuit of social justice within mathematics education is a complex and nuanced endeavor. It involves navigating through contentious debates that revolve around the potential negative or positive effects such initiatives may have on students' opportunities, access to resources, and equitable educational outcomes (Wright, 2017). To better understand and organize these intricate processes, we have delineated them into three distinct sub-themes: curriculum processes, pedagogical processes, and assessment processes. Each of these plays a pivotal role in the advancement of social justice in mathematics education. The specific categories within these sub-themes, along with their corresponding codes, are systematically outlined in Table 3.

Curricular Processes in Mathematics Education for Social Justice

The curriculum serves as a comprehensive guide that addresses the multifaceted questions of what, when, why, whom, and how to teach, as well as the assessment of students' mathematical understanding. It is shaped by diverse philosophical and theoretical perspectives, as depicted by various education scholars and practitioners (Belbase et al., 2022). Within the broad social, cultural, historical, and political contexts, a curriculum that strives for social justice must actively engage with these dimensions rather than remaining silent or neutral (Ndimande, 2010).

A spectrum of interest groups influences the development and ongoing refinement of mathematics curriculum, each viewing social justice through unique lenses, including mathematics teachers, educators, and stakeholders (German, 2021; Kumashiro, 2009). A socially just curricular process in mathematics education embodies high expectations for learning outcomes, ensures equitable access to rigorous and relevant mathematics for all students, and fosters a positive learning environment conducive to high achievement (National Council of Supervisors of Mathematics & TODOS, 2016).

To uphold the intrinsic value of mathematics for learners, the curriculum should cultivate positive beliefs and identities (Aguirre et al., 2013), demand high cognitive and affective engagement, and encourage creative mathematical thinking rooted in learners' knowledge and experiences (Bright et al., 2015; Turner et al., 2012). It should integrate content and assessment activities that promote civic engagement (Gutstein & Peterson, 2013; Martin et al., 2010) and facilitate participatory learning through open discourse and interaction (Horn, 2012).

Achieving such a curriculum requires a collaborative and inclusive approach to planning. This involves creating a shared platform where curriculum experts, mathematics teachers, educators, mathematicians, and supervisors can collectively contribute their beliefs, values, conceptions, and theories to shape the curriculum's aims and learning outcomes (Walker, 1970). The Walker model of curriculum planning, though classical, offers a more organic approach than the Tyler and Taba models. By incorporating social justice considerations into the evolving context of mathematics curriculum design, the Walker model can be adapted to develop and implement equity-based, inclusive standards and frameworks for school mathematics (Walker, 1971). In this process, teachers' insights and perceptions become instrumental in dynamically shaping the curriculum in action (Panthi et al., 2018a & 2018b).

Pedagogical Processes in Mathematics Education for Social Justice

In the realm of mathematics education, pedagogical processes are often categorized into traditional and constructivist frameworks, reflecting the distinct roles of teachers and students (von Glasersfeld, 1995). However, this binary classification may not fully capture the diverse array of teaching and learning practices found in mathematics classrooms (Bas & Kivilcim, 2021; Wood et al., 2001). A multitude of pedagogical approaches has been proposed to enhance students' mathematical learning, including

Table 3

Priorities of social justice in mathematics education

Primary Codes	Categories	Relevant Literature
CPr1. Curriculum images CPr2. Silent and neutral curriculum CPr3. Interest groups CPr4. Rigorous and relevant CPr5. Enduring value CPr6. Funds of knowledge CPr7. Civic engagement in math learning CPr8. Discourse and open interaction CPr9. Curriculum platform CPr10. Curriculum deliberation	Curricular process	Belbase et al. (2022), Ndimande (2010), German (2021), National Council of Supervisors of Mathematics and TODOS (2016), Aguirre et al. (2013), Bright et al.2(015), Gutstein & Peterson (2013), Horn (2012), Walker (1970)
PP1. Constructivist teaching and learning PP2. Varieties of teaching methods PP3. Community of practice PP4. Discourse-based teaching PP5. Realistic mathematics teaching PP6. Constructivist teaching PP7. Equity-based teaching PP8. Transformative teaching PP9. Culturally specific pedagogy PP10. Empowering marginalized PP11. Participation of students PP12. Opportunity of learning PP13. Empowering low achievers PP14. Teacher and student commitment PP15. Restrictions of race, class, and gender PP16. Students' needs PP17. Inclusive practices PP18. Social and environmental justice PP19. Build identities	Pedagogical process	Von Glasersfeld (1995), Bas & Kivilcim (2021), Carpenter et al. (2001), Franke & Kazemi (2001), Andersson & Valero (2016), Alro & Johnsen-Hoines (2016), Richardson (2001), Gutstein, (2006), Seda (2008), Sriraman (2008), Leonard (2019), Hytten & Buttez (2011), Hytten & Buttez (2011), Keddie (2012), Allsup & Shieh (2012), Sanjakdar et al. (2022), Buell & Shulman (2019), John (2020), Wright (2020), Aguirre et al. (2019), Fabrega (2019)
AP1. Effect of assessment AP2. Positive and negative sides of assessment AP3. Normative and formative assessment AP4. Assessment for learning AP5. Flexibility and fairness AP6. Developing creativity AP7. Project-based learning and assessment AP8. Inclusive assessment AP9. Transformative agency AP10. High-quality instructions and assessments AP11. Equity-based assessment AP12. Assessment for achieved curriculum	Assessment process	Wonnacott (2011), Autin et al. (2015), Bennett (2011), Mawarti & Nurlaelah (2020),McArthur (2015), Roman et al. (2021), McHugh (2015), Gutstein (2006), Turner & Font Strawhun (2013), National Governing Association Center for Best Practices & Council of Chief State School Officers (2010), NCTM (2000), Wager & Stinson (2012)

teaching for understanding (Carpenter et al., 2001), a community of practice learning (Franke & Kazemi, 2001), discourse-based and dialogic teaching (Andersson & Valero, 2016; Vithal, 2003), realistic mathematics teaching (Alro & Johnsen-Hoines, 2016; McNeal, 2001), facilitative teaching (Nelson, 2001; Richardson, 2001), constructivist teaching (Ernest, 2016; Richardson, 2001), equity-based teaching (Gutstein, 2006; Seda, 2008), transformative teaching (Sriraman, 2008), and culturally specific pedagogy (Leonard, 2019). These pedagogical strategies are linked to a socially just pedagogy, which encompasses a call for social change, a participatory vision of education, multicultural practices, empowerment of marginalized students, full participation of all students (Hytten & Buttez, 2011), and equal educational opportunities regardless of race, gender, or socioeconomic status (Keddie, 2012).

Equity-based pedagogy is particularly crucial for empowering marginalized and low-achieving students (Allsup & Shieh, 2012), as it aims to narrow the achievement gap by fostering an equitable environment, access, and participation in the classroom (Sanjakdar et al., 2022). This approach is shaped by institutional requirements, accreditation standards, and personal philosophies, leading to a curriculum and methodology that transcends the confines of race, class, and gender (Buell & Shulman, 2019) and challenges power dynamics, racism, and classism (Cochran-Smith, 1999).

To implement these changes, professional development for teachers is essential, focusing on promoting equity and the role of mathematicians in a democratic society (Buell & Shulman, 2019). Teachers who adopt a socially just pedagogy can tailor their teaching to meet students' needs through collaborative, discursive, and inquiry-based methods, establishing inclusive practices (John, 2020; Wright, 2020). They should design mathematical tasks that are cognitively stimulating, socially engaging, and methodologically varied, addressing key social and environmental justice issues (Aguirre et al., 2019; Freire, 1970; Gutstein, 2006). Ultimately, mathematics teaching should be child-centered, contextual, meaningful, and interactive, enabling students to construct knowledge, examine biases, and develop their identities within a culturally responsive educational framework (Panthi et al., 2018a & 2018b; Fabrega, 2019; Gay, 2002).

The Assessment Processes in Mathematics Education for Social Justice

Assessment practices in mathematics education are pivotal in shaping students' educational experiences and their development as citizens. These practices can have varied impacts based on students' race, gender, ability group, culture, and socioeconomic status, potentially influencing their opportunities and fair share of educational outcomes (Wonnacott, 2011). The assessment process can be viewed through two lenses—normative and formative. Normative assessments may segregate students by merit, adversely affecting those underperforming, while formative assessments offer a more interactive and reflective approach to learning, promoting social and economic well-being (Autin et al., 2015; Bennett, 2011).

Formative assessments, characterized by contextual tasks and activities, serve as tools for social justice by providing flexibility and fairness in the evaluation process, thereby enhancing students' mathematical learning (Mawarti & Nurlaelah, 2020; McArthur, 2015; Widjaja, 2013). Project work, for instance, can foster in-depth learning, mutual respect, and creativity among students (Roman et al., 2021). Project-based learning (PBL) bridges classroom learning with real-world applications, supporting inclusive pedagogy and empowering students with transformative agency (Gutstein, 2006; McHugh, 2015; Turner & Font Strawhun, 2013).

The curriculum manifests in three forms: intended, implemented, and achieved (Phaeton & Stears, 2017). The intended curriculum outlines a vision for incorporating social justice into teaching, learning, and assessment practices (UNESCO International Bureau of Education, 2022). The Common Core State Standards for Mathematics (CCSSM) emphasize problem-solving, perseverance, abstract reasoning, and logical argumentation (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), serving as guidelines for high-quality mathematics instruction and assessment.

The National Council of Teachers of Mathematics (NCTM) articulates a vision for social justice in mathematics curriculum standards, advocating for inclusion, equity, and high-quality support for all students (NCTM, 2000). Interest in social justice within mathematics education is increasing, though variations exist between the intended curriculum and its implementation and the achieved outcomes (Wager & Stinson, 2012).

The journey towards a socially just mathematics education is complex and challenging. Discrepancies between the visions of curriculum authorities and the practical understanding and implementation by mathematics teachers can create obstacles. Exam directives and priorities may exacerbate these challenges, leading to high-stakes outcomes that undermine the essence of process-based learning and raise concerns regarding social justice in mathematics education.

Problems of Social Justice in Mathematics Education

The integration of social justice into mathematics education, despite increasing interest and dedication, faces complex challenges in the reform and execution of classroom plans, policies, and actions. The process of aligning social justice principles with mathematics content and pedagogy presents several obstacles. To better understand these issues, they have been organized into two main categories:

Difficulties in Implementing Social Justice in Mathematics Education:

This category explores the practical hurdles educators face when attempting to incorporate social justice into their teaching practices.

Table 4

Problems of social justice in mathematics education

Primary Codes	Categories	Relevant Literature
DI1. Equity and fairness DI2. Critiques of social justice DI3. Political biases and provocation DI4. Implementing social justice DI5. Low performance DI6. Disadvantaged students DI7. Mainstream benefits DI8. Inclusion of racial minorities DI9. Tools and resources DI10. White middle class DI11. Issues developing countries DI12. Critical mathematical literacy DI13. No quick guide for equity pedagogy DI14. Student diversity	Difficulty in implementation	Alfrey & O'Connor (2020), Gewertz (2020), Harrison (2015), McName (2013), Seegmiller (2020), Turner, Gutierrez, Gutierrez (2012), Secada et al. (2003), Frankenstein (1997), Hodge (2006)
CT1. Teacher readiness CT2. Diversity and multicultural classes CT3. Different interests CT4. Different SES CT5. Lack of social justice in teacher education CT6. Decontextualized mathematics curriculum and textbooks CT7. Demands teacher dedication CT7. Mandated standardized tests CT7. Difficulty in maintaining balance in curriculum and context CT8. Daunting tasks for teachers CT9. Balancing math content with real-world application CT10. Complexity of social justice and equity in implementation	Challenge of teaching	Secada et al. (2003), LaDuke (2009), Acharya et al. (2021), Esmonde & Caswell (2010), Vomvoridi-Ivanovic & McLeman (2015), Luitel (2009), Luitel & Taylor (2005), Belbase et al. (2008), Bond & Chernoff (2015), Gregson (2013), Pennell (2019), Felton et al. (2016)

Challenges of Social Justice in Mathematics Education:

This category examines the broader challenges that impact the effectiveness of social justice initiatives within the educational system.

These categories, along with their associated codes, are detailed in Table 4, providing a structured overview of the complexities involved in realizing social justice within the context of mathematics education.

Difficulty in Implementing Social Justice in Mathematics Education

Social justice in mathematics education, defined as equity, fairness, and equality (Alfrey & O'Connor, 2020), is a multifaceted concept that hinges on the effective use of resources by teachers to ensure equitable student engagement. Achieving high levels of learning for all students while addressing social justice is challenging, with concerns that it may shift focus from cognitive to non-cognitive issues due to the politics of power, culture, and diversity (Gewertz, 2020). The integration of personal and political beliefs into mathematics teaching is fraught with the risk of political bias and controversy (Gewertz, 2020).

Teachers' understanding and skills regarding race, gender, and socioeconomic status are crucial in implementing social justice in mathematics education, as these factors influence the social constructs of equity, equality, and fairness within teaching (Harrison, 2015). Teacher education programs often promote a non-critical stance, yet social justice efforts tend to support marginalized groups, aiming to elevate disadvantaged students to equal educational standing (Harrison, 2015).

Curricula designed with social justice in mind must navigate the complexities of incorporating social, cultural, and political issues, which is a significant challenge

(McName, 2013). Many schools lack the necessary tools and resources for quality social justice teaching (Seegmiller, 2020). Additionally, teachers, particularly those from the majority class, may be ill-equipped to teach mathematics through a multicultural, equitable lens, complicating the implementation of social justice pedagogy (Turner et al., 2012). This difficulty is prevalent in developing countries, where introducing social justice into mathematics classes is problematic (Secada et al., 2003), possibly due to lack of resources, unfair distribution of available resources, and decontextualized mathematics teaching and learning.

Beyond teacher knowledge, other factors impede the realization of social justice in mathematics education. Even with adequate technology, resources, and culturally competent teachers, achieving true equity remains elusive (Colquitt, 2014). Classrooms must be equitable to foster critical thinking about mathematics from a social justice perspective (Colquitt, 2014). Critical mathematics literacy involves using mathematics to analyze and challenge injustices, and advocating for change (Frankenstein, 1997). However, there is a need for a comprehensive guide to equity pedagogy (Frankenstein, 1997). The distinction between equality and fairness is also highlighted, as students' diverse abilities, values, and motivations necessitate responsive teaching (Gutiérrez, 1999; Hodge, 2006; Rousseau & Tate, 2003), adding another layer of complexity to social justice in mathematics education.

Challenges of Teaching for Social Justice in Mathematics Education

In the current landscape of standards-based educational reform, intellectuals are finding it increasingly difficult to integrate a social justice agenda. The emphasis on high-stakes testing and standardized assessments tends to marginalize content that is not tested, stifling the creativity of both teachers and students (Um, 2019). The challenges of upholding social justice in mathematics education are manifold and include:

- Teacher Readiness: Educators must be prepared to apply socially critical pedagogies, a commitment that requires ongoing dedication (Bond & Chernoff, 2015; Secada et al., 2003).
- Student Diversity: Multicultural classrooms present unique challenges in aligning mathematics content with the varied personal and family interests of students (LaDuke, 2009; Acharya et al., 2021).
- Socioeconomic Factors: Students' socioeconomic backgrounds influence their access to mathematics education (Esmonde & Caswell, 2010).
- Teacher Education: Training programs need to equip teachers with the skills to address social justice issues within their teaching (Vomvoridi-Ivanovic & McLeman, 2015).
- Curriculum and Pedagogy: The content and methods of teaching must be contextualized to reflect social justice principles (Belbase et al., 2008; Luitel, 2009; Taylor & Luitel, 2005).

The pressure to prepare students for mandated tests complicates the implementation of teaching for social justice (Gregson, 2013). Teachers struggle to find mathematics content that aligns with social issues and often require more time to prepare equity-based lessons. Additionally, the transient nature of student populations in schools adds to the complexity of maintaining a consistent approach to social justice education.

Implementing an equity pedagogy, critical pedagogy, and social justice pedagogy in mathematics requires flexibility to adapt plans and activities to the evolving dialogues and debates in the classroom, which can be challenging for teachers (Pennell, 2019). Equity pedagogy in mathematics can be fairness in classroom participation, engagement, and assessment, and equitable access to school resources for all students. Critical pedagogy in mathematics education can be a method of teaching and learning mathematics that enables and empowers students to question unfairness and injustice in society through mathematical discourse. Likewise, social justice pedagogy can be an approach to mathematics teaching and learning promoting fairness, equity, equality, and justice. In this context, mathematics teachers who believe in the value of incorporating diverse cultures and experiences into their lessons are more likely to integrate considerations of race, class, gender, disabilities, and critical social issues into their teaching. However, fostering such beliefs is a challenge, necessitating teacher development programs and professional development activities that provide real-world context and examples (Evans, 2013).

Reflection, Implication, and Conclusion

In our reflection on the four paradigmatic themes of social justice in mathematics education, we delved into the implications and synthesized our perspectives. We have dissected the concept into four key areas: prospects, priorities, processes, and problems. Social justice as a movement within mathematics education is deeply rooted in historical social and political movements advocating for educational reform. This movement encompasses the standards movement, realist mathematics, and ethnomathematics, which have significantly influenced teacher awareness but have had a limited impact on actual teaching practices and student outcomes, particularly for marginalized groups. To foster equitable pedagogy, mathematics teachers must employ innovative teaching methods, integrating technology and other resources to ensure fairness, equality, and equity.

Curricular and pedagogical reforms are central to social justice in mathematics education. Efforts across various countries have led to changes in content alignment and distribution across grade levels. However, these reforms have inadvertently perpetuated segregation and performance-based student division, failing to bridge gaps across race, gender, and economic status. Despite the transformative intentions of these reforms, their full potential has yet to be realized in classroom pedagogy. The processes supporting social justice in mathematics education are underpinned by curricular, pedagogical, and assessment practices. The curriculum, often viewed by teachers as a neutral document, outlines content without considering the broader social context. A robust and relevant curriculum should facilitate high-quality mathematics learning, drawing from students' backgrounds and knowledge. Teachers should engage students in civic responsibilities, encouraging them to address community issues through mathematics. Collaborative platforms are essential for stakeholders to share beliefs and priorities, shaping a curriculum that responds to job market needs, scientific advancements, and societal challenges.

