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A Century of Leadership in Mathematics and Its Teaching

Growth through Reflection in Mathematics Education

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AIMS AND SCOPE

The Journal of Mathematics Education at Teachers College (JMETC) is a recreation of an earlier publication by the Program in Mathematics and Education at Teachers College, Columbia University. As a peer-reviewed, semiannual journal, it is intended to provide dissemination opportunities for writers of practice-based or research contributions to the general field of mathematics education. Although each of the past issues of *JMETC* focused on a theme, the journal accepts articles related to any current topic in mathematics education, from which pertinent themes for future issues may be developed.

JMETC readers are educators from pre-kindergarten through 12th grade teachers, principals, superintendents, professors of education, and other leaders in education. Articles appearing in the *JMETC* include research reports, commentaries on practice, historical analyses, and responses to issues and recommendations of professional interest.

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PREFACE

The spring 2019 issue of the Journal of Mathematics Education at Teacher College features five articles that focus on aspects of teacher growth and development. In particular, a theme of "reflective practices" emerges throughout the articles—how can educators, through introspection and evaluation, become better practitioners of their craft? Whether considering the relationship between secondary education and universities, how preservice teachers grow and change through preservice experiences, the role of culture in educational practices, or how to develop and measure teacher competencies, each article examines a different aspect of how educators of mathematics learn and grow. Included in this is a biography of a recently deceased mathematician that serves as an inspiration and role model to future mathematics educators and scholars.

Gujarati explored how the Child Mathematics Inquiry Portfolio (CMIP), a semester-long project in an elementary mathematics methods course involving repeated observation and reflection on one child's mathematical development, influenced preservice teachers. This structure was chosen in place of several short independent observations over the course of the semester. The CMIP increased the preservice teachers' understandings of mathematics teaching and learning and its relation to teacher student interaction, as well as their mathematical confidence. This work shows that reflective practice can have a positive impact on preservice teachers as they head into their own classrooms.

In her biographical article Gibbons examined the life of mathematician Maryam Mirzakhani through the lens of a mathematics educator. Mirzakhani, the first and only female winner of the Fields Medal, died in 2017 of breast cancer. In this brief biography the author discussed aspects of Mirzakhani's personal history, professional life, and mathematical contributions. Gibbons also argued that both Mirzakhani herself and her mathematical discoveries could serve as an inspiration to students, helping to reveal the humanistic aspect of mathematics and mathematical learning.

Johnson, Nebesniak and Rupnow investigated how collaborations between school districts and local universities led to the creation of district specific graduate courses for teachers. The courses focused on curricular and instructional reform and had key components of research, practice, and leadership. Teachers who took these courses were able to become curriculum leaders in their respective schools and felt more able to improve their own teaching practices. Benefits flowed in both directions as faculty members also felt more informed about curriculum and practice trends. This work can serve as a model for creating professional development for teachers tailored to their local school districts.

PREFACE (Continued)

Thomas and Berry synthesized qualitative research on Culturally Relevant Pedagogy (CRP) and Culturally Relevant Teaching (CRT) in relation to mathematics teaching and learning in primary and secondary schools. Twelve articles met the criteria for inclusion and were analyzed according to the framework of a qualitative metasynthesis. The findings were that CRP and CRT include five major components: caring, knowledge of contexts and teaching practices using contexts, knowledge of cultural competency and teaching practices using cultural competency, high expectations, and mathematics instruction/teacher efficacy and beliefs.

Ahuja described the theory of professional competence in teaching mathematics by examining a cross-cultural collection of teaching practices of teachers in India and the United States. By comparing different case studies, the author develops a rich conceptualization of the relationship between knowledge base and competence in mathematics teachers. The author's theory attempts to explain how teachers develop and display competence, and it justifies competence as a dynamic interplay of knowledge situated in the context of classroom, school, and wider social culture.

Together these five articles serve as a framework for the ways that we as educators can investigate our own practice. Looking from the perspectives of pre-service teacher, secondary educator, college professor, and even curriculum developer, these articles provide examples of ways the mathematics education field is continuing to develop in the 21st century. Taking the time to reflect on all aspects of our practice is a necessary component of this development.