Implementing social justice in mathematics education is fraught with challenges, including teacher readiness, student diversity, socioeconomic factors, and the nature of teacher education programs. High-stakes testing and standardized assessments further complicate the landscape, limiting teachers' ability to foster creativity and critical thinking. Pedagogical practices must empower students, providing equitable learning opportunities and dismantling barriers related to race, class, gender, and ability. Assessment practices should be reformed to evaluate students holistically, reflecting their capabilities and potential rather than conforming to a uniform standard.

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Secondary Mathematics Teacher Decision-Making and Their Selection of Digital Materials

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ABSTRACT As online spaces for mathematics content and lesson plans become more accessible, teachers are exploring these websites to make decisions about instructional materials for their courses. These resources then have an impact on what is enacted in the classroom and what opportunities students have to learn mathematics. One of the most popular online spaces teachers frequent is Teachers Pay Teachers (TpT). Using the Remillard and Heck's model on curriculum enactment, this study investigates secondary mathematics teachers' decision-making process and the heuristics they rely on when choosing mathematics content from online spaces, namely TpT due to its extensive use by educators. Findings suggest that these participants exhibit reliance on the availability, representativeness, anchoring and adjustment, and groupthink heuristics.

KEYWORDS *mathematics teacher education, digital curriculum resources, heuristics, TeachersPay Teachers (TpT), secondary mathematics instruction*

Introduction

Teachers across the United States are increasingly using resources from the internet as they plan their lessons, despite the fact that many states, districts, and schools have policies in place listing approved or mandated curricular materials, such as textbooks, prescriptive curricula, and district-level materials (Gewertz, 2015; Pittard, 2017; Timberlake et al., 2017). A 2019 RAND Corporation *American Instructional Resources Survey* (AIRS) of a nationally-representative sample of teachers showed that 88% of the survey's respondents consult online sources as they plan for instruction, including Google, Pinterest, Facebook, and Teachers Pay Teachers (Tosh et al., 2020). In another survey, 56% of mathematics teacher respondents specifically referenced Teachers Pay Teachers (TpT) for digital materials on a weekly basis (Doan et al., 2020). These resources have an impact on what is enacted in the classroom and what opportunities students have to learn mathematics.

Teachers point to a host of benefits from online resources, including improved content knowledge and pedagogical content knowledge, increased self-efficacy, greater understanding of social justice and equity issues, and relationships with educators outside of their schools and districts (Carpenter et al., 2020; Shelton & Archambault, 2018). Yet, researchers have found evidence that unofficial resources from online spaces may not be standards-aligned, developmentally appropriate, culturally relevant, or even accurate (Gallagher et al., 2019; Greene, 2016). For instance, Sawyer and colleagues (2019) found that within the 500 most popular elementary mathematics "pins" on Pinterest, 98% were of low cognitive demand and focused on classroom aesthetics. This implies that teachers need to be vigilant about the quality and alignment of unofficial materials they find on online spaces, so that students are not exposed to inaccurate or inappropriate curricular materials.

Teachers make many decisions throughout their lesson planning process based on their content and pedagogical content knowledge to determine which curricular resources to use in their classrooms. There is still much to understand about the ways in which teachers make these decisions using online spaces. This paper aims to investigate teachers' decision-making processes and the heuristics (i.e., mental shortcuts) they rely on to identify suitable materials to use with their students from online lesson plan sharing websites (Doan et al., 2020; Kaufman et al., 2020; Sawyer et al., 2019; Shapiro et al., 2019).

Background

As online spaces for mathematics content and lesson plans become more prevalent, teachers are exploring these websites to make decisions about instructional materials for their courses (Greene, 2016; Pittard, 2017; Tosh et al., 2020). These online spaces may include state-sponsored websites like EngageNY or third-party sites such as Pinterest, Instagram, Facebook, Twitter, and TpT (Sundrani, 2021). The types of content teachers search for range from downloadable scavenger hunts to complete year-long curricula with notes, activities, homework, and assessments. The sources mathematics educators reference may vary based on need, leading them to use a variety of spaces as indicated by Prado Tuma and colleagues (2020; see Table 1).

One of the most popular online spaces teachers frequent for lesson plans is Teachers Pay Teachers. TpT is a platform that allows teachers to share their original content on the website for other educators to view and download, either for free or for a listed price. The materials posted range from single worksheets to an entire year's worth of units and assessments for the subject area. Teachers search key terms to identify what they are looking for, read a description for the listing once clicking on it, view snippets of the content through the "preview" feature, and then buy the material they feel matches their aims. TpT allows teachers to create and share original content that they feel may be more appropriate for their students than those disseminated by their district or school. Currently, the site does not vet the lessons that are posted for purchase or free download, so all ratings, downloads, and reviews are wholly based on teachers' experiences with the materials (Gallagher et al., 2019; Greene, 2016). However, this also means teachers are unable to truly identify the depth at which these items cover the concepts they hope to teach, nor is there any verification of alignment to their state standards (Polikoff & Dean, 2019). Unfortunately, teachers may not know this until they have already chosen to purchase these curricular materials or may not have the content or pedagogical knowledge to assess the quality of the materials selected. Time constraints can also complicate teachers' decisions to download certain materials (Gallagher et al., 2019). Despite these issues, TpT has gained more visibility over the past decade as they report that "more than 2 out of 3 U.S. teachers" use their platform (About Us | Teachers Pay Teachers, n.d.).

Table 1

Mathematics Educators Most Used Digital Planning Sources in the 2018-2019 Academic School Year (N=2,015)

Online Space	Relative Frequency
Teachers Pay Teachers	56%
Search Engine (e.g., Google)	35%
Pinterest	23%
Common Core State Standards Initiative	22%
Kuta Software	18%
State Department of Education Websites	14%

Theoretical Framework

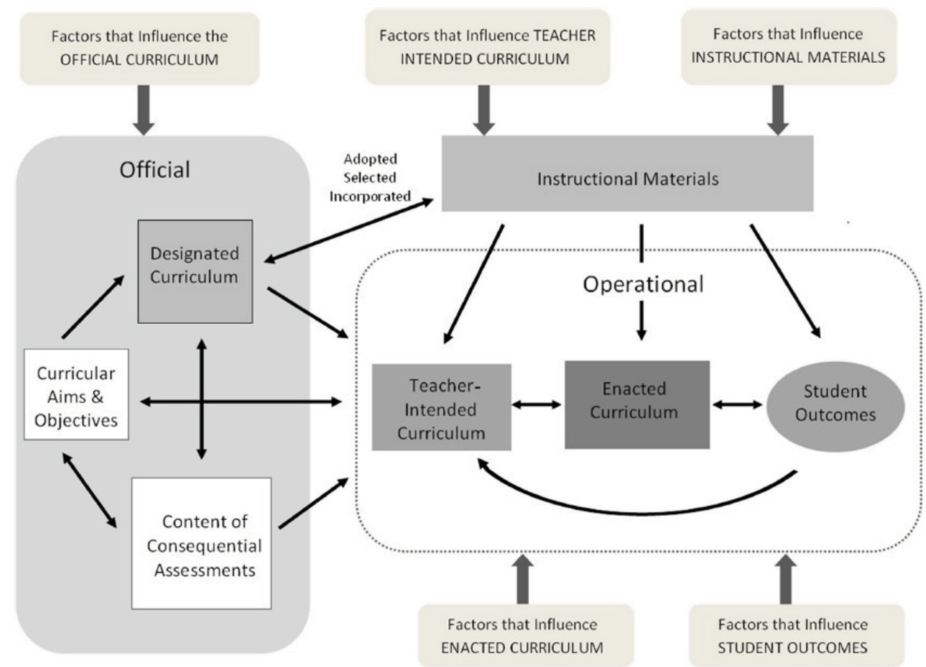
This study draws from the theoretical framework of teacher decision-making to investigate how teachers choose and use online curricula. Remillard and Heck's (2014) model of the curriculum enactment process provides further insight into which factors may guide teachers' decision-making when using their selected materials.

Teacher Decision-Making

Teachers are constantly making decisions related to their curriculum and instruction (Herbst et al., 2016; Holstein & Keene, 2013; Schoenfeld, 2015; Shavelson & Stern, 1981; Smith et al., 2018). During the planning phase of the curriculum enactment process, teachers' decisions focus on balancing activity flow, predictability during the lesson, choice of content, student needs, and instructional style, all while taking into account external pressures from state and local policies, administrators, or other educators (Shavelson & Stern, 1981). According to Remillard and Heck (2014), these external pressures represent the official curriculum; each has a direct or indirect effect on what materials the teacher uses in their classroom. The operational curriculum is where teachers are able to engage in cognitive processes and factors that are influenced by their beliefs, content knowledge,

Figure 1

Model of Curriculum Enactment Process (Remillard & Heck, 2014)



pedagogical content knowledge, and judgement (Shavelson & Stern, 1981). The complexity of official and operational curricula are represented in Remillard and Heck's (2014) model on the curriculum enactment process (see Figure 1). This paper focuses on the factors that influence the teacher-intended curriculum that feed into the operational curriculum within this model. Teacher-intended curriculum focuses on the planning phase of curriculum enactment where teachers take any components of the official curriculum and align them with their curricular materials and pedagogical goals.

Mathematics Educators Decision-Making

Mathematical decisions are "those decisions that influence students' opportunity to learn mathematics, and teachers' reasoning for those decisions" (Dingman et al., 2019, p. 44). Researchers have explored what decisions mathematics teachers make during the planning and enactment of lessons and the drivers of these decisions, including mathematics content knowledge, pedagogical content knowledge, and teacher beliefs (Bush, 1986; Choppin, 2011; Dingman et al., 2019). However, external factors can have an impact on mathematics teachers' decision-making in planning and implementation of lessons. For instance, teachers may be encouraged by other teachers or administrators to focus on basic skills to improve standardized test scores (Lu et al., 2021), or teachers may have difficulty planning for mathematics instruction that balances the various content objectives with the amount of time they have with their students (Leong & Chick, 2011).

Potential Obstacles to Teacher Decision-Making

Educators are expected to make a multitude of decisions throughout the curriculum planning process. Unfortunately, teachers often are not provided adequate planning time and resources (Tichenor & Tichenor, 2019), which may lead them to rely on heuristics including availability, representativeness, anchoring and adjustment, and groupthink when making decisions (Jaeger, 2020; Tversky & Kahneman, 1974). Pulled from the field of cognitive psychology, these heuristics can be applied to teachers' lesson planning processes. See Table 2 for a definition of each decision-making heuristic.

The availability heuristic places importance on the perceived frequency of an event or stimulus, which may lead individuals to incorrectly predict an outcome of an event based on previous experiences, salient memories, or other readily available examples (Greening et al., 1996). This heuristic has been used in a variety of

Table 2

Definitions of Decision-Making Heuristics

Heuristic	Definition
Availability	Individual places importance on the perceived frequency of an event based on recent experiences or salient memories
Representativeness	Individual links the occurrence and characteristics of an event to another based on existing prototypes or stereotypes
Anchoring and Adjustment	Individual begins with an initial value or assumption and adjusts that value to get to a final answer or result
Groupthink	Individual forms a unified approach due to external pressures, such as other peoples' actions or beliefs.

studies about decision-making such as financial analysis (Kliger & Kudryavtsev, 2010; Lee et al., 2008), recall and prediction of events (Greening et al., 1996; Kamiya & Yanase, 2019), and politicians' judgements (Vis, 2019). For instance, Eisenman's (1993) study revealed that although the rate of drug usage in the United States since 1972 was decreasing, college students believed that this rate was actually increasing because of other messaging. In the context of education, this heuristic may lead teachers to use lessons with contexts they are familiar with through teaching strategies and sources they have been exposed to recently, such as teaching slope through the analogy of two runners in a race showing up in multiple teacher-created materials.

The representativeness heuristic links the occurrence and characteristics of an event to another. In a social context, the representativeness heuristic leads individuals to compare individuals or events to a prototype or stereotype (Jones, 1995). Anglada-Tort (2019) asserts that musicians may make evaluative claims about certain compositions based on their preconceived notions about musical genres, as opposed to the composition's merit. Using this heuristic, a teacher may be drawn to the strategy of "drill-and-kill" because they were exposed to that type of environment throughout their schooling. This belief could lead to a reproduction of a behaviorist approach to teaching mathematics.

The anchoring and adjustment heuristic involves adjusting an initial value to get to the final answer (Parmigiani, 2012; Tversky & Kahneman, 1974). If the anchor is based on weak evidence or a faulty assumption, the adjustment will result in an incorrect answer (Fortune & Goodie, 2012). For example, Siddiqi (2018) references the top three information technology firms in the S&P 500 index, the most successful of which is Apple. He asserts that "a typical investor may start from Apple and then attempt to make appropriate adjustments to form judgments about the other two firms" (Siddiqi, 2018, p. 250). Because of the large gap between the earnings of Apple versus the other two firms, these adjustments fall short of appropriately understanding the earning potential of these other two companies. An educator could enact this heuristic by making faulty assumptions about students' ability levels and planning a lesson that does not meet the students' needs.

It is well-documented that teachers have been shortchanged on common planning opportunities (Ross, 1993; Tichenor & Tichenor, 2019). Therefore, teachers may hasten their decision-making process in collaborative spaces by using "groupthink" – the individual's desire to form a unified approach due to external pressures (Jaeger, 2020). Clark (2013) found that while preservice social studies teachers were able to articulate sound pedagogical reasons for using graphic novels to engage their students in conversations about social issues, they admitted that they would likely not utilize them out of fear of rejection by teachers and parents. This desire to conform was so pronounced that the preservice teachers did not even consider the idea of using graphic novels, despite no indication of who their future colleagues and parents would be.

Technology serves as an additional dimension within decision-making, as teachers may connect with individuals in online spaces when accessing digital curricular resources. Given the complex nature of the curriculum enactment process and decision-making that must occur prior to instruction, applying the four aforementioned heuristics to decision-making in online spaces may assist researchers in understanding teachers' rationale for choosing instructional materials.

The purpose of this study is to understand secondary mathematics teachers' decision-making process when choosing mathematics content from online spaces. This study will focus on TpT due to its extensive use by educators. The following research questions guided this exploration.

1) How do secondary mathematics teachers choose what online materials to use to teach and/or supplement their curriculum?
2) How do secondary mathematics teachers rely on heuristics to guide their decision-making process when choosing online materials from Teachers Pay Teachers for their instruction?

Methods

To answer the research questions, this study employed a multiple case study method (Yin, 2009) with three participants. The TpT platform was chosen as the focus of each participant's heuristic use in this study as its use is most prevalent amongst mathematics educators (Tosh et al., 2020). Recruited participants self-reported using TpT at least once per instructional unit.

Table 3

Participant Details

Participant	Details
Patrick	Secondary Geometry and Algebra II teacher at a public school in Texas
	White male under the age of 40 and holds a secondary mathematics teaching certificate and bachelor's degree
	School district provides teachers with mathematics curriculum but does not require educators to use them
	Uses Teachers Pay Teachers because he believes it is a time saver and can fill in any gaps he has in instruction, such as providing additional practice for students or mathematics-related games
Nancy	Secondary mathematics teacher at a public K-12 school in Illinois, where she teaches a number of grades 6-12 mathematics subjects including Algebra I and Statistics
	White female under the age of 40 and holds a secondary mathematics teaching certificate and a master's degree
	School uses a teacher-selected set of textbooks and accompanying online modules that she helped choose, so she uses this set for most of her mathematics curriculum
	Uses Teachers Pay Teachers because she believes it is a safe and reliable site to find supplementary materials
Jack	Secondary Algebra I teacher at a public high school in Nevada
	White male over the age of 40 and holds a secondary mathematics teaching certificate and a master's degree
	School provides a textbook and the district maintains a website that houses curricular materials, but teachers are not required to use any of these materials
	Uses Teachers Pay Teachers because of the types of activities that are sold on the website that he cannot find through his district and school-provided content

Participants

Three in-service secondary mathematics teachers (see Table 3) were selected to take part in a semi-structured interview through voluntary participation. Participants for the study were solicited through Facebook Groups including "Secondary Math Teachers" and "Algebra Teachers" and through snowball sampling. All names are pseudonyms.

Data Analysis

To answer the posed research questions, participants were asked about instructional resources provided by their school or district, how they use the internet to plan for instruction, and how they use TpT to plan for instruction. The interview transcripts were then open coded at the response level to find instances related to decisions made in online spaces and within the TpT platform to plan for instruction (Saldaña, 2013). Responses were then categorized to identify which online spaces the participants used, the specific types of materials selected, and their process for selecting these materials. In a second round of coding, excerpts were labeled using the a priori codes of availability, representativeness, anchoring and adjustment, and groupthink based on the heuristic definitions provided in Table 2. The operationalizations of these codes were guided by the literature and participant responses.

Decision-Making Heuristics in Education

Because availability, representativeness, anchoring and adjustment, and groupthink have been traditionally used in the field of cognitive psychology, the following operationalizations for the field of education determined by the literature and respondent excerpts are provided (see Table 4). The availability heuristic consists of responses that associate certain types of activities and lessons with other educators or recent examples, whereas the representativeness is operationalized as quality of materials in reference to or alignment with instructional objectives. The anchoring and adjustment heuristic is presented as additional contextual characteristics not directly associated with pedagogy that may impact a teacher's decision to download certain digital materials or not. Lastly, groupthink is operationalized as any reference to other individuals and their knowledge or insights when selecting digital materials to purchase and use.

Table 4

Operationalization of Online Decision-Making Heuristics

Heuristic	Observable Descriptions
Availability	I like to use scavenger hunts from Teachers Pay Teachers because I've seen they get students up and moving.
	I was looking for a specific lesson I've seen another teacher use before.
Representativeness	I was looking for worksheets with a lot of problems, so the students could practice how to solve multi-steps equations.
	The listing's description stated that the assignment focused on the standard I was getting ready to teach.
Anchoring and Adjustment	I had a feeling my students were going to be tired after taking a standardized test, so I was looking for a fun activity to do with them.
	My department chair told me that the students in my class were at risk for failure, so I wanted a lesson at a lower level.
Groupthink	The lesson I was looking at had a lot of really great reviews saying things like they were glad they purchased it.