Paul Gray
Sarah Nelson
Guest Editors

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Reflecting on Action: Implications from the Child Mathematics Inquiry Portfolio

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Brown University

ABSTRACT This article examines how the Child Mathematics Inquiry Portfolio (CMIP), a semester-long field experience project attached to an elementary mathematics methods course, can impact preservice teachers' understanding about teaching and learning mathematics and set them on more positive mathematics teaching journeys as they prepare to enter the field. This action research study, grounded in reflective practice, was contextualized in undergraduate and graduate elementary mathematics methods courses with 92 participants over two academic years. Data came from preservice teachers' written reflections on the child mathematics inquiry process. Findings reveal that the CMIP impacted the preservice teachers' mathematics teaching and learning in four major areas: bridge between mathematics methods course, textbook, and actual classroom experience; mathematical confidence; greater understanding of how teachers shape students' mathematics dispositions; and personal. Implications for mathematics teacher education are discussed.

KEYWORDS *preservice elementary teachers, mathematics inquiry, action research, reflective practice, field experiences*

Mathematics scores from students in the United States are regularly compared to those in other countries whose mathematics achievement is greater on international assessments such as Trends in International Mathematics and Science Study (TIMSS) and Program for International Student Assessment (PISA). The results have shone a spotlight on areas for mathematics improvement. Consequently, there has been a great push toward increasing student mathematics achievement in the United States (NCTM, 2014). Beginning with the mathematics standards set forth by the National Council of Teachers of Mathematics (NCTM, 1989; 2000) and continuing with the Common Core State Standards (CCSSO, 2010), there has been a focus on what students need to learn in order to be successful in college and careers in the 21st century. With increased emphasis on these standards comes increased responsibility and accountability for teachers to meet them. Competency in both content and pedagogical content knowledge is essential for teachers to effectively teach their students (Shulman, 1987) and meet those standards. However, mathematics is a content area that can be challenging for early childhood and elementary teachers (Buss, 2010; Phillip, 2007) since they may not feel as confident in teaching it as other areas (Gujarati, 2013). In an extensive examination of the Teacher Preparation Programs of approximately 100 institutions in the northeast region of the United States leading to initial teacher licensure in either early childhood or elementary education, Pimentel (2018) found that most preservice elementary teachers are only expected to take one mathematics methods course, which covers all K-6 content in one semester, to prepare them for all their mathematics teaching responsibilities that lay ahead. In some instances, that one methods course is integrated with science methods, and relies on prior undergraduate preparation in mathematics which varies widely. Mathematics educators, therefore, need to maximize time with preservice teachers in their one semester to ensure that they are prepared with appropriate content and pedagogical content knowledge. This preparation should also entail reflective practice since the degree to which preservice teachers' practice improves depends on how well and how frequently they reflect on their practices (Artzt, Armour-Thomas, Curcio, & Gurl, 2015). Reflection is a meaning-making process that can move preservice teachers to the next level of understanding so they can become more effective teachers and decision makers (Rieger, Radcliffe, & Doepker, 2013). Intentional and structured reflection needs to be built into preservice teachers' experiences more, as research indicates this is currently lacking (Rieger, Radcliffe, & Doepker, 2013).

This article presents an action research study which examines the impact of the Child Mathematics Inquiry Portfolio (CMIP), a semester-long field experience project attached to an elementary mathematics methods course which preservice elementary teachers can engage in to maximize learning how to teach mathematics and learning more about themselves as teachers, within the parameters of only one semester. For the CMIP, preservice elementary teacher candidates are required to observe one child (and teacher/classroom) in grades 1-6 during mathematics lessons/activities, collect and analyze several work samples (artifacts) from that child, document what they learned about that child relative to mathematics, and reflect on how this semester-long project benefits their learning to teach mathematics. The research questions which inform this study are: (1) What can preservice elementary teachers learn through engaging with the CMIP to aid with their mathematics teaching and learning? and (2) How can that information be used to inform how I structure my mathematics methods course to use the time most effectively? The voices of preservice teachers are prominent in this study because it is the teachers and their practices which impact student achievement and dispositions in the discipline.

Frameworks

Action Research

This study is grounded in teacher action research which is a systematic, intentional study of one's professional practice (Cochran-Smith & Lytle, 2009). At the heart of teacher action research is teachers seeking change by reflecting on their practice (Dana, 2013; Foulger, 2010); it is change research (Pine, 2009). Action research assumes that teachers are the agents and sources of educational reform and not the objects of it (Pine, 2009). The cyclical nature of action research involves identifying an area of focus, collecting data, analyzing and interpreting data, and developing an action plan (Mills, 2014). Action researchers engage in multiple cycles of inquiry as they reflect on each phase.

Reflective Practice

Since action research incorporates a reflective stance into daily routines and a willingness to critically examine one's teaching in order to improve or enhance it (Mills, 2014), this study is also grounded in reflective practice (Dewey, 1933; Schön, 1983). Teacher reflection is a vehicle for knowledge growth. While thinking is automatic and unregulated, reflective thinking has a purpose, a goal. "[Reflective thought] emancipates us from merely impulsive and merely routine activity…It enables us to know what we are about when we act. It converts action that is merely appetitive, blind, and impulsive into intelligent action" (Dewey, 1933, p. 17).

According to Dewey (1933), there are two phases of reflective thinking: (1) State of doubt, hesitation, perplexity, mental difficulty, in which thinking originates and (2) Act of searching, hunting, inquiring, to find material that will resolve the doubt, settle and dispose of the perplexity. By thinking reflectively, a person can "transform a situation in which there is experienced obscurity, doubt, conflict, disturbance of some sort, into a situation that is clear, coherent, settled, harmonious" (Dewey, 1933, pp. 100-101). One can think reflectively only when one is willing undergo the trouble of searching which is an active process that involves open-mindedness, wholeheartedness, and responsibility (Dewey, 1933).

Schön (1983) posits the notions of reflection-in-action and reflection-on-action. The former is sometimes described as "thinking on [one's] feet." It entails building new understandings to inform one's actions in the midst of the situation that is unfolding. The latter is done after the encounter. It enables one to spend time exploring why one acted as one did. In doing so, one develops sets of questions and ideas about one's activities and practices to propel one forward. The power of these processes is when reflection-in and on-action are taken together since action is an integral component of reflective practice as reflection is not a singular retrospective act but an ongoing process to prepare teachers to take action. If reflection does not translate into taking action, it cannot be considered reflective (Reynolds, 2011).

Methodology

Sites and Participants

The site purposefully selected for this study was Greenville College (pseudonym), a small liberal arts college located in the northeast region of the United States. The study was contextualized in two undergraduate and four graduate elementary mathematics methods courses (six sections of the same course) which I taught, with 92 preservice teachers ranging in age from 20 through mid-50s and who were working toward their initial teaching licensure in childhood education (grades 1-6) during the 2011-2012 and 2012-2013 academic years. So that I had a sense of who these teachers were relative to mathematics, their initial course assignment was to write a mathematics autobiography in which they detailed their beliefs about what mathematics is, the evolution of those beliefs, and their relationship to this content area.

As part of the semester-long elementary mathematics methods course, teacher candidates at Greenville College were expected to complete eight hours of a field experience since the state in which they were enrolled required observation hours prior to student teaching. As the action researcher/professor of the mathematics methods course, I was particularly interested to know if the way I structured the observation hours was beneficial. Unlike many of my colleagues who chose to have eight short disparate assignments to fulfill the requisite observation hours, I chose to assign one comprehensive integrated project, which spanned the semester. I designed the CMIP to accompany and extend what I was teaching in the methods course. Teacher candidates selected an elementary school of their choice, observed a child (and teacher) in one elementary classroom (grades 1-6) during mathematics lessons/activities during the semester, collected and analyzed student work samples (artifacts), and reflected on the process of this child's mathematics inquiry regarding their current and future teaching practices (see Appendix for the CMIP's guidelines). The major difference between the expectation of the undergraduate and graduate teacher candidates was the collection of the number of artifacts, with graduate teacher candidates collecting and analyzing two additional artifacts.

For the CMIP, teacher candidates chose a school setting and grade level they were interested in teaching in the future. They could choose a child according to an area of interest (e.g., special education, English language learners, gifted and talented). There did not have to be a particular reason for the selection of the focal child, but I encouraged them to select a child based on an area within mathematics they were genuinely interested in studying. School sites included suburban, urban, public, private, inclusive, and self-contained settings, with the majority being public, inclusive, suburban classrooms. Table 1 shows the breakdown of grade levels which teacher candidates chose as the sites for their child mathematics inquiry.

Table 1

Breakdown of Grade Levels for CMIP Sites

Grade Level	Number of Teacher Candidates
First	23
Second	11
Third	19
Fourth	17
Fifth	14
Sixth	8

Data Sources and Analysis

The CMIP was comprised of four parts: (I) Introduction; (II) Artifacts; (III) Integration and Interpretive Analysis; and (IV) Reflection on the Process (See Appendix for the CMIP's detailed guidelines). The data collected for this study primarily came from Part IV, Reflection on the Process, in which teacher candidates had to reflect on how the CMIP informed their understanding and teaching of mathematics, the usefulness of it when they assume their own classrooms, and any questions which remained.