Results

In the following sections, I will present results related to each teacher's use of online spaces overall and highlight the ways the participants exhibited each of the four decision-making heuristics.

Teachers' Use of Online Spaces

Aside from TpT, the only other online spaces the participants identified as planning resources were Google, Facebook, and Desmos. When using Google, Patrick indicated that he used simple searches and clicked on results that he found interesting. On Facebook, Jack is a part of educator groups that discuss curriculum and instruction. When members of these group write posts, they appear on Jack's Facebook Timeline. When posts with materials attached or linked appear on his timeline, he downloads those resources. On Desmos, Jack uses the search bar and filters to look for lessons that align with his topic.

However, on TpT, the teachers have more control in their searches. Patrick and Jack stated they begin with a topic and sometimes select an activity type filter. Then, they filter their selections by topic alignment and view individual item listings. Patrick uses the preview provided for the listing to see if the item looks organized and appropriate for his classroom. Finally, Patrick references reviews on the listing. Nancy identified TpT as the only online space she frequents to find supplementary materials for her classroom. When asked about her reasons for using TpT, Nancy stated that she believes that TpT is a safe and reliable website. She elaborated by giving the example of a YouTube video that might align with her objectives for the first two to three minutes, but then "something in your classroom pops up, and it's unacceptable." While Nancy did not elaborate, it was apparent that she felt there she did not have enough control over what would be presented to students through YouTube. Nancy also pointed to the multiple filters on TpT that allow her to look for resources based on subject area and standard.

Teachers' Use of Decision-Making Heuristics

Each of the teachers in this study approached their selection of digital materials from TpT in unique ways and therefore exhibited the four decision-making heuristics in distinct ways and to varying degrees. In the sections below, I detail how each teacher relied on these heuristics based on their pedagogical goals.

Availability

During his interview, Patrick exhibited traits of the availability heuristic in two ways. The first way was that Patrick pointed to specific types of resources that he looked for on TpT, including mathematics mazes and scavenger hunts. He associates these types of activities and other games with opportunities for students to practice certain skills. Patrick also often selected lessons from the "All Things Algebra" TpT page, as that seller is often at the forefront of his mind, showing up as one of the top results in his TpT searches; he specifically mentioned this profile twice during the interview but did not point to any other sellers on TpT. One of the most salient ways Nancy exhibited the availability heuristic was through her desire to find resources that could be used online, especially Google Forms. Because her required curriculum has an online component, she is drawn to supplementary materials that use the same modality. Lastly, Nancy uses the "favorite" button on TpT to mark items that she has purchased and enjoyed using, so she can return to those sellers to find additional materials for use in her classroom; in this way, she prioritizes sellers based on previous materials and associates them with high quality materials. Throughout his interview, it was apparent that much of Jack's decision-making process relied on the availability heuristic. Most of his focus when using online lesson plan sharing websites was the availability of activities – scavenger hunts, gallery walks, and games. He believes that activities like these are more interesting, as well as "a little different and [give] a little freedom." He also uses TpT to purchase test banks for ExamView, as using these test banks have saved him time while creating assessments for his students.

Representativeness

During his interview, Patrick stated outright that the most important aspect of searching for curricular materials from online spaces such as TpT is alignment to instructional goals. From there, he looks for resources that he likes and believes will "fill a hole" in his lesson based on previews of items. Patrick admits that, "I probably have passed on good resources because the preview is garbage, but if I can't tell what it is, I'm not spending money on it." If he perceives a resource to not be representative of his conception of a good quality material (e.g., aligned to standards, building on existing lessons and assignments), he does not purchase the item. Nancy also places importance on the representativeness heuristic, as she focuses her attention on TpT lessons that are aligned to her standards. Therefore, any resources she chooses are representative of appropriate content for her students. Additionally, she methodically reviews the description and preview of any listing from her search results as she makes the decision to purchase the item. She believes that the preview gives her a more realistic view of what she is purchasing, and it is this information that is the most important on a TpT listing. Lastly, Nancy believes the ability to provide instantaneous scores and feedback is representative of a timely and efficient resource, which Google Forms allow her to do. This way, she can provide additional help to her students and allow them to redo the assignment to improve their content understanding. While the representativeness heuristic was not a major guiding factor in Jack's decision-making process, Jack mentioned difficulty levels of problems multiple times throughout the interview as a proxy for high-quality instructional materials, signaling that his perception of the rigor of an activity is representative of a high-quality activity.

Anchoring and Adjustment

The primary way Patrick relies on the anchoring and adjustment heuristic is his concern for appearance of TpT items. Based on the listing's preview, Patrick assesses the activity's flow, pedagogical approach, logic, and readability. Patrick does admit that he may still end up choosing a digital resource on TpT that is not aligned with his pedagogical goals. However, TpT is the first website Patrick visits during his lesson planning process, indicating that his perception of high-quality materials is set based on the types of curricular resources offered on the TpT platform. As stated earlier, Nancy follows sellers on TpT based on her experiences with previous materials she has purchased and enacted in her classroom. If she sees these sellers in subsequent searches, she is more likely to purchase materials from them because of her past interactions with them. Therefore, Nancy considers these sellers to be producers of high-quality resources and returns to their profiles for future materials instead of starting entirely new searches. Interestingly, in this way, Nancy relies on both the availability and anchoring and adjustment heuristics simultaneously when selecting TpT content based on seller profiles. Jack did not show significant signs of using the anchoring and adjustment heuristic during his interview.

Groupthink

During his interview, Patrick referred to himself as a novice teacher and points to materials on TpT as "leveraging like other people's experience and knowledge that you just don't have yet." Patrick is drawn to TpT because it is a website solely devoted to "good teachers who have been doing it [creating instructional resources], helping out those who need the support." This view of TpT suggests that Patrick uses the website as a way to connect with more experienced educators and to borrow their instructional materials and methods. He also believes that other teachers have already vetted these materials, so he is more likely to find what works based on other people's reviews. Furthermore, Patrick does not rely on star ratings that TpT includes on their website because he prefers to preview the materials and read about other teachers' experiences using the resources instead of un-contextualized star ratings. While Nancy did not mention anything related to the groupthink heuristic, Jack made many statements that support his use of the groupthink heuristic when making decisions to use curricular materials from TpT. He mentioned on several occasions that he uses ratings and reviews to make quick decisions to download materials. He stated that there have been a few times when he has not liked using materials he purchased, but a majority of the time believes he has found high-quality materials. When asked to elaborate on how he uses ratings and reviews, Jack said that as long as the activity is aligned with his topic and has a rating of four stars or higher, he will purchase the item. If the rating is below four stars, then he will read reviews to understand why and make his final decision to download an item listing or not. This, coupled with his attraction to using the same sellers' content, shows that Jack places most of his trust on other teachers' opinions.

Discussion

Overall, much of the participants' selection process using lesson plan sharing websites differed by the affordances and constraints of each type of website. For instance, Google, while a helpful tool to search for information, was seen as leading to an overwhelming number of results. Desmos can also provide interactive and rigorous lessons, but the search tool on the website did not always include all pertinent results. Part of the appeal of Teachers Pay Teachers may stem from its user-friendly interface. Compared to other lesson plan sharing websites, the TpT user experience was seen as more customizable and streamlined, aimed at leading teachers to their desired instructional material.

In all three interviews, the participants, reflecting the availability heuristic, voiced a desire to find certain types of activities to use in their classrooms because of a perceived benefit based on previous experiences. In addition, all three participants referenced returning to certain sellers to purchase more materials from their storefronts because they had positive experiences using those sellers' resources in the past.

The representativeness heuristic most often appeared as an implicit or explicit alignment to learning objectives or standards. Additionally, in Patrick and Jack's interviews, both participants stated that they look at item previews to ascertain the rigor level of the resources and whether the problems included are representative of their students' ability levels. Interestingly, each respondent has very specific, albeit different, goals associated with the types of activities they searched for on TpT: Patrick generally looks for activities that will provide additional practice for his students; Nancy prioritizes activities that provide instantaneous feedback, while Jack places importance on when, during a specific unit, the activity will be used (e.g., on the second day of practice with a skill).

The anchoring and adjustment heuristic was initially operationalized to include statements that focus on the use of certain materials based on anchor item previews or descriptions. While this was true in Patrick's case, this initial operationalization did not fully capture how Nancy and Jack exhibited this heuristic. These two participants anchor themselves to specific sellers on TpT and then often adjust their searches based on who they follow and place more trust in their resources over those of other sellers.

Although Nancy did not show a reliance on the groupthink heuristic beyond returning to specific sellers' profiles, Patrick and Jack mentioned in their interviews that they do look at other teachers' ratings of sellers' materials. Patrick places more importance on reviews, while Jack is more attracted to overall ratings of items. Ultimately, each of the participants exhibited this heuristic in different ways. Additional research may be needed to fully understand the social dynamics of online lesson plan sharing websites so that teachers can become more critical consumers of the content they find in these spaces.

It is also important to note that interview responses indicated each heuristic is not mutually exclusive. For instance, Patrick named types of activities he looks for from TpT and also gave insight into how he believes scavenger hunts are representative of high-quality materials in the same statement. Future research may delve deeper into a teacher's rationale for using specific types of activities from online spaces to understand which heuristic(s) are at play.

Conclusion

Many mathematics teachers use the internet to assist them in planning for instruction, often when their current curricular resources do not provide enough content to fully cover the topic at hand. When used appropriately, the internet can help teachers make decisions on how to engage students in content, plan and enact culturally responsive lessons, and implement meaningful assessments. This study adds to the research base that teachers may find certain online spaces more appealing than others due to their user interface. In addition, secondary mathematics teachers may rely on heuristics when choosing content from online lesson plan sharing websites but could do so to differing degrees. Future research should continue to explore the evolving landscape of lesson plan sharing websites, the ways secondary mathematics educators use these spaces to prepare and enact lessons, and the decision-making processes teachers use when selecting materials from online spaces, so that mathematics teacher educators and teacher leaders can prepare pre-service and in-service teachers to be critical consumers of content found in these spaces.

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NOTES FROM THE FIELD

The Spring 2025 issue features three *Notes from the Field* articles that provide grounded perspectives from distinct contexts in mathematics education. In the first piece, Fernandez, Lahiere, & Leszczynski describe how they adapted the Launch-Explore-Summarize instructional routine for an online setting for a synchronous online geometry course for middle school teachers. In the second piece, Fletcher et al. provide support to early scholars in mathematics education by sharing a curated list of programs and opportunities, along with personal reflections and practical guidance regarding navigating the beginning years of a career as a mathematics education scholar. Lastly, Mogilski & Parry describe how they implemented a reflective homework system in undergraduate mathematics courses, encouraging more formative, student-centered assessment practices in postsecondary settings.

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NOTES FROM THE FIELD

Online Investigations of the Quadrilateral Hierarchy Using "Launch-Explore-Summarize"

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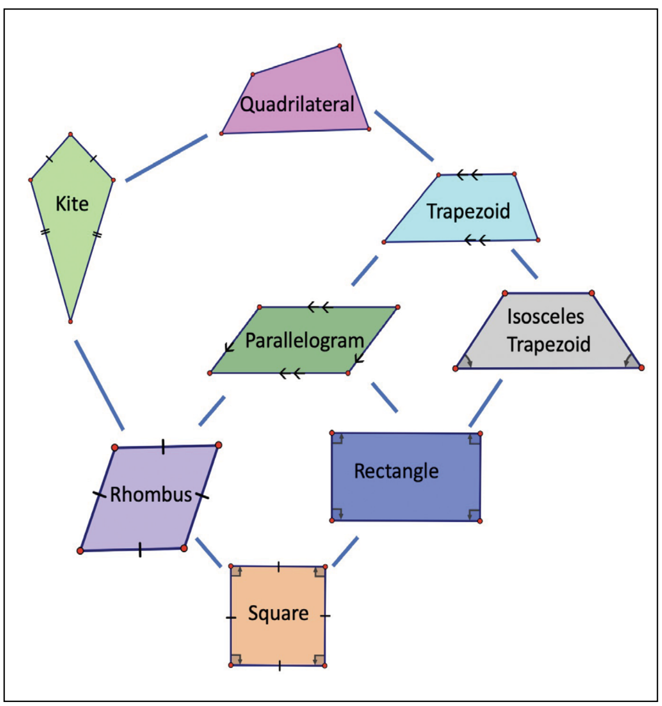
KEYWORDS Launch-Explore-Summarize (LES), online teacher education, quadrilateral hierarchy

Introduction

As three mathematics educators who have been working for years within different facets of online teaching, we often wonder about transitioning face-to-face methods to online platforms (Fernández & Leszczynski, 2019; Lahiere, n.d). For us, upholding constructivist philosophies that center on "the personal construction of meaning" and the negotiation of learners' "taken-as-shared" meanings (Wood et al. 1991, p. 591) presents new challenges when participants are interacting from distinct locations, introducing additional temporal and technological concerns. Nevertheless, we appreciate the opportunities that online learning provides to busy practicing teachers pursuing continuing education (Huang & Manouchehri, 2019).

FIGURE 1
Quadrilateral Hierarchy



One of our favorite face-to-face teaching frameworks is "Launch, Explore, Summarize" (LES) (Shroyer, 1984), which embraces constructivist principles by supporting inquiry and investigations of mathematical ideas through classroom interactions (Michigan State University, 2018). In the first part of this paper, we describe considerations for moving a face-to-face LES lesson on creating a *quadrilateral hierarchy* (Figure 1) to an online format in two geometry classes of 24 and 25 middle-school teachers. The mathematics teacher educator (MTE) (an author) used her institution's *synchronous* conferencing platforms (Conferences and Zoom) to run the lessons. In the second part of this paper, we use the online lesson recordings to describe implementation attempts, learning opportunities, and challenges that arose in the lessons to inform others who are interested in such transitions for their own practice.

Moving the Lesson Online

Table 1 outlines the MTE's lesson transitioning, including preparation of resources. The overall idea for the lesson was for teachers to acquaint themselves with a smaller portion of the hierarchy (Launch), use that exercise as a foundation to create the general hierarchy in cooperative groups (Explore), and discuss and debate findings in a whole-class setting until a final hierarchy was reached (Summarize).

Launch

A Launch phase is initiated when an educator reviews prior ideas and presents new ones, preparing for the Explore task. In both the face-to-face and online lessons, the Launch included reviewing quadrilateral definitions and investigating a whole-class *mini-challenge* to determine whether a square is a rectangle or a rectangle a square. In the online classes, the MTE utilized screen-sharing capabilities to lead these discussions. Using a Powerpoint (PPT) slide containing a moveable rectangle and square, she employed PPT's *editing* mode to insert text reflecting teacher thinking, move shapes, and insert line segments between shapes to indicate hierarchical relationships suggested by teachers. In their respective locations, the teachers could use the definitions sheet and arrange the paper cutouts to reason about the challenge. Depending on the conferencing platform, teachers raised their hands digitally or physically or used chat messaging to communicate (these features were also utilized in the Summarize phase).

Explore

In the Explore phase, learners interact cooperatively to investigate an activity initiated by the Launch. The educator supports these interactions by listening in, raising questions, and gathering information for the Summarize phase. Although this is readily accomplished in face-to-face settings, the conferencing programs compelled other arrangements. In the first class, the MTE's limited familiarity with Conferences' breakout rooms meant that teachers worked *individually* with paper cutouts. In the second class, the Zoom Breakout Room feature provided an analogue of face-to-face groups. The MTE supported this groupwork by preparing Jamboard slides containing moveable quadrilaterals, and instructing teachers in breakout rooms to arrange (and rearrange) shapes to create a hierarchy (some teachers preferred paper cutouts). The MTE invited teachers to elicit help in all settings, and visited some but not *all* breakout rooms.

Summarize

The Summarize phase is devoted to sharing conjectures, and organizing and generalizing findings into a culminating message. Throughout the phases, the educator is

Table 1

Moving LES from Face-to-Face to Online

	Face-to-Face	Online, Spring 2020	Online, Fall 2022
LMS & Platform Used	Classroom	Canvas & Conferences	Canvas & Zoom
Resources Provided	Paper handouts of shapes & definitions distributed on day of lesson	Powerpoint (PPT) slides of printable shapes & definitions posted to LMS in advance of lesson	PPT slides of printable shapes & definitions posted to LMS in advance of lesson; Jamboards containing moveable shapes
LAUNCH: MTE facilitates whole-class discussion on "Is a rectangle a square?" mini-challenge	MTE arranges shapes on doc cam and writes on whiteboard	MTE shares screen with a rectangle and square, rearranging shapes on a slide in PPT's editing mode	
EXPLORE: Teachers create full hierarchy using the challenge completed during the Launch phase as a model	MTE circulates the classroom; teachers work in small groups using paper shapes	Teachers arrange hierarchy with paper shapes, working individually in their respective locations	Teachers work in small breakout rooms arranging shapes in Jamboards
SUMMARIZE: MTE facilitates whole-class discussion based on work from the Explore phase, building a hierarchy as participants navigate to consensus	MTE arranges shapes on doc cam and writes on whiteboard	MTE shares screen of PPT slide containing all eight quadrilaterals, rearranging shapes on the slide in PPT's editing mode	

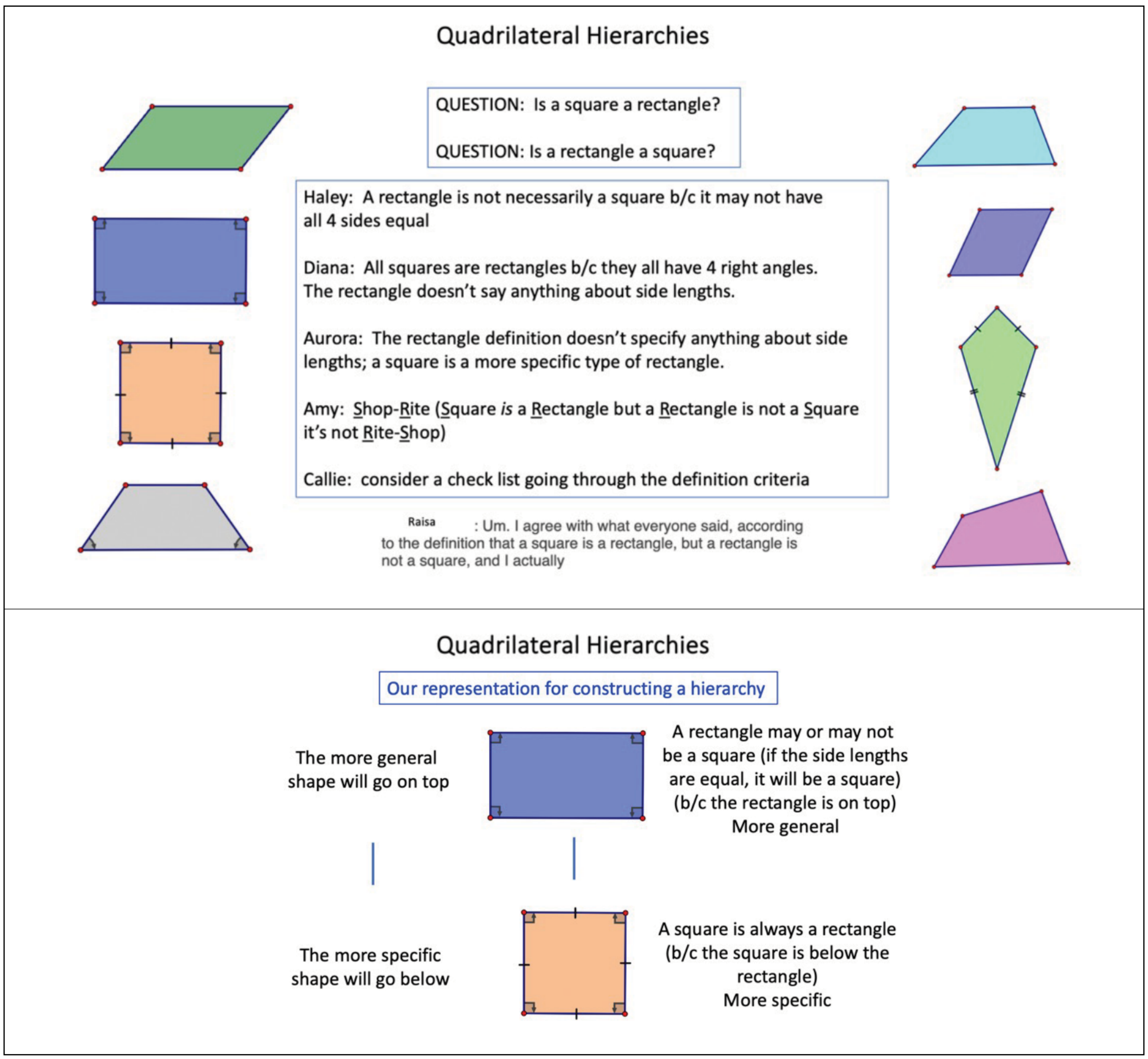
discouraged from "telling," and instead encouraged to guide and challenge learners as they question and justify their mathematical findings. Irrespective of format, the MTE utilized turn-taking to elicit a single quadrilateral location and the corresponding reasoning from an individual teacher or group spokesperson. As teachers provided their contributions, the MTE displayed their suggestions using a shared PPT slide with moveable quadrilaterals for both online lessons. The MTE intended the evolving shared representation as a point of reference to provoke thinking. She listened to teachers, moving quadrilaterals where they indicated, regardless of correct placement. Subsequent groups could challenge a quadrilateral's location, but the challenge needed to be justified and could be rebutted. The MTE continued the discussion until a shared hierarchy was reached.

Implementing the Lesson: Opportunities and Discoveries

The MTE noted that "nothing was lost" in moving the LES lesson online. Whether using chat or verbal communication, whole-class discussions were active, lively, and filled with naturally occurring questions, challenges, confirmations, and argumentation. During the Launch, teachers' insights provided the basis for a hierarchical representation used throughout the Explore and Summarize phases (Figure 2).

FIGURE 2

Launch Slides Co-created between MTE and Teachers



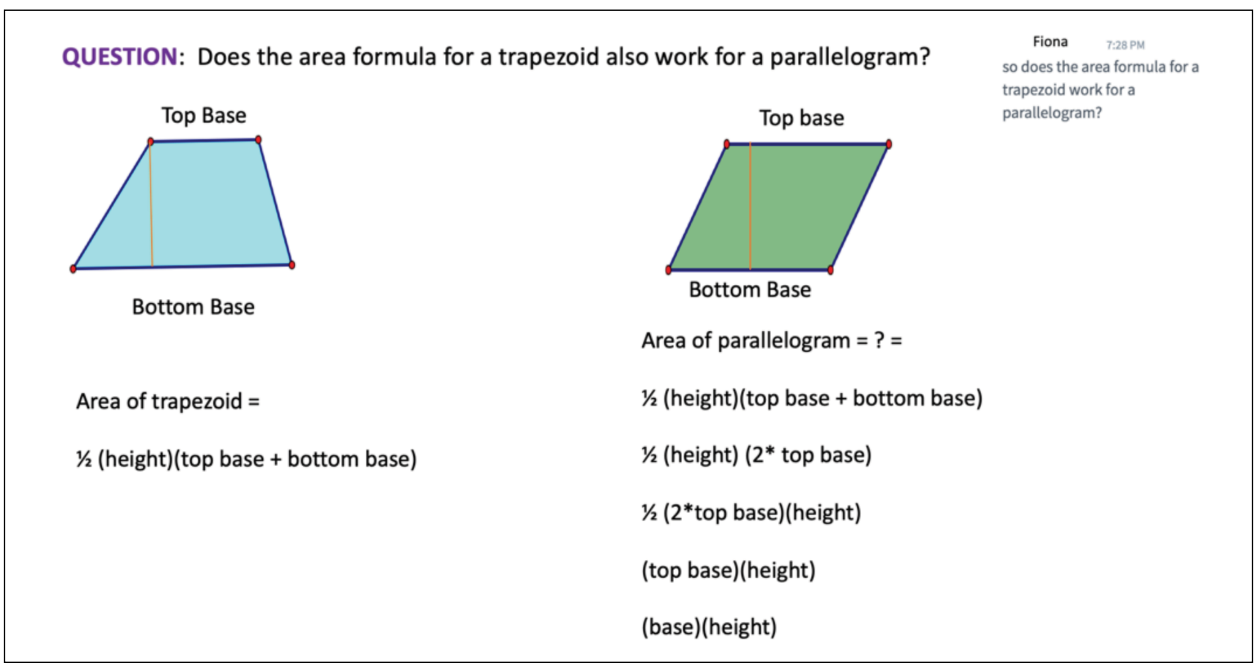
In the Exploration phase, the Jamboard activity mirrored the collaborative nature intended for groupwork. For the reader wondering whether teachers were authentically engaged in breakout rooms that are hidden from the MTE's sight, Jamboard's version history function provided an accounting of teachers' activities. During the 24 minutes of groupwork, multiple teachers per group rearranged shapes and typed notes into Jamboard slides. Four out of ten groups created mostly accurate hierarchies (missing one or two shapes). Teachers transitioned between placing shapes above or below the connections they had already established (eg., placing a parallelogram above a rectangle or a rhombus below the kite). Two groups inverted the hierarchy (placing the square on top) and one group struggled with horizontal arrangements (eg., placing a rectangle and parallelogram alongside each other).

During the Summarize phase, the online discussion was as rich as any face-to-face implementation. Some teachers' contributions stemmed from the lesson design. For example, the definitions for a rhombus (four equal sides) and rectangle (four right angles) do not immediately suggest they are parallelograms. This purposeful design prompted teachers to identify missing implications as they reasoned with *only* the given definitions. In both classes, teachers questioned the rhombus-parallelogram connection, with Daniel typing, "the implications of equal sides, would be something with parallel lines, right?" In the other class, Sandra wondered aloud, "I still don't know what, if I feel like the rhombus and the parallelogram should be connected. We connected it, but it just, it doesn't match the definition. So it's bothering me." These teachers are seeing or remembering the rhombus as a parallelogram, but realizing the need for a rigorous argument based only on the provided information. Their scrutiny reflects their mathematical prowess, and is precisely the outcome the MTE hoped for. Moreover, it provides a teacher-initiated springboard to prove that a quadrilateral's four equal sides or angles imply it is a parallelogram.

Other teachers' observations went beyond the lesson design highlighting constructivism in action and the theory's recognition of the value of unexpected questions (Wood et al, 1991). For example, Fiona raised the issue of inherited hierarchical properties when she typed into the chat, "So does the area formula for a trapezoid work for a parallelogram?" This question was a pleasant surprise for the MTE, and generated a rich discussion in which teachers proposed multiple promising methods for investigation. Fiona was able to resolve the conjecture when she noted that "bases are the same length in a parallelogram" and the MTE picked up this chat entry. If $b1 = b2$, this suggested that the trapezoid's formula ½ $h(b1+b2)$ becomes ½ $h(2b)=bh$. As teachers typed their excitement and encouragement at this discovery, Ariana courageously articulated, "I'm sorry I'm really confused." The MTE reassured her and highlighted salient points about Fiona's argument in a

FIGURE 3

Quadrilateral Hierarchy



new PPT slide (Figure 3). Ariana's responsive chat messages during the MTE's explanation ("ahhh", "wow", "I'm very impressed") indicated her understanding.

In this online setting, the richness and value of this unexpected investigation culminated in two teachers describing a newfound belief in the hierarchy's integrity: Fiona typed, "I did not believe a parallelogram was a trapezoid and now I do" and Carmen followed with, "I feel like that further supports the hierarchy (...) I had a hard time believing it too." These observations speak to the teachers' unvoiced doubts about the relationships represented in their hierarchy, which were only discovered through this unexpected investigation.

Challenges and Conclusion

The MTE noted that challenges, like navigating streams of incoming text messages or worrying whether teachers were on-task during individual or Jamboard sessions, were comparable to other synchronous and face-to-face lessons. A novel online challenge concerned the advanced preparation and sharing of handouts to give teachers a week's notice to print (or draw) and cut shapes. The MTE further recognized the compromised collaborative experience during the first Explore session, but noted that this did not adversely affect the lesson's Summarize discussion. Moreover, with evolving technologies and familiarity, the collaborative activity was restored during her second online iteration. The MTE appreciated that technologies that typically present final products (e.g., PPT, Jamboard) can be repurposed (see Mishra & Koehler, 2006) to reflect and document evolving and responsive thinking. Such technologies, along with an appropriate activity, can serve to uphold critical LES features in online synchronous settings, suggesting the promise of LES to the work of online mathematics teacher education.

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NOTES FROM THE FIELD

Programs and Opportunities for Early Career Mathematics Education Scholars

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ABSTRACT Early career mathematics education scholars face several challenges as they navigate academia. Some of these challenges relate to disparities in access to information and encouragement to apply to certain opportunities. In this paper, a diverse group of early career mathematics education scholars presents programs that have supported their work. The purpose of the paper is to serve as a repository of some of these programs for advisors, mentors, and scholars. Based on their experience securing these opportunities, the authors also offer advice and encouragement for future applicants. Ultimately, the paper contributes to demystifying academia and facilitating the search for opportunities for those joining the field.

KEYWORDS *early career, mentoring, professional development, academic community*

Introduction

Academia can be difficult terrain to navigate and comes with a plethora of challenges. For early career faculty, these challenges can include unclear tenure expectations, lack of work-life balance, and lack of community or collegiality at their institution (Greene et al., 2008). Additional challenges include developing a research program while learning to balance teaching and service expectations (Reys et al., 2009) and feelings of loneliness or competition (Rice et al., 2000). Isolation can also be a significant challenge. In the mathematics education field, some programs graduate multiple doctoral degree holders each year, while others only have one doctoral graduate every few years; on the faculty side, some programs may have up to 10 mathematics education faculty, while other programs only have one (Reys et al., 2019). Diggs and colleagues (2009) identify further barriers faced by faculty of color, including marginalization, racism, lack of power or status, lack of role models and colleagues with shared backgrounds, experiences of cultural dissonance, and issues with development of one's "academic identity within and against the existing institutional structure" (p. 321). Women of color in STEM (and elsewhere) face the formidable barrier of simultaneous racism and sexism (Ong et al., 2011), what Malcom et al. (1976) have called "the double bind."

Attrition rates for both doctoral students and early career faculty indicate a clear need to address these challenges. The PhD Completion Project found an overall US PhD completion rate of approximately 57% (Council of Graduate Schools, 2007), and in a longitudinal study tracking attrition in STEMM (science, technology, engineering, mathematics, and medicine) disciplines, almost 60% of those in STEMM had left the field or stopped publishing within nine years (Kwiek & Szymula, 2024). The need for supporting early career scholars is clear, but significant inconsistency has been identified in the support received by untenured faculty (Greene et al., 2008). Many universities lack the resources or climate to adequately support doctoral students or early career faculty in developing research skills and productivity, particularly at less

* Authors two through eight contributed equally to this article and are listed alphabetically.

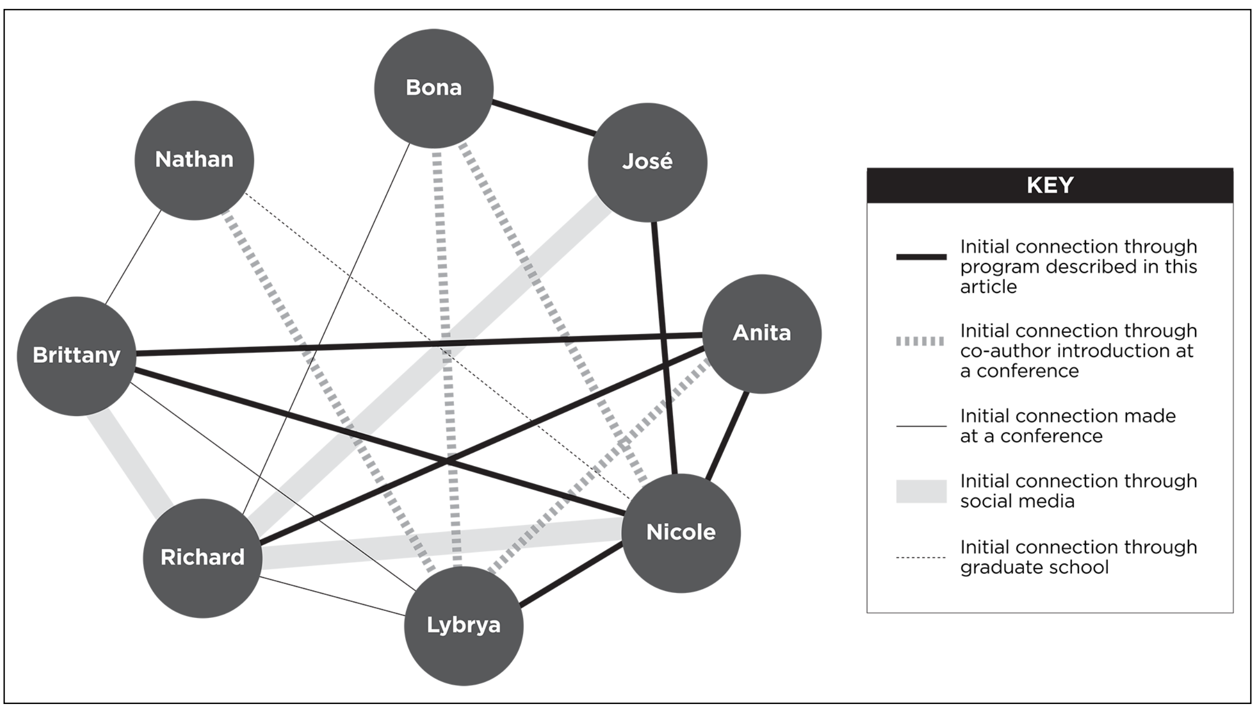
research-intensive universities (Levine, 2007). Furthermore, though mentorship may be an important support for navigating challenges in academia, Greene and colleagues (2008) found that 58% of the 96 untenured faculty they surveyed reported having an assigned formal mentor, and of those with an assigned mentor, only 17% found the relationship to be helpful.

Researchers studying the experiences of early career scholars have proposed structures for support. For example, Greene and colleagues (2008) developed a comprehensive support system for junior faculty, including research support (e.g., research collaborations with senior scholars or peers with established research programs, writing groups, research development centers), mentorship (along with mentorship training), limited service responsibilities, clear expectations, feedback on progress towards tenure, research start-up funds, additional conference travel funds, and a welcoming, collegial environment. In Borum and Walker's (2012) study examining undergraduate and graduate experiences of 12 Black women mathematicians, mentorship networks, research opportunities, and support systems contributed to the women's academic success. The authors noted that mentors who had shared identities with the women as well as mentors who did not have the same gender or racial identity were instrumental to their academic development.

Within mathematics education, various initiatives and programs have been created to provide early career scholars with access to support structures, mentorship, skill development, resources, and networking opportunities. These programs have been created by professional organizations, funding agencies, or groups of faculty who have obtained grant funding. Though a number of these programs exist, many early career scholars are unaware of these opportunities for various reasons: they may have attended a smaller doctoral program, they may not have funding to attend conferences, their mentors or colleagues may be in a particular academic lane, or their mentors or colleagues may not be active in professional organizations or mathematics education social media or listservs, all of which can limit access to expanded professional networks and to information about opportunities. The authors of this article are a group of early career mathematics educators who have participated in and benefited from such programs. We are a diverse group in many ways–doctoral programs, location, type of role and institution, race, and gender– and we have all experienced how these programs can provide important opportunities for mentorship, skill building, access to financial and other resources, and development of professional networks, community, and academic kinship. Figure 1 demonstrates how we came to be connected to one another, with early career programs providing the most connections amongst the author group, followed by conferences, social media, and graduate school.

FIGURE 1

Sociogram of author connections



In this article, we share our knowledge of and experiences in these programs with other early career mathematics educators, as well as the faculty and administrators who advise them, so that we may serve as *gateways* rather than *gatekeepers* of opportunities in the field. In particular, we describe eight different programs or opportunities, which include fellowships, grants, service opportunities, and workshops (Table 1, included in the appendix, summarizes the programs described in this article). Each section is written by one of the article authors who participated in the program and includes a summary of the opportunity and a reflection on the author's experience in the program. Some programs are ongoing and others are not, but we include descriptions of these programs to demonstrate the types of programs that exist and to document the benefits that such programs have had on our early career development.[1] Table 2 (included in the appendix) provides a list of additional opportunities that can benefit early career scholars in mathematics education.