Using qualitative research design (Creswell, 2013; Guba & Lincoln, 1989), analysis of data began as soon as it was collected. Data were manually analyzed within and across cases utilizing Grounded Theory (Strauss, 1987; Strauss & Corbin, 1998) where theorizing grows from the data rather than from a pre-existing framework used to confirm or disconfirm a theory. Analytic memos (Saldaña, 2016) were utilized in this process. The CMIP reflections were analyzed by reading and re-reading them, paying close attention to what was learned from the process of reflection on the experience to understand what preservice teachers learned/took away from it to assist them on their teaching journeys, and what questions still remained after this experience. Inductive coding methods were primarily utilized during the first and second cycles of the coding process. To ensure trustworthiness of the data, member-checking (Guba & Lincoln, 1989) was utilized. I asked participants any clarifying questions, as needed, to ensure that I understood the intention behind their quotes in the reflections.

Findings

Analysis of data reveals that the Child Mathematics Inquiry Portfolio (CMIP) impacted the preservice teachers' mathematics teaching and learning in four major areas: Bridge between Mathematics Methods Course, Textbook, and Actual Classroom Experience; Mathematical Confidence; Greater Understanding of How Teachers Shape Students' Mathematics Dispositions; and Personal.

Bridge between Mathematics Methods Course, Textbook, and Actual Classroom Experience

The preservice teachers overwhelmingly felt that the CMIP brought what they were learning in our mathematics methods course to life. The strategies which were shown and discussed in our methods course and read about in the Van de Walle, Karp, and Bay-Williams (2010) textbook were being implemented in actual classrooms. Noted one preservice teacher observing in a second grade classroom, "When reading about certain strategies in a textbook, they may seem productive and useful, but you cannot know that until you apply them to your students." The area that most preservice teachers saw in practice was differentiated instruction. At the heart of differentiated instruction is tailoring instruction to meet diverse student needs. Nearly all of the teacher candidates felt that the process of engaging in a child mathematics inquiry over the course of an entire semester was such a valuable experience because they witnessed the importance of differentiated instruction first-hand. In their portfolios, they emphasized that it was a start to hear about differentiated instruction in the mathematics methods course, but it was a vastly different experience witnessing and experiencing it in their field experience classrooms.

The preservice teachers had a greater sense of what it means to differentiate instruction for particular learning styles to really get to know who a student is on a deeper level. Here are two reflections about learning styles on different parts of the spectrum; the first on a first grade child who struggled with mathematics and the second on a fifth grade child classified as gifted and talented:

> Being able to observe mathematics lessons being taught in both a general education classroom setting and a resource room setting was eye opening. I was amazed at what a different child Sara (pseudonym) was once placed in a small group setting. She went from struggling in math to being able to shine in the resource room setting. This made me realize that sometimes academic issues are not always what they seem. I first believed that Sara did not have the cognitive ability to complete the grade level work she was given. I went on to later find that Sara just needed a different type of instruction than what she was being given.

> Often, gifted and talented students are forgotten or ignored in the classroom. When teachers need to turn their attention to struggling students, the gifted and talented students receive less one-on-one attention. By focusing on Caitie's (pseudonym) growth and development, I have thought more about students who require less remedial work and more challenges. This case study has also helped me to realize that challenging gifted and talented students does not have to take much time. Instead, it requires extra thoughtfulness about a specific student's interests and needs. This project will help me design appropriate instruction so that I can support my students' mathematics development at their own *zone of proximal development* (Vygotsky, 1978) no matter which zone that is.

From this experience of engaging in the CMIP, the preservice teachers became much more aware of the range of student abilities within each classroom. These teacher candidates now realized just how critical it is to observe students, their behaviors, and learning processes to inform their own instruction. They further learned how important it is to know each individual student and check in with each student regularly. This concept, echoed by nearly all teacher candidates, was nicely summarized by one teacher candidate observing in a fifth grade classroom:

> This project will definitely serve as a reminder throughout the upcoming years, especially when first beginning my teaching career, to make sure I pay attention to every student in my class and to get to know them and their learning styles—learning their weaknesses in order to counterbalance it with their learning strengths. It will also serve as a great reminder to never be the teacher that I encountered on so many occasions growing up who could not be bothered to pay attention to the students who did not understand the material, and who expected everyone to work at a set pace in order to get through the curriculum in an efficient manner.