Fellowships

Service, Teaching, & Research (STaR) in Mathematics Education Fellows Program–Bona Kang

The "STaR Program" (Service, Teaching, and Research (STaR) in Mathematics Education) is an early career professional development program offered by the Association of Mathematics Teacher Educators (AMTE, n.d.). This program is led by a rotating team of advanced faculty and offers an opportunity for participants to expand their network with other early career mathematics education faculty across the nation through workshops and informal discussion sessions that support the navigation and understanding of various faculty expectations. Any tenure-track or continuing track mathematics education faculty in their first or second year can apply in late fall to be considered for a cohort of approximately 20 fellows the following summer. Accepted fellows are expected to attend a 5-day summer institute (usually in Park City, Utah) and the following AMTE conference, arriving the day before for a workshop day and reception. While the lodging and a few selected meals at the summer institute are covered by the program, participating fellows are expected to use their home institution's endorsement funds and any supplementary funds to cover their own travel expenses for the summer institute and the costs entailed in participating in the AMTE conference the following year. Accepted fellows also become connected to a community of over 440 STaR fellows that are invited to the annual STaR reception at the AMTE conference, and are added to an email listserv through which they may receive or send various items, such as job calls, invitations to research studies, links to online depositories of data, etc.

I am an Asian immigrant and the only assistant professor focusing on mathematics education in an undergraduate teacher preparation program at a predominantly white small liberal arts institution. After my first year, I realized that I needed to pivot my research agenda to focus on preservice teacher education because my high teaching load limited my ability to continue classroom research in elementary schools. I also needed support in designing and teaching the sole mathematical preparation course to which elementary preservice teachers had access in the program. In the process of looking for an academic community, I decided to apply to the STaR program. I participated as a STaR fellow in 2023, and it has been one of the most positive and impactful experiences that has significantly expanded my previously limited professional network to one full of lively subgroups of collaborators and friendly colleagues I regularly connect with at conferences. The summer institute provided abundant opportunities to informally spend time with fellows every day outside of the planned events. These opportunities accelerated establishment of rapport and provided the foundation for deeper connections during sessions for manuscript feedback, topic-specific research and teaching groups, and informal discussion sessions on topics such as applying for grants, navigating the field as BIPOC scholars, balancing work-life expectations, preparing for promotion and tenure, establishing productive collaborations, and managing course loads. The program committee members, led by a group of advanced faculty in the field, also provided specific resources and dedicated their time to support the early scholars' success. While the outcomes may vary by fellow and cohort, such

[1] Shortly after we wrote this article (January 2025), federal mandates have dramatically reduced funding for some agencies and have terminated numerous active grants, including some of the grant-funded opportunities described in this article. These mandates jeopardize funding for many of these mentorship, grant, and service opportunities. The options available to early career mathematics education scholars may shift accordingly, and executive orders targeting diversity, equity, and inclusion initiatives may limit access to equity-focused opportunities in particular. This elevates the importance of ensuring that colleagues have information on resources available and demonstrates a need for professional organizations, universities, and senior scholars to begin exploring alternative funding sources to support programs and opportunities for early career mathematics education scholars. As we all try to navigate the uncertain landscape of education and education research, doctoral students, postdoctoral fellows, and early career faculty need the mentorship and support of our mathematics education community now more than ever.

resources provided me with two active, collaborative projects, and I have looked forward to informal reunions with my cohort at each subsequent conference. Fellows also shared various professional opportunities as well as resources for teaching, scholarship, service, and funding. Through this network, I obtained a position as a reviewer on the editorial board of a journal for the first time and received substantial support as I developed a new mathematics content course and updated my elementary methods course. While the personal and institutional context will impact individual fellows' experiences, I recommend this program for faculty who seek new and multiple opportunities to generate ideas and collaborative projects with other supportive faculty.

Community for Advancing Discovery Research in Education (CADRE) Fellows Program–Anita Sundrani

CADRE Fellows is part of a National Science Foundation-funded program that provides early career scholars the tools to be successful researchers in the field of STEM education. Over the course of six months, Fellows network with accomplished scholars, engage in learning activities, and are mentored through the Discovery Research PreK-12 (DRK-12) proposal review process. This opportunity is open to early career scholars, ranging from year one doctoral students to postdoctoral researchers, and would be a good fit for individuals with limited access to mentorship or a network of STEM education researchers at their home institutions or organizations. However, the eligibility criteria may differ from year to year. For instance, the 2025 cohort of CADRE Fellows must be located "in EPSCoR jurisdictions and rural areas" (CADRE, n.d.).

I first learned about the CADRE Fellows program through my doctoral institution's research newsletter. I was interested in the opportunity to expand my network and learn more about grant funding from experienced researchers while preparing to graduate with my PhD. The program took place between January to June and the organizers offered four key experiences for Fellows during this time:

1) CADRE Fellows orientation, where Fellows introduced themselves and shared their research interests and career goals with the group. The organizers also led a discussion around diversity, equity, inclusion, and justice and set the foundation for community building and learning experiences.
2) Monthly sessions on topics ranging from career pathways to developing partnerships for research. During these sessions, Fellows read about the speakers and prepared questions in advance.
3) Peer-led personal writing projects focusing on one of four potential areas—an article for publication, grant proposal, research agenda, or cover letter for a position. Fellows selected their topic of interest and met once a month to work on their project. These meetings were peer-led using a provided structure that included a discussion prompt, time for writing, and debrief.
4) Mock NSF proposal review with an NSF program officer at the annual DRK-12 Principal Investigator (PI) meeting to gain a deep understanding of the review process and the multiple layers involved from proposal submission to awarding a grant.

One of the most impactful components of this program was learning about numerous opportunities that exist in the field of education. The monthly sessions provided space to learn about different careers in education and funding agencies. The program also provided an in-depth exploration of the NSF funding process through direct accounts of the proposal writing process and an opportunity to interact with DRK-12 awardees at the annual DRK-12 PI meeting. After completing the program, I felt better equipped to write a grant proposal and developed relationships with colleagues who could support me as I pursue funding for my work. I also gained a clearer understanding of career options adjacent to academia while I was on the job search, including research organizations and research centers connected to universities. I was then able to broaden my search to other roles that would allow me to apply the skills I acquired during my doctoral studies and still stay connected to the community I have built over the past five years. I eventually found a position at a research center based out of a school of education and was able to extend my research agenda and continue to pursue my personal research projects.

Quantitative Research Methods for STEM Education (NSF QRM) Scholars Program–Lybrya Kebreab

The National Science Foundation-funded Quantitative Research Methods for STEM (Science, Technology, Engineering, and Math) Education Scholars Program (NSF QRM) developed early-career education researchers' skills in design, measurement, and data analyses (University of Maryland, 2024). Throughout the yearlong program, NSF QRM Scholars participated in several virtual workshops, a 2-day institute at the University of Maryland (UMD), as well as one-to-one mentorship from experienced, UMD quantitative methodologists. The program assigned mentors to participants based on our accepted research proposals' areas of interest and prospective methodologies. Additionally, we received access to advanced computational and statistical software to design and implement our respective studies, which were presented publicly to each 20-person cohort. The collegial atmosphere encouraged multidisciplinary innovation and collaboration, offering various opportunities for feedback to improve our statistically complex research proposals on issues of equitable STEM.

The virtual workshops and in-person institute supported QRM scholars in learning to lead studies which necessitate advanced data analyses and state-of-the art computational software. Further, the mentors guided us in the process for developing strong proposals for NSF funding for our research proposals and future research. The workshops and feedback offered unique opportunities to engage with quantitative research methodologies not typically found in dissertations. Currently, we are a part of a small but influential network of quantitative researchers who are passionate about using sophisticated data analyses to solve important equity-related issues in STEM. The advanced statistical knowledge and skills gained during my year as a QRM scholar have been invaluable to my development as an early-career researcher.

As a first-generation PhD, the NSF QRM program has been integral in cultimating the development of my mathematics education researcher identity. I have transformed from identifying as an avid reader merely consuming mathematics education research to generating theories and knowledge in service of the field. Being the first in my family to complete a terminal degree, I found that establishing a sense of belonging and an authentic, niche identity in academic circles was sometimes an arduous, disheartening process. It is difficult to put into words the shock one experiences at the realization your childhood, young adult, and pre-doctoral STEM experiences may not have adequately prepared you for the unspoken expectations of the hidden curriculum in academia.Nonetheless, I persevered with the unending support of my dissertation chair, committee, and other mentors who supported me through the unexpected challenges to my identity development as a mathematics education researcher (Covey, 2020). This program helped refine the nuanced aspects of my identity and belonging as a unique contributor of advanced statistical studies in mathematics education.

I am incredibly blessed to have been guided by my advisor to apply and participate in the wonderfully challenging NSF QRM Scholars Program. Hopefully, sharing these experiences will encourage senior faculty and researchers to (a) continue organizing more programs like this in their future NSF-funded endeavors, and (b) inspire kids in STEM from underrepresented groups to continue growing the field with their brilliance. My heartfelt appreciation goes out to NSF QRM Scholars Program's principal investigators, mentors, and staff who invested in an underprivileged Black woman passionate about science and mathematics. These experiences had a profoundly positive impact in my postdoctoral, university work as a STEM institute Project Manager/Mathematics Education Lecturer.

Institute on Mixture Modeling for Equity-Oriented Researchers, Scholars, and Educators (IMMERSE) Fellowship–Nathan N. Alexander

The Institute on Mixture Modeling for Equity-Oriented Researchers, Scholars and Educators (IMMERSE) is a year-long training fellowship that supports research scholars and educators in developing the necessary competencies to leverage modeling and statistical methods to conduct equity-oriented scholarship in education and human development (University of California, Santa Barbara, 2024). The scholarship on mixture models is rapidly expanding given its use in addressing issues of heterogeneity. Specifically, a recent set of studies have begun to exemplify the use of mixture models in equity-oriented literature to challenge the longstanding concerns about quantitative methods and researcher bias, and those methods historically used to extend features of "gap gazing" in mathematics and science education, which has been used as a deficit-oriented focus in statistics used to compare white students to non-white groups (Gutiérrez, 2008). In response, mixture modeling has been used to directly challenge ideas of homogeneity among populations grouped by race and ethnicity, as shown in the works of Slominski et al. (2024) and Suzuki, Morris, & Johnson (2021). This body of research in mixture modeling is part of a broader contemporary effort among quantitative scholars to integrate critical theories into education research and is one application of scholarship in the critical quantification and QuantCrit (Quantitative Critical Race Theory) traditions (Frisby, 2024; Fong & Irizarry, 2025). Despite these recent advancements, there is still much potential for using mixture modeling to help reduce issues of bias and equity in educational statistics and quantitative modeling.

The IMMERSE fellowship program is a useful program for both research scholars and educators during any period of their career, and especially those scholars earlier in their careers. As junior scholars, we may not be fully attuned into focused programs that provide technical support over an extended period of time. Programs of this nature provide an opportunity for scholars new to the academy to see ideas change and others take hold. The program is for those who wish to expand their knowledge of latent variable modeling and the associated class of statistical methods grouped under the umbrella of mixture models. A fellowship application is required for acceptance into the program, and fellows are required to demonstrate both their interests in equity-oriented quantitative research and their background in statistical methodologies. There are two options for the program, one for education researchers conducting discipline-based research and another program for equity-oriented researchers, funded by the National Science Foundation (NSF) and the Institute for Education Sciences (IES).

I started this program as the director of a research lab focused on developing a series of modules that would go on to inform future work teaching research skills to undergraduate and graduate students. However, participating in this program as a junior faculty member provided me with early training and preparation in a cohort of fellow researchers focused on expanding their data analysis in ways that attend more skillfully to our histories and cultural realities, our identities, and collective educational experiences. The year-long program begins with a series of pre-training modules that help to support participants with the setup of various technical apparatuses that will be utilized throughout the program and to build community. A weeklong in-person training is held to support fellows in the further development of their competencies in mixture modeling, and to establish research question(s) and collaborations. Upon completion of the in-person training, fellows are provided with extensive weekly support throughout the following academic year including a host of integrative activities. These activities range from guest speakers conducting research that leverages the mixture modeling methods, to writing group meetings, access to online tools, and resources for additional support. Refer to the appendix and the program website for more information.

Grants

Equity in Math Education Research Grants (EMERG) Program–Richard Velasco

In 2023, the National Academy of Education launched the Equity in Math Education Research Grants (EMERG) Program with generous donation from the Gates Foundation (National Academy of Education, n.d.). EMERG provides early-career faculty (referred to as EMERG scholars) with the opportunity to conduct robust research on equity in mathematics education, particularly focusing on historically marginalized student populations. During the EMERG program's inaugural cycle, ten proposals were funded, with each EMERG scholar receiving $113,000 to carry out their projects over a two-year period starting in the 2024-2025 academic year. A unique feature of the EMERG program is its mentorship structure. Proposals were not required to be fully developed at the time of submission because each EMERG scholar was paired with a primary mentor and grouped with two secondary mentors, forming an advisory triad. These mentors, all seasoned education scholars and members of the National Academy of Education, formed the EMERG scholar's advisory committee. Scholars were given the opportunity to select their mentors based on who they believed could best support their project goals. Although there will not be a 2025-2026 EMERG funding cycle, the EMERG Executive Board has announced plans for another cycle beginning in the 2026-2027 academic year, with applications due in Fall 2025. Eligible applicants will be early-career scholars (defined as having obtained a Ph.D. or equivalent within the past seven years) whose research addresses equity in math education, particularly for historically marginalized student populations. Ideal candidates will either belong to these populations or have significant experience working with them.

I first learned about the EMERG program when I was in my second year as a tenure-track assistant professor at the University of Oklahoma. I found out about it through the social media platform formerly known as Twitter, where a colleague from another institution shared the call for proposals. After reviewing the eligibility criteria, I realized I fit the profile and already had a project idea in mind. Before learning about EMERG, I had been in discussions with a teacher in Guam who expressed interest in engaging more Pacific Islanders—specifically students in Micronesia—in data science. Reviewers found my proposal promising, and, after navigating the application and interview process, I was selected as an EMERG scholar and awarded the grant. Now one year into the project, I deeply value the mentorship provided through the program. The advisory triad has offered invaluable guidance, while the peer mentorship among my fellow EMERG scholars has been equally enriching. We provide mutual support and celebrate one another throughout the program. From my perspective, the program's key benefits include expanding professional networks and gaining experience managing large, grant-funded projects. This experience has significantly increased my confidence in applying for larger grants in the future.

Faculty Early Career Development Program (CAREER)–José Martínez Hinestroza

The Faculty Early Career Development Program (CAREER) is an award granted by the National Science Foundation (NSF) (National Science Foundation, 2022). The purpose of this award is to support early-career faculty in developing a foundation to become leaders in their fields and institutions. Two of the main characteristics of this award are that no co-principal investigators are allowed and that the proposed project must integrate research and education. The CAREER award provides funds for five years during which the principal investigator integrates research and education activities. Institutions of higher education, as well as certain non-profit, non-academic institutions located in the United States can submit proposals on behalf of a faculty member. The NSF has established criteria for tenure-track equivalency. To be eligible, the principal investigator must hold a doctoral degree, be an untenured assistant professor, and not have received the CAREER award before. Applicants can submit a proposal up to three times. In short, early-career scholars in the US interested in advancing and integrating research and education may find it particularly relevant.

I learned about this opportunity as a graduate research assistant in a CAREER project and in a doctoral course where we read funded proposals and met with former awardees. Later on, I attended the NSF CAREER webinar where program officers described this opportunity and answered questions. Serving in a review panel for other NSF grants, as recommended by program officers during the webinars, contributed substantially to securing funding. NSF proposals, including CAREER proposals, are evaluated in terms of intellectual merits and broader impact. Being part of review panels enhanced my understanding of how reviewers evaluate these criteria in practice. It was also useful to read the proposal and advice that several colleagues who had received the CAREER grant generously shared. Locally, my home institution also offered workshops for potential applicants led by former program officers or CAREER awardees. Importantly, working closely with the communities that were central to my proposed research and educational activities was essential. This ongoing collaboration informed my proposal and allowed me to demonstrate the community support that was necessary to implement the project. Once I was ready to share an initial idea, a program officer met with me virtually and answered my questions.

My experiences as a Spanish-speaking immigrant from Colombia have strengthened my interest in collaborating with Latinx, bilingual children, pre-service teachers, and in-service teachers. My research project entitled “CAREER: Affirming bilingual children’s participation in mathematics (ABC-Par)” has provided the resources and time necessary to develop this work. This includes disseminating findings with teachers and undergraduate research assistants locally, nationally, and internationally. The grant has also allowed me to consult with leading scholars in my area of research who serve as advisory board members. Ultimately, this has been a transformational experience through which I continue to refine my thinking, reaffirm my commitment to critical mathematics (teacher) education, and learn with mathematically creative bilingual Latinx children, teachers, and undergraduate students.

Service Opportunities

AERA Special Interest Group for Research in Mathematics Education (SIG-RME) Graduate Student Representative–Brittany L. Marshall

I served as one of two graduate student representatives on the American Educational Research Association’s (AERA) Research in Mathematics Education’s Special Interest Group (SIG-RME) executive board during the final two years of my PhD program. As an African American woman who started my doctoral studies a bit later in life, I sought to expand my professional network and engage in leadership roles. I had been to several SIG-RME events and found the community extremely engaging and supportive, so when my doctoral advisor suggested I apply for the position, I agreed. Every year, the SIG posts their upcoming vacant positions, the call for a short letter outlining your qualifications, and what you can contribute to the organization (American Educational Research Association, 2025). Any graduate student in mathematics or STEM education is qualified to apply, with preference given to students who have completed at least one year of graduate study. Instead of having the entire SIG membership vote, the current junior student representative chose the new representative as they would have to work closely with each other.

Depending on the initiatives they decide to enact, this position is often more demanding than expected for a graduate representative. During my term, I led a weekly writing group while my colleague implemented a book club. We also hosted a week of mentoring workshops every spring open to all mathematics education scholars, intended for graduate students and early career professionals who could not attend AERA's annual meeting. It involved identifying the topics we believed mattered to current doctoral students (e.g., publishing, funding, the job market, etc.) and then recruiting scholars in the field willing to participate in publicly accessible virtual discussions on these topics. Over the two years I served as a graduate student representative, we had various successful outcomes with our program initiatives, highlighted by the mentoring workshops.