Mathematical Confidence

From reading their mathematics autobiographies at the onset of the semester, at least three-fourths of the preservice teachers said they had a negative relationship or self-image with mathematics based on experiences in their formative years which left them feeling not as confident to teach it as other areas. However, the majority of the teacher candidates felt that the CMIP helped them gain much needed mathematical confidence. By the end of the semester, mathematics was not such "a foreign and scary subject" because it was taught in a "more real-world and comprehensive way" both in the methods course and in the field experience classrooms. As one preservice teacher observing in a fifth grade classroom noted:

> This project has allowed me to change my prior negative view of teaching math into a positive one. The knowledge I have gained on techniques and strategies of teaching mathematics has shown me how to teach mathematics in an exciting and effective way.

Stated by another teacher candidate observing in a fourth grade classroom:

> I have always been hesitant to teach math because I know that in order to teach a subject one has to have an understanding of the subject itself. This project, while allowing me to focus on how one student comes to learn and deeply understand a topic, gave me incredible insight into the necessity of providing students with a few strategies to utilize when solving a problem; giving them the ability to make math a creative subject rather than one based on strict and structured rules.

These teacher candidates gained a greater appreciation for mathematics, a new perspective on the subject, and confidence to teach students a variety of strategies using more manipulatives. As another teacher candidate mentioned "I will take away the confidence that there is a way to reach every student; you may just have to think about and present the concept another way." Overall, the CMIP deepened teacher candidates' understanding of current practices for teaching mathematics which led to greater confidence when they assume their own classrooms.

Greater Understanding of How Teachers Shape Students' Mathematics Dispositions

Many preservice teachers came away with a heightened understanding of the role they play in shaping students' mathematics dispositions for life. Since many candidates had expressed in their autobiographies that they had negative experiences with their own elementary mathematics teachers and that they perceived as ineffectiveness of some of their teachers, the CMIP allowed for more positive experiences and more positive role models than they experienced growing up. It hit home for many that they have a large role in shaping student dispositions, and that this role is important in not perpetuating a negative cycle of mathematics affect. Reflected one teacher candidate observing in a third grade classroom:

> I discovered that the role of the math teacher is very different from the role that many of my elementary math teachers assumed. Teaching math is not simply about memorizing formulas and solving problems; it is about its application to the real world and its use in critical thinking development… Overall, I learned new strategies for teaching math as an integral life process rather than as a set of facts and problems. Math is often a subject that many people struggle in but if it is taught in a hands-on way that makes connections to students' real worlds, it can make a huge difference in how students think about and approach math. As a math teacher, I will now assume a larger responsibility of teaching math for life.

Reflected another teacher observing in a fourth grade classroom:

> Math is a subject that is very difficult, confusing, frustrating and disliked by many students. I believe that many of these negative feelings toward math have developed because students are often not taught mathematical concepts using their strengths or through strategies which meet their individual learning styles and needs. After this experience, I now intend to incorporate as many different teaching strategies and activities as possible in every math lesson I teach. By assessing my students and their learning styles, I will plan my lessons to include different elements which will appeal to all different types of learners.

Personal

For many teacher candidates, the CMIP impacted them in highly personal ways. One teacher candidate found a new colleague (and role model) in her field experience teacher and plans to keep in touch. Another teacher candidate came away from the experience inspired to emulate some interactive PowerPoints in her future sixth grade classroom as a result of what she successfully saw enacted in the observed class. Yet another felt confident that she chose the right career as a result of her experience. For others, it solidified their interest in teaching a particular grade level. Still others learned more about specific learning styles such as gifted and talented and language based disabilities. Each candidate had something personal to take away but as one teacher candidate observing in a fifth grade classroom said, engaging in the CMIP helps to develop a personal teaching philosophy:

> It is my belief that the purpose for projects and assignments such as these, coupled with field experience and observation, is to help teacher candidates to develop our own philosophy in teaching, and that each methods course helps us to further investigate our philosophies in specific subjects, such as math.

Teacher candidates felt more invested in this project than in ones for other courses where they may have just observed more generally, as one teacher candidate observing in a sixth grade classroom asserted:

> This project forced me to take on the teacher's role of tracking a student's progress and being able to identify their individual strengths and weaknesses. This was the first time I felt involved in a student's development and it changed my whole perception on observations; I was more invested in the process…As a teacher you can make such a difference in a child's life and seeing this first hand is so remarkable; it's a feeling I hope to carry with me as I begin to teach in my own classroom…The CMIP has let me look at how I can be a better teacher; how I can change the ways that I teach children to better benefit the ones who need more help than others.

Enduring Questions

Even with the tremendous learning experience the CMIP afforded them, these preservice teachers had questions to ponder as they begin to enter the teaching profession, as a result of their observations and experience with the child inquiry process. Most of the questions fell into three areas: differentiation; parental involvement; and mathematics methods in the current educational climate. A question representative of each area follows:

> How does one teacher provide the students with extra support or enrichment? How does a teacher, without additional assistance, effectively differentiate within a classroom to provide all students with the most beneficial and positive learning experience?

> How can teachers educate parents on the new methods being used for math in the classroom? What is being done in schools to help solve this issue and what is the best method to help parents become more involved in the curriculum provided at their children's school?

> After being introduced to all the exciting and motivating ways to teach children math through inquiry and exploration, I wonder how realistic it is to think I will be able to teach this way out in the real world. As a result of NCLB and standardized testing, this seems to be the age of prepackaged curriculum that leaves very little to the teacher's imagination. Therefore, is it realistic to believe a teacher (especially a new one) is going to be given the freedom to use her own teaching philosophy and present math through inquiry ideas in a classroom? Although we all like to think that we are willing to do whatever it takes to do what is best for our students, how realistic is it to think that a new teacher can venture away from the package of curriculum, lesson plans and activities she will most likely be handed when she starts teaching?

Discussion

The Child Mathematics Inquiry Portfolio (CMIP) not only encouraged preservice teacher candidates to observe, it also caused them to reflect on their own practices. As mentioned earlier, reflection is purposeful; it has a goal. In this case, the goal was to gain a heightened awareness of mathematics teaching and learning in today's classrooms through this formative assessment project. With continuous and sustained reflection resulting from this semester-long project, preservice teacher candidates learned more about teaching mathematics in elementary classrooms and more about themselves in relation to mathematics in their authentic field experience settings and appeared to begin to move from phase one to phase two of reflective thinking (Dewey, 1933). In short, they began to move from a state of doubt to search for ways to resolve the doubt, specifically pertaining to mathematics teaching and learning.

Prior to this child mathematics inquiry project, from reading the portfolios and earlier mathematics autobiographies, it appeared that the preservice teachers had many doubts about their teaching responsibilities, particularly in their confidence to teach mathematics. However, by the end of the project/semester, they all reported being more confident to teach as a result of learning about differentiated instruction, the role of the teacher in shaping students' mathematics dispositions, and strategies in authentic classrooms. They better understood mathematics teaching as they had sustained time in authentic contexts. These preservice teachers were asked to reflect predominantly "on action" (Schön, 1983) by reflecting on each individual artifact (Part II), across the collection of artifacts (Part III), and on the inquiry process as a whole (Part IV). The multiple opportunities for reflection enabled them to understand the ways this project was beneficial to their future teaching careers. It is unlikely that this would have happened if they had not been asked to engage in this project and instead only relegated to learning through a textbook and an on-campus mathematics methods course. However, although the teacher candidates appeared to move toward phase two of reflective thinking, questions about teaching and learning mathematics still remained, as described earlier. This is natural for a novice teacher just beginning the teaching journey. As one preservice teacher noted, "many questions can really only be answered through experience over time."

For myself as the action researcher and mathematics methods professor, the enduring questions provide potential directions for future coursework. Having completed several cycles of inquiry over two years, patterns of questions emerged which are now incorporated into the methods courses I teach.

Educational Significance

Teachers are at the forefront of this study because it is necessary to look beyond simple curriculum reform efforts. Instead, we must focus on the actual practices of teaching mathematics in order to help identify the additional support systems needed to bring about the desired changes. We must also look at challenges which can hinder change or high-quality mathematics teaching. It is important to listen to preservice teachers' voices and hear what they have to say in their own words because they represent the future of the profession. Often teachers' voices become muted in larger educational conversations, but this study puts their voices front and center.

As the action researcher, this study had personal significance for my methods courses in that it validated how I used the requisite observation hours. I could have had eight separate short assignments as many of my colleagues did, but chose one comprehensive, sustained project and the preservice teachers found value in the way the project was structured—namely that they were more invested in it since it represents what teachers are expected to do around formative assessments. The CMIP has allowed me to restructure my coursework to tap some of the enduring questions especially surrounding differentiated instruction.