Being a graduate student representative for SIG-RME was highly beneficial for my early career development. Being in this role allowed me to connect with many scholars nationally and internationally. It also allowed me to strengthen and demonstrate my planning and organizational skills in front of many scholars in the mathematics education community. This role greatly supported my academic job search, as I was able to obtain a highly sought after tenure-track position just prior to graduation.

Workshops

Workshop on Rehumanizing Mathematics–Nicole Fletcher

T he Workshop on Rehumanizing Mathematics is a two-week workshop led by Dr. Rochelle Gutiérrez through the Park City Mathematics Institute (PCMI), an initiative of the Institute for Advanced Studies (IAS). The workshop is designed for mathematics teachers, mathematicians, and mathematics education professors to expand their understandings of issues connected to equity, identity, and power in mathematics and to further their work towards rehumanizing mathematics (Gutiérrez, 2024). Participants learn about various theories and frameworks related to equity and social justice in mathematics, including the Rehumanizing Mathematics framework (Gutiérrez, 2018), and examine their own work in their respective professions in relation to equity and rehumanizing mathematics goals. Throughout the workshop, participants work on a relevant project (e.g., revising a course, designing teaching materials or assessments, developing an initiative), receive feedback from fellow participants and the facilitator, and prepare to further develop and carry their work forward after the workshop in their professional contexts.

PCMI takes place annually in July in Park City, Utah and includes programs for undergraduate and graduate mathematics students and mathematics researchers (IAS, 2025), as well as an equity-focused workshop with rotating facilitators who are established scholars committed to equity. Prior iterations of PCMI included a Teacher Leadership Program for teachers of 7th-12th grade mathematics. Participants are selected through an application process, and PCMI supports participation by providing a travel allowance, housing, some meals, and a stipend.

Though the Workshop on Rehumanizing Mathematics is not strictly an early career opportunity, it was instrumental for my early career development. I am an assistant professor of elementary education at a teaching-focused university, and I am the only faculty member with a full-time appointment in the Educational Studies and Teacher Preparation department who focuses on mathematics education. The workshop brought together people across K-12 mathematics teaching, mathematicians, and mathematics teacher educators—a diversity of perspectives I have not experienced in other workshops or conferences. Academia is so often siloed; interacting with people who work across mathematics teaching, learning, and research introduced me to experiences and points of view I would not have been aware of otherwise, allowing me to learn about the challenges and possibilities of equity and social justice work across corners of the mathematics community. Reflecting on our practices within a group with such diverse perspectives helped us to see our work in a new light, allowing us to identify areas of strength and recognize where we can focus our efforts to develop further and fill in gaps in our work to rehumanize mathematics. Engaging in brainstorming conversations and discussing project feedback within our diverse group gave us new insights that helped each of us to make significant progress on our project action plans. For my own project, I developed a Social Justice Mathematics Lesson project for my elementary mathematics methods course. Our learning, work, brainstorming, and feedback opportunities throughout the workshop helped me to move from an early idea to a clearly-defined action plan, enabling me to develop the project further that summer and implement the project in my course the following fall.

Dr. Gutiérrez was intentional about building community throughout the workshop, helping us to develop the relationships necessary to be vulnerable, take risks, and engage deeply and thoughtfully in our work. Because of Dr. Gutiérrez's intentional community building, our group developed strong, close bonds over the course of the workshop that continue today. We have an active group chat and gather for periodic zoom calls, and we support one another through personal and professional challenges and celebrate each other's accomplishments. Further professional collaborations have also come out of our group. Being in community and solidarity with this incredible group of people has provided me with support, connection, and love that have allowed me to grow during this important time of my early career development.

Conclusion

In this paper, we offered resources and insights into opportunities for early career mathematics education scholars. The authors describing each of these opportunities come from diverse backgrounds and institutions, which highlights the options available for scholars with different interests and with varying expectations for research, teaching, and service. Our intention is to demonstrate that all of these options are possible to achieve and that the benefits are far-reaching. Specifically, participating in these opportunities can help further knowledge of research methods and boost research programs, enhance teaching practices and resources, expand professional networks, and, importantly, provide community support. Moreover, these programs allow early career scholars to contribute to the field and their communities, which is needed to continue moving mathematics education forward. We encourage graduate advisors, mentors of early career faculty, and scholars themselves to use this resource as a reference and as an inspiration to pursue these and similar opportunities[1] As others have advised us before: The only application that is never accepted is the one you do not submit.

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Appendix

Table 1

Early Career Scholar Opportunities Discussed in Article

Name	Type	Eligibility	Funder/Organizer	Website
Service, Teaching, & Research (STaR) in Mathematics Education Fellows Program	Fellowship	Tenure-track or continuing track math education faculty members in the U.S. who are in their first or second year at the time of application	Association of Mathematics Teacher Educators	https://amte.net/star
Community for Advancing Discovery Research in Education (CADRE) Fellows Program	Fellowship	Early career scholars (e.g., doctoral students, postdoctoral researchers) from institutions in EPSCoR jurisdictions (NSF, n.d) and rural areas	National Science Foundation	https://cadrek12.org/about-cadre-fellows-program
Quantitative Research Methods for STEM Education (NSF-QRM) Scholars Program	Fellowship	Focused on research surrounding access and equity for marginalized populations in PK-20 STEM settings: open to full-time faculty and postdoctoral fellows within 10 years of terminal degree completion	Funded by National Science Foundation; organized by Measurement, Statistics and Evaluation Program Faculty at University of Maryland, College Park	https://education.umd.edu/academics/departments/hdqm/research/nsf-qrm-scholars-program
Institute of Mixture Modeling for Equity- Oriented Researchers, Scholars, and Educators (IMMERSE) Fellowship	Fellowship	Early career scholars and postdoctoral fellows; advanced graduate students also considered	Funded by Institute of Education Studies; Organized by University of California, Santa Barbara faculty and collaborators	https://www.aera.net/SIG087/Research-in-Mathematics-Education
Equity in Math Education Research Grants (EMERG) Program	Grant	Early career scholars (within seven years of obtaining a doctoral degree)	National Academy of Education	https://naeducation.org/emerg-program-home-page/
Faculty Early Career Development Program (CAREER)	Grant	Tenure-track assistant professors (or equivalent title)	National Science Foundation	https://new.nsf.gov/funding/opportunities/career-faculty-early-career-development-program
AERA Special Interest Group for Research in Mathematics Education (SIG-RME) Graduate Student Representative	Service	Graduate students	American Education Research Association Special Interest Group for Research in Mathematics Education	https://www.aera.net/SIG087/Research-in-Mathematics-Education
PCMI Workshop on Rehumanizing Mathematics	Workshop	Mathematics education professors, mathematics teachers, mathematicians	Institute for Advanced Study/Park City Mathematics Institute Mathematics	https://www.ias.edu/pcmi/programs

Table 2

Other Early Career Scholar Opportunities

Name	Type	Eligibility	Funder/Organizer	Website
AERA Outstanding Dissertation Award	Award	Awarded doctoral degree within the past 2 years but no later than November prior to the award date	American Education Research Association	https://www.aera.net/SIG168/Awards/Outstanding-Dissertation-Award
NAEd/ Spencer Dissertation Fellowship	Fellowship	Doctoral candidate with all pre-dissertation requirements completed, plan for completing dissertation within 1-2 years, and education research project	National Academy of Education, Spencer Foundation	https://naeducation.org/naed-spencer-dissertation-fellowship/
NaEd/ Spencer Postdoctoral Fellowship	Fellowship	Pre-tenure, doctoral degree within past 1 to 6 years, education research experience and project	National Academy of Education, Spencer Foundation	https://naeducation.org/naed-spencer-postdoctoral-fellowship/
MET-AMTE NCTM Early Career Research Grant	Grant	Early career math educator (doctorate within past 5 years) or advanced doctoral student in math education and current member of NCTM and AMTE	National Council of Teachers of Mathematics, Association of Mathematics Teacher Educators, Mathematics Education Trust	https://www.nctm.org/uploadedFiles/Grants_and_Awards/Grants/2024_Winter/Info/Early_Career_Research_Euguene_Clara_Smith_Fund_Application.docx.pdf
TODOS-NCTM Mathematics Teacher Education Grant	Grant	A PK-12 school seeking to promote multilingualism in mathematics; math coaches, teachers, or administrators can apply	TODOS: Mathematics for All, National Council of Teachers of Mathematics	https://www.nctm.org/uploadedFiles/Grants_and_Awards/Grants/2024/fostering-support-of-mathematics-learning-for-multilingual-learners-description.pdf
NCTM Mathematics Education Trust	Grants scholarships, and awards	Varies by opportunity	National Council of Teachers of Mathematics	https://www.nctm.org/Grants/
CPM Exploratory Research in Mathematics Education Award	Grant	Education researcher, awarded doctoral degree in mathematics education (or related field), and employed by a university or research organization	CPM Educational Program	https://cpm.org/exploratory-award/
AMTE Early Career BIPOC Faculty Mentoring Program	Mentorship	Early career BIPOC faculty (tenure-track & professional/ teaching tracks), completed doctorate within the past ten years	Association of Mathematics Teacher Educators	https://amte.net/content/amte-early-career-bipoc-faculty-mentoring-program
AMTE Community Circles	Professional Learning Community	AMTE member	Association of Mathematics Teacher Educators	https://amte.net/content/amte-community-circles
AMTE Manuscript Review Group	Professional Learning Community	AMTE member, submit manuscript in advance, attend AMTE Conference	Association of Mathematics Teacher Educators	n/a
PME-NA Working Groups	Professional Learning Community	Attend PME-NA conference	North American Chapter of the nternational Group for the Psychology of Mathematics Education	https://ed.psu.edu/pme-na-2025
Black Womxn in Mathematics Education	Professional Learning Community	Mathematics educators who identify as Black womxn	Black Womxn in Mathematics Education (BWXME)	https://bwxme.com/
MAA Project NExT	Professional Learning Community	Recent doctorate in a math-related field, minimal post-secondary teaching experience, full-time math teaching position in US or Canadian university	Mathematical Association of America	https://maa.org/maa-project-next/

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NOTES FROM THE FIELD

A Reflective Homework System in Mathematics Courses

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KEYWORDS *reflective abstraction, metacognition, formative assessment, calculus education*

Introduction

Reflection is a valuable way of thinking that is an important part of learning (Moon, 2013) and professional development (Schön, 2017). Students often write reports, compile personal development portfolios, write reflective essays/journals, etc. When completing such reflective assignments, the students must analyze what they did, how they did it, and why they did it. In this process they also think about what they learned and how their knowledge and skills developed.

The authors have taught at a total of nine higher education institutions between them. In our experience teaching mathematics at those institutions, assessments in traditional mathematics courses are typically not reflective or have no reflective component. Typical assessments consist of regular homework assignments and/or quizzes at least partially graded for correctness, chapter or midterm exams, and a final exam. All these assessments are summative in that they grade for correctness and check whether learning has already occurred (Blankman, 2024). Homework assignments are peculiar here because they occur during learning and students have very little time to acquire mastery before they receive a permanent grade. Instead, homework assignments should be formative and allow students to make mistakes and identify ways to improve without risking permanently damaging their grade (Blankman, 2022; Vatterott, 2018). Summative assessments should be reserved for their traditional role in other subjects as exams or quizzes given at the end of a chapter, unit, course, or topic.

In order to ensure that homework assignments are formative and include a reflective component, we propose the adoption of a reflective homework system, which we will describe in detail in the next section. The homework format described can be used and adapted to many different contexts and courses as desired by the instructor. This homework system possesses a reflective component that follows the theoretical framework of reflective abstraction described by Dubinsky (1991). Reflective abstraction is the ability to arrive at new knowledge by reflecting on knowledge one already possesses without the need for additional information.

After elaborating on our rendition of a reflective homework system, we turn our attention to investigating potential student experiences with such a homework system in the realm of mathematics education. This is a good setting for such a homework system, since mathematics is perceived as one of the most difficult subjects among students, particularly those who are not mathematically inclined (Kajamies, et al., 2010). Previous research has indicated that important issues to address in mathematics education are retention (see e.g., Rohrer & Taylor, 2006) and student anxiety (see e.g., Adamu, 2014). Thus, there is a need to develop homework systems that strengthen knowledge retention and reduce student anxiety. We also think that it would help to provide a procedure that is easy for students to follow and increases student motivation in completing homework.

We sought to better understand the impact that a reflective homework system had on our students at our home institution. A more rigorous quantitative study would be needed to establish a causal relationship between the observations we made here and the homework method proposed.

To this end, we administered a reflective homework system in a number of our undergraduate calculus courses and then implemented anonymous surveys to evaluate the impact.

Specifically, the reflective homework system was administered in the undergraduate courses Calculus 1 and 3 during the Spring 2022 and Fall 2022 semesters. These courses took place at a large university in the Western United States, and each course had roughly 30 students. These courses were chosen for two reasons: (1) the curriculum structure of these courses is similar; and (2) there is a different level of mathematical experience between students in each course.

A survey was administered four times during the semester that each course was offered. While each of the four surveys was different, all four surveys had common questions that allowed the progression of student responses throughout the semester to be tracked.

Calculus 1 and 3 have a similar curriculum structure in the sense that they cover the same notions but in two different settings: Calculus 1 strictly analyzes *single-variable* functions while Calculus 3 focuses on *multivariable* functions. The curricular similarity between Calculus 1 and 3 was relevant to structuring the four surveys so that each survey contained calculus-specific questions uniformly across the two courses.

Considering different levels of mathematical experience is also relevant to this discussion. First, including experiences of students of varying levels of mathematical exposure provides valuable information to determine whether the observed effects of students the homework method were due to mathematical experience or the method itself. Second, comparing the responses of students with different levels of mathematical experience justifies considering the implementation of a reflective homework system in different levels of the undergraduate mathematics curriculum. This will be discussed in more detail later.

The Reflective Homework System

As mentioned above, the authors observed in teaching at several institutions that it seems that most assessments typically given in mathematics courses at all levels are summative assessments. Summative assessments are high-stakes assessments that check for student learning and penalize students for falling short. These are used in most subjects as periodic checks for student competence that occur after learning (Blankman, 2024). Because of the high-stakes nature of these assessments, students are more inclined to find shortcuts or even to cheat to improve the grade they will receive on the assessment. Setting the cheating issue aside, looking for shortcuts or cramming to improve performance on a summative assessment does not seem to encourage students to reflect on or improve their understanding. As a result, in a course with only summative assessments, a student's likelihood of producing deep learning about a subject appears limited (Vicente, et al., 2021). Even worse, the high-stakes nature of these assessments tends to increase student anxiety, at least about the topic at hand and even potentially about the entire academic subject as well (Ismail, et al., 2022).

Formative assessments, on the other hand, are low- or no-stakes checks for student learning that offer opportunities for students to make mistakes without penalty, encourage students to recognize their own learning deficiencies, and provide a chance for students to improve their learning. These assessments are designed to encourage deeper learning about a subject, teach students how to identify their own weaknesses and improve, and encourage a growth mindset (National Council of Teachers of Mathematics, 2013).

In many subjects, students are assessed via a combination of formative and summative assessments. Some assessments, such as quizzes and exams, are typically summative regardless of the subject, but the homework and practice assessments are usually formative (Mogboh & Okoye, 2019). However, in traditional mathematics courses, even homework assignments are almost always summative. In such courses, a typical homework assignment consists of a list of problems for students to solve (or in more advanced settings, sometimes a list of theorems for students to prove). Students are graded for accuracy in these assignments, lose points if they get problems wrong, and typically do not have any means of improving those grades after the fact. While instructors may intend homework assignments to be practice for students, homework becomes a high-stakes performance to exhibit learning rather than a tool to induce learning when the grade is based on accuracy. This is akin to permanently deducting points from a drama student's grade if that student does not perfectly perform a part in a play after just one rehearsal. It is no wonder that many students exhibit math anxiety.

This inability for the typical mathematics homework assignments to encourage and induce student learning can be further complicated by a delay in receiving feedback if grading takes a long time. When a student submits a homework assignment, there can be a multi-day delay before they receive their graded assignment back, with not just a score, but hopefully some written feedback from the instructor for each problem the student did incorrectly. Writing feedback that is effective for student learning takes time, and high student-to-instructor ratios can yield a large number of assignments to grade, resulting in several hours of grading for each homework assignment. Depending on the complexity of the course, students may even turn in the next homework assignment before they receive feedback on the previous one. As a result, students may continue to make similar mistakes from one homework assignment to the next. Furthermore, the delay in receiving feedback means students must try to process that feedback well after they completed an assignment and have moved on to thinking about other problems. Thus, the feedback students get may not be on problems that are recent enough to be remembered clearly or as meaningful to them as when they first attempted to solve them.

Given these drawbacks to the typical mathematics homework model, we sought an alternative that would be more beneficial to mathematics students. Reflecting upon our own experiences as successful mathematics students, as well as current research on the subject (e.g Booth, et al., 2013), we noted that one of the things that helped us was the ability to see a worked-out solution to a problem that we just attempted. This allowed us to check whether our understanding was correct and, if it was not, to search for an error and understand how we made that error. This helped us to (1) identify misconceptions that we had about how to solve a problem and correct them before they became a habit, (2) internalize the kinds of mistakes that we might be prone to, especially if we noticed a pattern, which would help us avoid those mistakes in the future, and (3) learn how to evaluate our own work and look for common errors even when there were no solutions available. We thought then that the best way to do homework in a mathematics class was to create a formative assessment that encouraged students to have a similar experience.

To facilitate that similar experience for students, homework assignments were designed in the following way. This method can be summarized as having students *Attempt, Compare, and Revise* their homework. Each part of the process is described below identified by the emphasized words in each corresponding paragraph.

First, students were provided with a list of problems to try in the form of a worksheet and instructed to *attempt* each problem on their own. They were encouraged to use whatever resources for help they needed such as their textbooks, instructor office hours, working together, tutoring, etc.

Second, students were provided a set of completely worked out solutions to each problem. These solutions were intended to be instructive and so were quite detailed in their explanation, even providing potential alternate solutions to given problems. Students were instructed to *compare* their completed first attempts to the solutions and "self-grade" their work. Students then identified any errors they made and made comments about where they went wrong. Students were encouraged to do this in a different color than their original work or on a separate piece of paper to distinguish the comments from the original work. If students found that they did the problem completely correctly, they were instructed to indicate that they did so to make it clear that they compared their correct work to the solutions.

One might argue that the solution key might not be sufficiently self-contained, and that the student might need additional information to successfully arrive at new knowledge via the process of reflection. Sometimes students may find that they cannot identify their error even though their answer is incorrect, or students may have another question about a problem to which they do not know the answer. In these cases, students were instructed to write their questions down in a different color to draw attention to the question when the instructor reviewed the work later. The instructor answered these questions directly when reviewing student work, which allowed the instructor to provide targeted feedback where it was most optimal in helping students. This element of allowing students to easily access direct feedback from the instructor either utilized reflective abstraction by reminding students of information they already learned in class but were not connecting to the problem at hand or it filled any information void that students were not able to fill themselves through this process. Moreover, instructors were able to provide this feedback while grading for completeness which only adds minimally to the time required to grade the assignment.

Students were usually provided with the list of problems and solutions at the same time. For their first attempts, students were instructed that they were not allowed to use the solutions even though they were encouraged to use any other resource. The advantage to this approach is that students can access the solutions immediately after their first attempts allowing them to get immediate feedback while the problem is still recent enough to be remembered clearly.

Third, students were instructed to *revise* all their first attempt solutions that they found were incorrect. Revisions were required to be separate from first attempts and should have resulted in complete and correct solutions. This ensured that students reinforced the correct way to solve problems more than just simply reading a correct solution. Furthermore, this entire process ensured that students considered each homework problem deeply at least twice, and three times if their initial attempt was incorrect. This part of the system is particularly designed to strengthen retention of new knowledge and skills. It is noteworthy that there is research that supports that such a practice will help with retention: (Brown, et al., 2016) showed that students that had the correction opportunity for a mid-term exam performed better on the final exam compared to students that were just given an answer key for the mid-term exam.

Finally, students collected their first attempts, comparisons/corrections, and revisions together and submited them for grading. Instructors responded to all questions indicated by students in their comparison comments and graded the assignments for completeness, that is, so long as the students performed all the above steps for each problem, they receive full credit for the assignment. Since students have a completed set of solutions, the need for detailed instructor feedback is significantly reduced. Instead, instructors needed only to identify that students completed the work and to answer students' specific questions, providing feedback that was more targeted to where students needed help the most. This minimized the time from student submission to instructor response. Moreover, the fact that the homework was graded for completeness is the key to making these assignments formative and low-stakes. Students were not penalized for making mistakes or for getting answers wrong. Instead, students were rewarded for spending time learning and improving their ability. Because instructors also answered questions that students had, students received active feedback on problems that they cared about most and hence were more likely to review that feedback afterward.

Figures 1 and 2 provide an example of a student response to a homework question using this method. The first image of Figure 1 depicts an actual first attempt and comparisons/corrections from a student, and the second image is the same student's subsequent revision with the correct answer. The first image of Figure 2 is an example of an instructor-provided solution, while the second image is an example of actual student work that includes a question for the instructor to respond to.

This is not dissimilar to how many writing classes employ multiple rounds of editing and revision when assigning writing projects for students. Thus, the pedagogical idea of this method is not novel and certainly has been used for many years in non-mathematical contexts (Graham, Harris, & Hebert, 2011). However, this method *is* novel in mathematics courses and yields a new way of providing formative assessments to mathematics students.

FIGURE 1

The Compare and Revise Portion

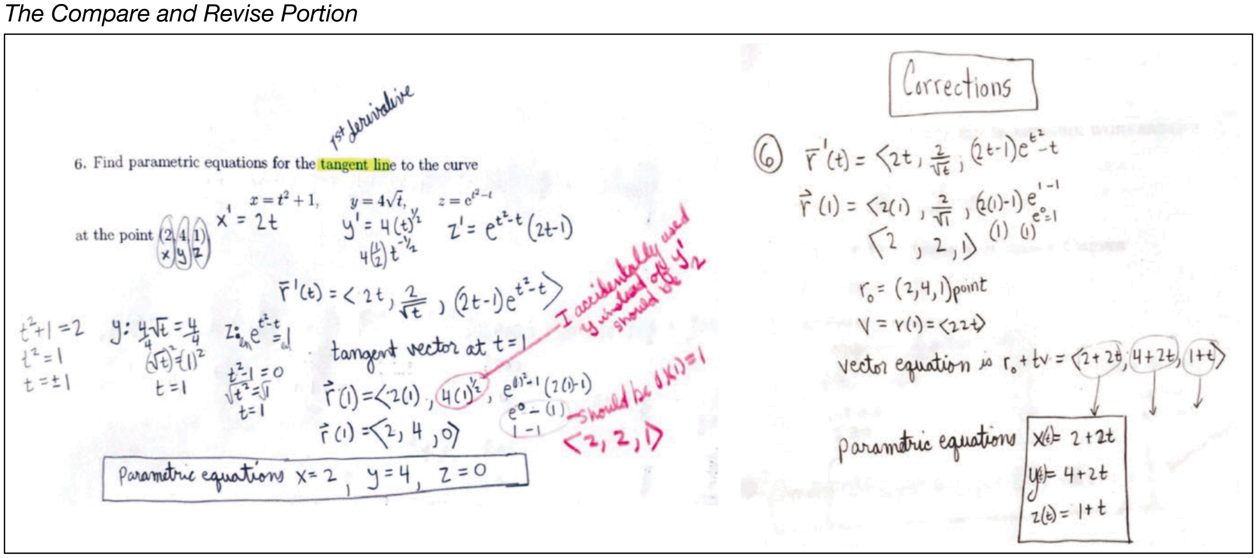
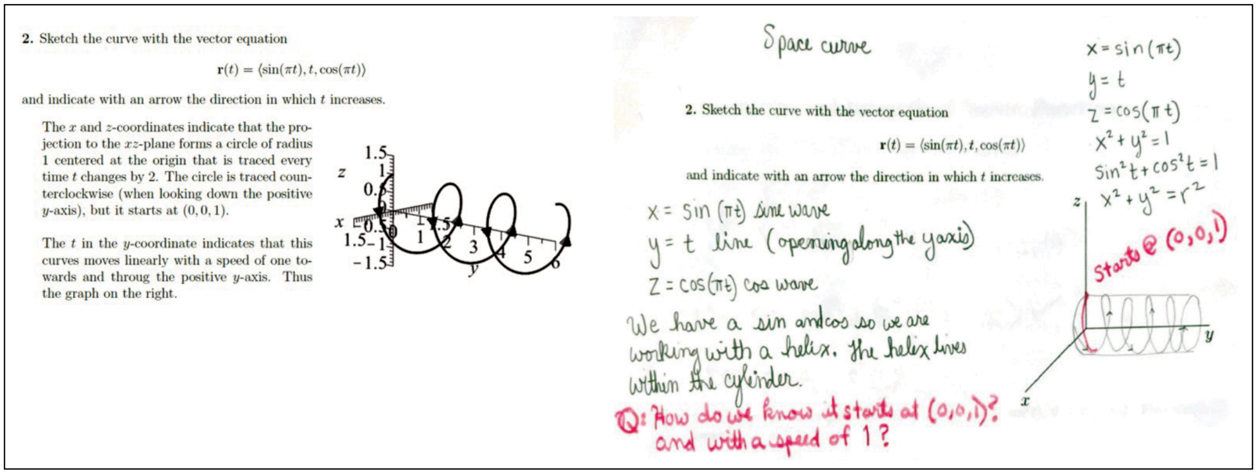


FIGURE 2

In-line Student Question



Surveys and Progress Checking

To assess student perception and experience with the homework system, anonymous surveys were implemented in the courses Calculus 1 and 3 during 2022 at a large Western open enrollment institution. In each class, the surveys were administered four times during the academic semester. The surveys had a proper disclaimer indicating that they were anonymous, optional, and did not impact student grades. The four surveys were titled: Beginning of Semester Survey, Differentiation Survey, Integration Survey, End of the Semester Survey.

Each survey consisted of 11-13 questions, all of which have the same answer options: Strongly Agree (1), Agree (2), Neutral (3), Disagree (4), Strongly Disagree (5). While each survey was different and contained specific topic-related questions, there were seven questions repeated across the surveys. The purpose of this approach is to provide longitudinal analysis: how student responses evolved over the course of the semester as the students gained more experience with and exposure to the homework system.

The following are the questions repeated across the surveys to track progress. For reference and discussion purposes, a label was assigned to each of these questions.

- (Q1: innovation) I find the homework system to be innovative.
- (Q2: learning advancement) The homework system is designed to help me better learn the material.
- (Q3: content retention) The self-grading and revision portion of the homework system will help me retain the material.
- (Q4: feedback) The fact that I can directly ask the instructor questions within the homework assignments is helpful.
- (Q5: anxiety) The grading scheme for the homework system alleviates anxiety.
- (Q6: motivation) The homework system in this course motivates me to complete the homework assignments.
- (Q7: student success) The homework system in this class sets me up for success.

Q1 was implemented to see whether students have been exposed to a similar type of homework system before. This is important to consider, as previous exposure might impact student perceptions and survey responses. Of particular interest were new student experiences.

Q2 and Q3 were chosen to see whether students felt that they were better able to learn and retain the course material. Retention is the cognitive information processing of the learner which involves understanding, information processing, and storing within memory (Lutz & Huitt, 2018).

Q4 was incorporated to see if instructor feedback in the homework system would lead to positive student experiences. This would provide evidence that having the instructor fill any information void would help students successfully arrive at new knowledge via the process of reflection. It would also solidify the instructor's importance as part of the process.

Q5 and Q6 were crucial to the surveys as they deal with student anxiety and motivation.

Anxiety in mathematics is a well-known issue and well-researched problem. For example, Hembree states that, “Mathematics anxiety is related to poor performance on mathematics achievement tests. It relates inversely to positive attitudes toward mathematics and is bound directly to avoidance of the subject” (Hembree, 1990). If students felt that this homework system relieved anxiety, then just that itself may justify implementing it.

Motivation in mathematics is just as well-studied. Hannula explains that “Motivation is conceptualized as a potential to direct behavior through the mechanisms that control emotion. As a potential, motivation cannot be directly observed. It is observable only as it manifests itself in affect and cognition, for example as beliefs, values, and emotional reactions” (Hannula, 2006). This suggests that a good way to observe whether the homework system impacts motivation is by including a question concerning the topic in our surveys.

Finally, Q7 was included to see if students perceived the homework system as promoting success in the course. Since typical calculus courses like these are heavy on examinations (summative assessments), success in such a course can be quantified by performing well on exams. We believe this is how students will measure their success in the courses.

In addition to the seven questions that repeated throughout the surveys, each survey contained specific questions, some of which were related to content covered in calculus. Because Calculus 1 and Calculus 3 both cover a similar curriculum in different flavors, these topics were uniform across both courses. The reason for including course content specific questions is due to the importance of calculus in a typical STEM curriculum. Calculus courses have been described as “gateway” courses in many recent calculus education research studies (e.g. Bressoud, Carlson, Mesa, & Rasmussen, 2013). A gateway course serves as a stepping-stone for students to complete their primary degree plan. Because of the obvious significance of promoting student success in calculus, content-specific questions were added to see if implementing an innovative formative assessment would help students learn the challenging topics covered in these courses. These additional questions for each survey are listed below.

Beginning of Semester Survey (Survey 1)

- I expect to do well in this class.
- The course has a balanced grading scheme between tests and homework.
- I expect this course to strengthen my mathematics computational skills.
- I expect this course to strengthen my conceptual and theoretical mathematics skills.
- I expect this course to introduce me to real-life applications of Calculus.
- The procedure in the homework system is easy to follow.

Differentiation Survey (Survey 2)

- The homework strengthened my computational skills with problems involving differentiation.
- The homework had a good balance of conceptual problems and computation problems involving differentiation.
- The homework had a sufficient amount of applications of differentiation.
- I feel confident with the differentiation portion of this course.

Integration Survey (Survey 3)

- The homework strengthened my computational skills with problems involving integration.
- The homework had a good balance of conceptual problems and computation problems involving integration.
- The homework had a sufficient amount of applications of integration.
- I feel confident with the integration portion of this course.

End of Semester Survey (Survey 4)

- I did as well as I expected to do in this course.
- This course strengthened my mathematics computational skills.
- This course strengthened my conceptual and theoretical mathematics skills.
- This course exposed me to real-life applications of Calculus.
- The homework system directly impacted my performance in this course.
- The homework system helped me perform better on the written examinations.

Many of these questions were chosen to see if the homework system was well-designed, for example, to see if the instructions for the homework were easy to follow. The supplemental questions were also used to see if students found the chosen homework problems as sufficient for the mastery of the given topic. Note that in the “End of the Semester” survey, two final questions were added to see if students perceived the homework system as positively impacting their performance in the course.

Survey Results

In this section, the results of the surveys are summarized. We start with the results of the repeated questions (Q1-Q7), which are central to this article. Recall that participants had the following answer options: Strongly Agree (1), Agree (2), Neutral (3), Disagree (4), Strongly Disagree (5).

Figure 3 shows the means of the responses to the repeated questions across all the courses in which the survey was administered. Observe that all the means are below 2.0 (Agree). This indicates a positive reception to all aspects of the homework system. Furthermore, there is a significant trend of increased agreement in the responses to Q5 and Q6 across the surveys. Figure 4 illustrates the distribution of responses to these two questions across the surveys via a box-whisker plot.

In Table 1 are listed the means of the responses to other selected survey questions. These indicate that students find the procedure of the homework system straightforward and find that the homework system benefited their course performance.

To compare and contrast the two courses, the Calculus 1 and Calculus 3 groups were separated and considered individually. This helped gauge the reception of the homework system between students of different levels of mathematical experience. Figure 5 shows the collective distribution of student responses to Q1-Q7 in each course. Note that both histograms have roughly the same shape. However, it appears that overall student perception and attitude was more positive in the Calculus 3 group.

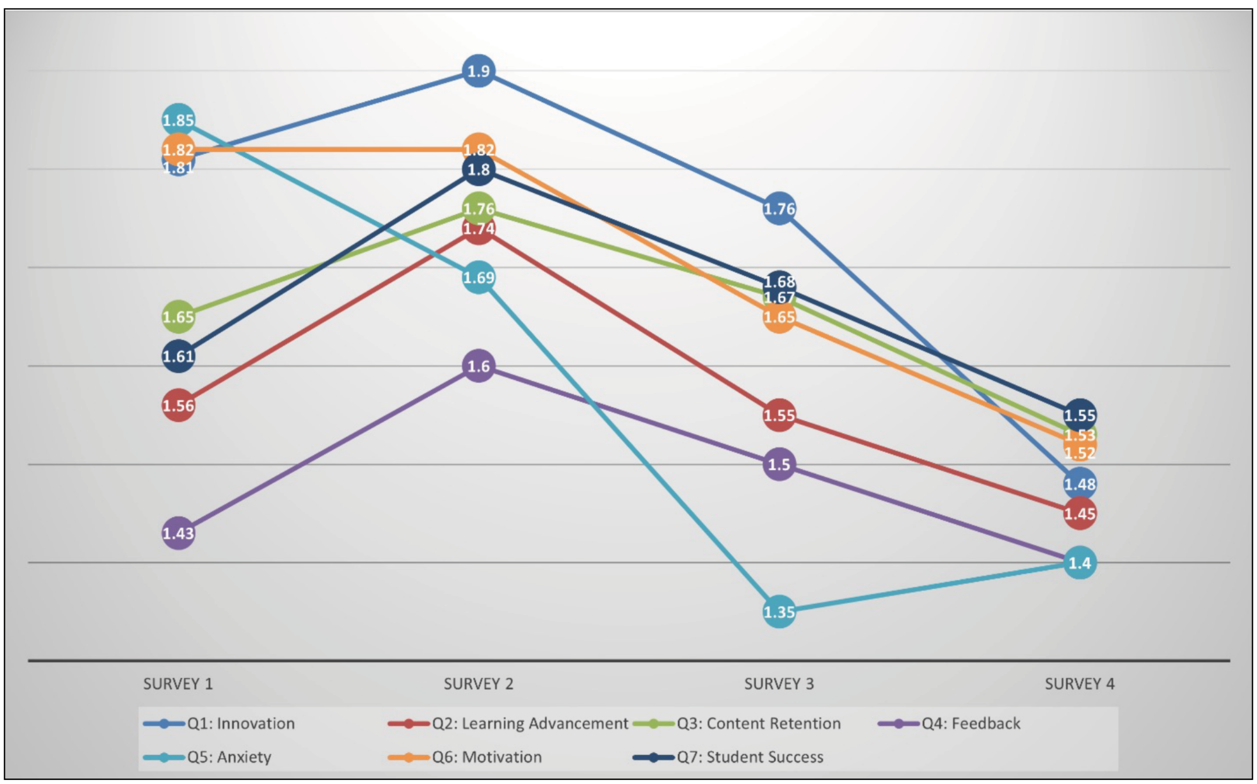
Table 1

Student Responses to Survey Questions about Homework System.

Survey questions regarding the homework system and its procedures	Descriptive statistics"
The homework system directly impacted my performance in this course.	*Mean: 1.48* *Std Dev: 0.83*
The homework system helped me perform better on the written examinations.	*Mean: 1.62* *Std Dev: 0.95*
The procedure in the homework system is easy to follow.	*Mean: 1.54* *Std Dev: 0.76*

FIGURE 3

Mean Trends of Repeated Questions



Figures 6A and 6B shows the mean trends for Q1-Q7 in both courses separately.

Note that among the Calculus 3 group, each of Q1-Q7 shows a significant trend toward increased agreement, especially near the end of the semester.

On the other hand, among the Calculus 1 group, all questions except Q5 appear to initially trend toward greater disagreement and then stabilize to roughly the same level they had originally. Thus, the trend for those questions is essentially flat. The initial trend toward greater disagreement might be explained simply by the initial shock some students experience in Calculus 1 due to it commonly being the first highly conceptual mathematics course they have taken. However, this survey does not provide enough evidence to address that claim.