As Pimentel (2018) found, most preservice elementary teachers only have one mathematics methods course to prepare them for all their mathematics teaching responsibilities in grades K-6. Looking beyond my courses, teacher educators can maximize their time by engaging in the CMIP and framing the work of the semester within real world mathematical contexts. Furthermore, they can learn valuable information about their students and what is learned from this experience in conjunction with the methods course; these can give preservice teacher candidates valuable glimpses into what they will do as teachers even before they begin their student teaching practicum. In today's educational climate, data-driven instruction to inform teaching practices is a major classroom responsibility. This project provides teacher candidates an experience with data-driven instruction as they analyze and interpret students' mathematics work samples (artifacts). Several preservice teacher candidates' reflections noted that they were then able to extrapolate their experience with one student to a larger scale with the whole class.

Reflective practice is critical in preservice teacher education. The CMIP underscores how much preservice teachers can learn about teaching mathematics by engaging in and reflecting on real world/authentic experiences, and not just busy work to fulfill a degree requirement. The beauty of the CMIP is that it can be done anywhere in the world; it is easily transferrable because it is not country-specific. Teacher candidates just need a classroom to conduct observations, a child to observe, and a purpose for the observations, which can vary according to specific needs a professor may see and want to explore further. Overall, the CMIP is a valuable project to prepare teachers for authentic, real world, teaching experiences and set them on more positive mathematics teaching journeys in which they can positively impact student achievement and mathematics dispositions.

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Appendix

Guidelines for the Child Mathematics Inquiry Portfolio

Field Experience Assignment: Child Mathematics Inquiry Portfolio

Teacher candidates are required to complete ***8 hours*** of fieldwork for this course.
Each teacher candidate is required to fill out ONE field experience log documenting their hours. The major assignment for this fieldwork experience is a Child Mathematics Inquiry Portfolio.

This semester-long project provides an opportunity to develop close observational and authentic assessment skills, document a child's growth and development over time, and carefully interpret observational data to inform your teaching.

Create a portfolio for one child in your field experience setting (grades 1-6). Observe and record this child's mathematics behavior, document activities, and systematically collect and analyze artifacts that demonstrate his/her individual strengths, needs, interests, and achievements.
Artifacts may include: student work samples, checklists, journal writings, informal interviews, curricular materials, anecdotal records, conference notes, end of unit tests, etc.

This semester-long project will be regularly discussed during class sessions so that ongoing feedback and suggestions can be incorporated.

Please note: No real names of children, teachers, or school settings should be written. Please create pseudonyms for all names used.

Outline for the Child Mathematics Inquiry Portfolio

I. **Introduction** [Approximately 2 pages]

- Context/background information which includes:
  - The type of educational setting (e.g., urban, suburban, public, private, grade level, school demographics)
  - Classroom environment (e.g., number of students, physical layout, mathematics décor)
  - Mathematics curriculum used (include title and publisher) and mathematics approach in the classroom (e.g., teacher's philosophy)
  - A brief description of the child
  - Rationale for choosing the particular child

II. **Artifacts** [Approximately 2 pages ***per artifact***]

- Approximately 4 or 5 pieces of student work (artifacts) and descriptive analysis
- Each artifact should contain the following information ***clearly labeled***:
  - Artifact # and a brief title (e.g., Artifact #1: Subtraction Fun)
  - Date and time artifact was observed/collected (e.g., October 13, 2011 from 9:30 – 10:15am)
  - A description of the activity/lesson observed
    - Give a brief summary of the lesson/activity
    - What were the directions given? Student expectations?

- ❍ Child participation in the activity/lesson
  - ■ Give an *objective* description of how the child approached the lesson/activity
- ❍ Reflections of the child's response to the activity/lesson
  - ■ These are *subjective* interpretations. For example, why do you feel the child may have approached the activity/lesson as he/she did? What are your impressions of the child's participation in the activity/lesson?

III. **Integration and Interpretative Analysis** [Approximately 3 pages]

- In analyzing the artifacts, what have you learned about this child and his/her mathematics thinking/skills/behavior? For example, is this child particularly strong or weak in a certain content strand? What type of mathematics learner is this child? (e.g., auditory, visual, kinesthetic, spatial). Please cite specific evidence.
- As a teacher candidate, if this were a child in your class, what might you have done the same/different to accommodate this child's mathematics needs?

IV. **Reflection on the Process** [Approximately 2-3 pages]

- A reflection on the child mathematics inquiry process. Consider the following questions:
  - ❍ How has the child mathematics inquiry project informed your understanding and teaching of mathematics?
  - ❍ How might this project be helpful when you assume your own classroom? In other words, what will you take away from this experience?
  - ❍ What questions still may remain?