Q5 in the Calculus 1 group dealt with student perceptions of their level of anxiety. This question is the only one in the Calculus 1 group where the student responses showed an overall trend toward increased agreement. This is similar to the trend observed for Q5 among the Calculus 3 group. Thus, while this homework system may not have had as pronounced of an effect in other aspects for Calculus 1 students compared to Calculus 3 students, it did lower their anxiety towards the course. As mentioned before, the reduction of anxiety on its own may make this system worthwhile, especially since it does not seem to adversely affect any of the other questions in Q1-Q7. Since Q5 shows a significant trend toward increased agreement in both courses, respective response distributions of Q5 are compared in Figure 7.

FIGURE 4

Overall Response Distribution for Q5-Q6

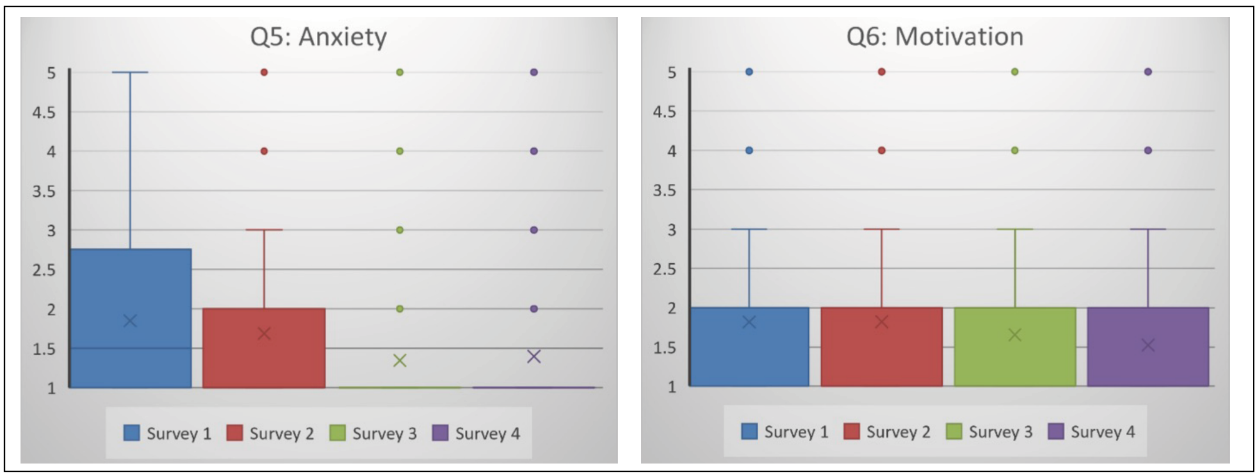


FIGURE 5

Q1-Q7 Collective Response Distribution in Calculus 1 and 3

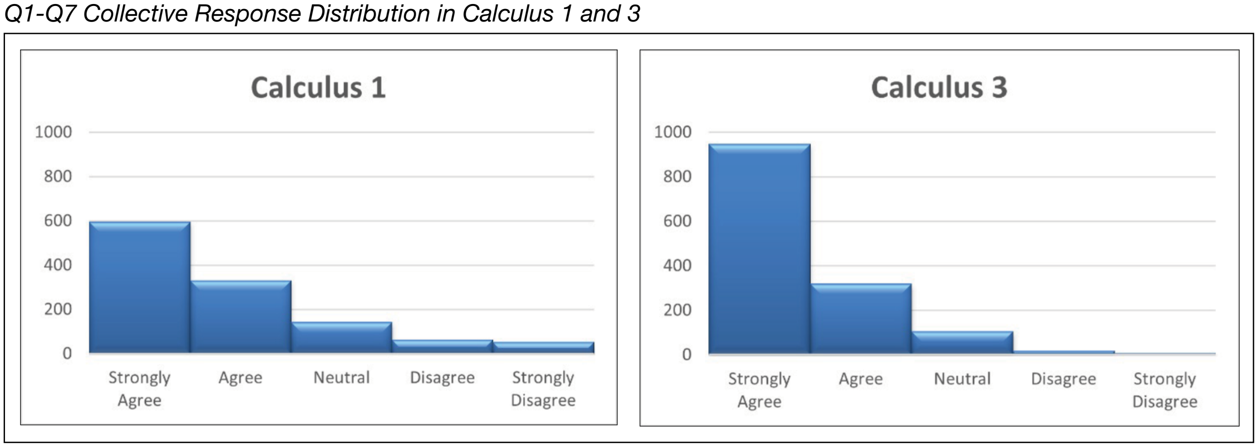


FIGURE 6A

Individual Course Survey Mean Trends

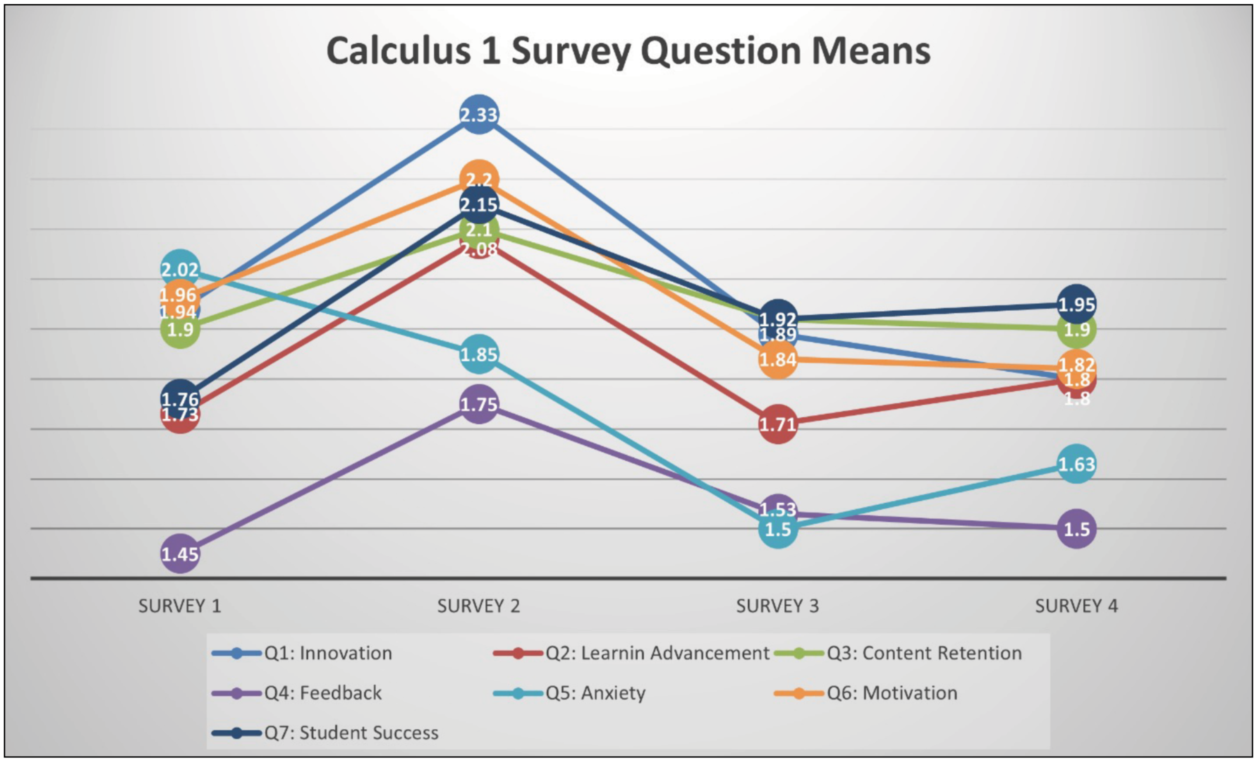


FIGURE 6B

Individual Course Survey Mean Trends

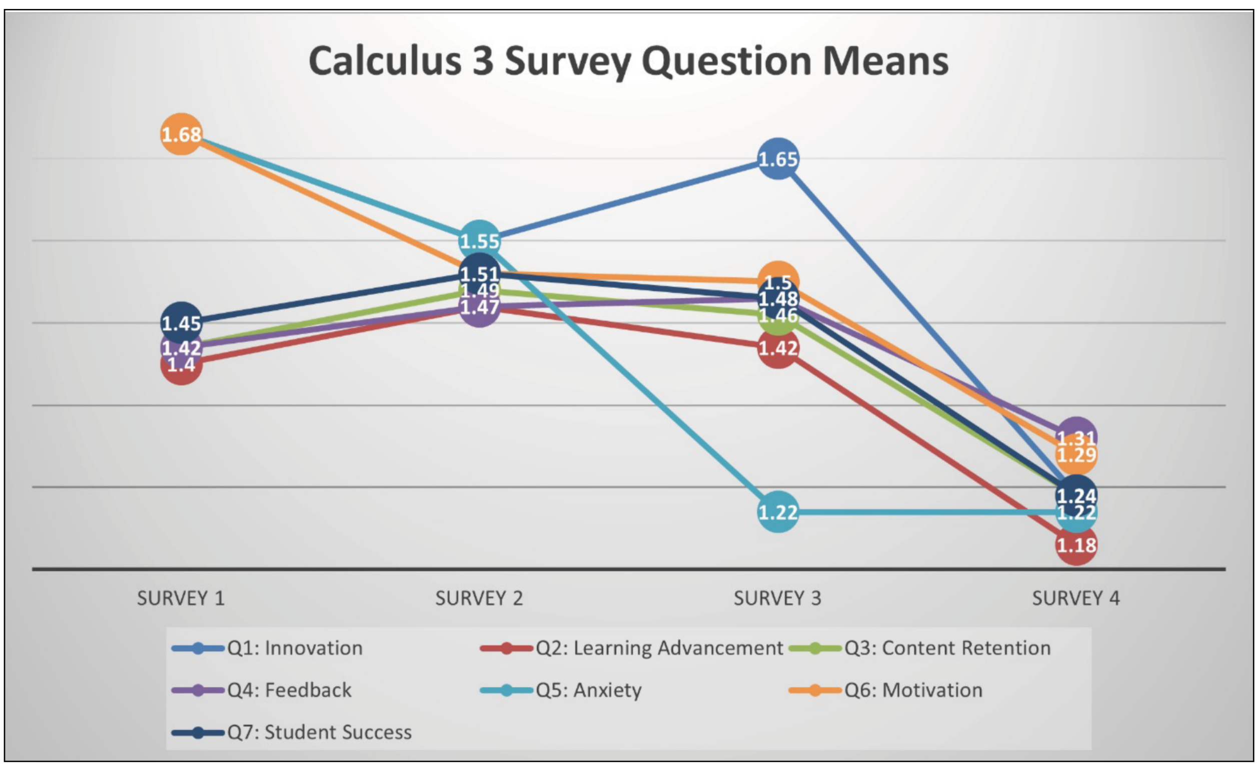
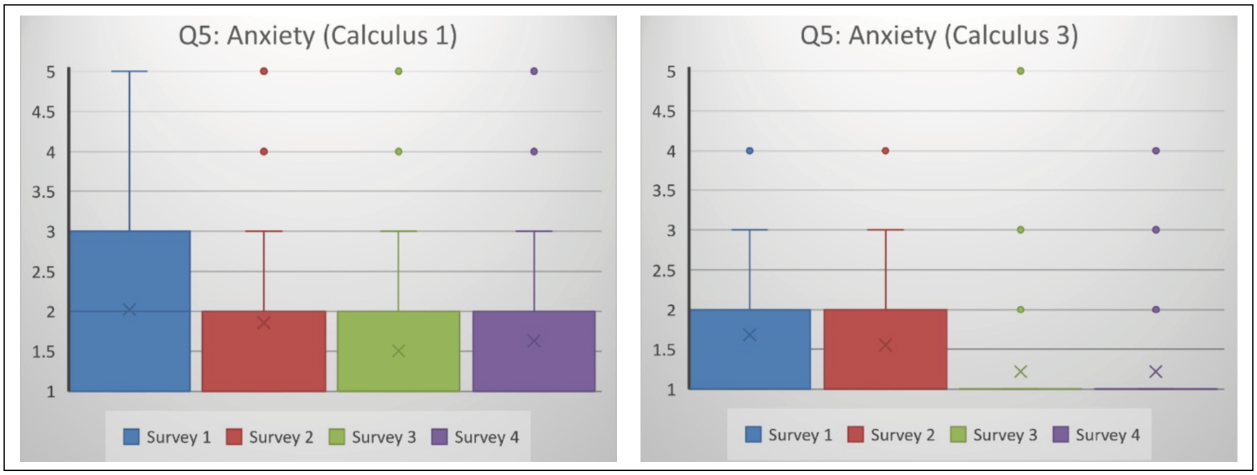


FIGURE 7

Overall Response Distribution for Q5-Q6



Conclusion

We conclude from the surveys that the reflective homework system had a generally positive impact on student attitudes in those courses, though this trend is more pronounced among the Calculus 3 group. In each of Q1-Q7, the collective (Calculus 1 and Calculus 3 combined) means of the responses in the final survey arrived at a lower number (recall that lower is better) than the initial survey, though some decreased significantly more than others. More specifically, Calculus 3 had significant trends toward increased agreement in each of the Q1-Q7, while Calculus 1 only showed a significant trend toward increased agreement for Q5. Thus, both groups experienced a significant reduction in anxiety. Overall, while the reflective homework system was well received by both groups, our survey indicates that the system had a larger impact for the more advanced students (the Calculus 3 group).

The contrast between the effects of the reflective homework system between Calculus 1 and Calculus 3 could be due to several factors. We suspect the main culprits are the curriculum itself and how the reflective homework system was utilized by students.

Curriculum-wise, Calculus 1 is often the first highly conceptual mathematics course that undergraduate students take. Thus, this course initially has an adjustment period, and students typically require a few weeks of acclimation. As noted, this might also explain some of the initial upward trends that one sees in Figure 6.

The other suspected factor is the utilization of the reflective homework system by students. A large portion of students taking Calculus 1 are freshmen/sophomores, and hence they are still honing their study skills. This may mean that they are more prone to procrastination or are initially less comfortable with an assessment that requires more procedures to follow. Some modifications to the homework method could potentially address how students utilize it. For example, instructors could delay providing the solutions until after a first attempt is completed.

These surveys focused mainly on intangibles in student experience such as opinions on feedback, levels of anxiety, and motivation. Another question to be considered with this homework method is how it affects performance in a math class, such as exam scores or grades. While our survey included student perceptions of these, a study that better quantifies the effects of this homework method on academic performance would be an interesting next step toward understanding the impact of this homework method.

It is also an interesting question of how a similar homework method to this would function in a more advanced proof-based course. The authors utilize a similar method for those courses, but it does come with other considerations. These include the much wider variance of correct responses to a question asking students to prove a statement and the fact that seeing a solution too soon can potentially stifle the creative process needed to create a valid proof. The authors have had to make modifications to this reflective homework system to make it viable for these courses. A study similar to the one in this paper or a more detailed one that quantifies the impact of this kind of reflective homework system on those upper division courses would be useful.

While the results of these surveys do not prove that this method will produce better outcomes for students compared to more traditional homework methods, they do suggest that this homework method has a generally positive impact on student experiences, particularly in reducing anxiety and improving motivation. Given these positive results, we encourage mathematics teachers to try implementing a reflective homework system in their math classes akin to the one described in this paper.

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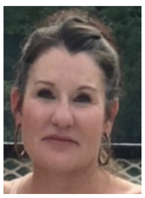

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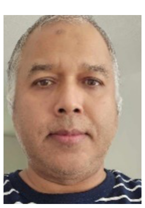

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ACKNOWLEDGEMENT OF REVIEWERS

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JOURNAL OF MATHEMATICS EDUCATION AT TEACHERS COLLEGE

CALL FOR PAPERS

This call for papers is an invitation to mathematics education professionals, especially Teachers College students, alumni, and associates, to submit articles describing research, experiments, projects, innovations, or practices in mathematics education. The journal features full reports (approximately 3500 to 4500 words) and short reports (approximately 500 to 1500 words). Full reports describe findings from specific research, experiments, projects, innovations, or practices that contribute to advancing scholarly knowledge in mathematics education. Short reports ("Notes from the Field") provide examples, commentary, and/or dialogue about practices out in the field of mathematics education or mathematics teacher education; examples from classroom experience are encouraged. Although many past issues of *JMETC* focused around a theme, authors are encouraged to submit articles related to any current topic in mathematics education, from which pertinent themes for future issues may be developed. Articles must not have been submitted to or accepted for publication elsewhere. All manuscripts must include an abstract (approximately 150 words in length) and keywords. Manuscripts should be composed in Microsoft Word and follow APA format. Guest editors will send submitted articles to the review panel and facilitate the blind peer-review process. Articles for consideration should be submitted online at jmetc.columbia.edu, and are reviewed on a rolling basis; however, to be considered for the Fall issue, articles should be received by **September 1, 2025**.

CALL FOR REVIEWERS

This call for reviewers is an invitation to mathematics educators with experience in reading or writing professional papers to join the review panel for future issues of *JMETC.* Reviewers are expected to complete assigned reviews within three weeks of receipt of the manuscript in order to expedite the publication process. Reviewers are responsible for editorial suggestions, fact and citations review, and identification of similar works that may be helpful to contributors whose submissions appear appropriate for publication. Neither authors' nor reviewers' names and affiliations will be shared with one another; however, reviewers' comments may be sent to contributors of manuscripts to guide revision of manuscripts (without identifying the reviewer). If you wish to be considered for review assignments, please register and indicate your willingness to serve as a reviewer on the journal's website: jmetc.columbia.edu.

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Do you know someone who would be a good candidate to serve as a guest editor of a future issue of *JMETC?* Students in the Program in Mathematics Education at Teachers College are invited to nominate (self-nominations accepted) current doctoral students for this position. Being asked to serve as a guest editor is a testament to the high quality and standards of the student's work and research. In particular, anyone nominated as a guest editor should be a current doctoral student whose scholarship is of the highest quality, whose writing skills are appropriate for editorial oversight, and whose dedication and responsibility will ensure timely publication of the journal issues. All nominations should be submitted to Ms. Juliana Fullon at jmf2213@tc.columbia.edu .