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The Life of Maryam Mirzakhani

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ABSTRACT Maryam Mirzakhani is the first and the only female winner of the Fields Medal since its establishment in 1936. She is arguably one of the greatest mathematicians of our generation. This biographical paper outlines her life and work. Her mathematical theorems and noteworthy accomplishments are just as impressive as her determination, imagination, and optimistic outlook on life. Mirzakhani's success came from her passion, creativity, and playful approach to mathematics. She felt the most rewarding part of mathematics was the enjoyment of understanding or discovering something. Mirzakhani's work was visual and focused on patterns and ideas rather than on numbers and algorithms. She constructed mathematics in an artistic way, and her goal was to always find truth and beauty in the world. Mirzakhani will forever be an inspiration to anyone who has studied her work and read her story.

KEYWORDS *curves, geodesics, hyperbolic geometry, moduli spaces, topology, biography, history of mathematics*

"The beauty of mathematics only shows itself to more patient followers." ~ Maryam Mirzakhani

Iranian mathematician and Fields Medal winner Maryam Mirzakhani dedicated her life to developing theoretical mathematics. She contributed especially to the fields of Teichmüller and Ergodic theory, hyperbolic and symplectic geometry, and moduli spaces (Myers & Carey, 2017). She also blended mathematical areas of diverse fields, from algebraic geometry to topology and probability theory (Riddle, 2017). Mirzakhani's work has been labeled as "the beginning of a new era" because of her major contributions to the beautiful blend of dynamical systems and geometry that we are able to enjoy today (Myers & Carey, 2017). Although she is most known for being a prominent female mathematician, Maryam Mirzakhani was also a devoted colleague, friend, teacher, learner, mother, and wife. It is important to document the life of arguably one of the greatest mathematicians of our generation. There have been studies of Mirzakhani's work done in the past (Klarreich, 2018; O'Connor & Robertson, 2017; Myers & Carey, 2017; Rafi, 2017; Riddle, 2017; Series, 2017). This biographical paper examines the life of Maryam Mirzakhani through the lens of a mathematics educator.

Early Life

Maryam Mirzakhani was born on May 3rd, 1977 in Tehran, Iran to Ahmad Mirzakhani and Zahra Haghighi. Mirzakhani's parents always encouraged her to have a meaningful profession and supported her in everything she did (O'Connor & Robertson, 2017). She grew up in Tehran at the time of the Iran-Iraq war. However, she felt fortunate to come of age after the war had ended and the political, social, and economic environment had stabilized (Myers & Carey, 2017). She believed this allowed her great opportunities.

Mirzakhani had a sister and two brothers. She first became interested in the sciences when her older brother came home from school one day and began telling her about what he had learned (O'Connor & Robertson, 2017). Her first memory of mathematics was when her brother told her about summing numbers 1 to 100 (Mirzakhani, 2014). He shared how Gauss was alleged to have solved the problem. The solution fascinated her even though she was unable to find it on her own at the time.

Growing up, Mirzakhani had no intention of being a mathematician. At eight years old she used to tell herself stories about a girl who achieved great things, such as becoming mayor or traveling the world (Klarreich, 2018). It was clear at a young age that Mirzakhani had a wonderful imagination. As a child, her goal was to read every book that she could find. Mirzakhani's passion was reading novels, and her dream was to become an established writer (Mehta, Mishra, & Henriksen, 2016). Furthermore, Mirzakhani enjoyed watching biographies of famous women, such as Helen Keller and Marie Curie, on television (O'Connor & Robertson, 2017). Their stories sparked her ambition to do something great with her life (Klarreich, 2018).

Around the time the Iran-Iraq war ended, Mirzakhani was completing her studies at elementary school. She sat for and passed an admissions examination to Farzanegan, the National Organization for Development of Exceptional Talents, or NODET (Zorich, 2015). NODET was a middle and high school in Iran for girls with exceptional talents. Mirzakhani was accepted and decided to attend. During her first year at NODET, Mirzakhani did not do well in mathematics. Her teacher at the time told her that she was not talented in the subject (O'Connor & Robertson, 2017). Reflecting on this experience, Mirzakhani said, "at that age, it's so important what others see in you. I lost my interest in math" (Klarreich, 2018). Luckily, Mirzakhani had an encouraging mathematics teacher the following year. This greatly bolstered her confidence. She quickly began to once again show her talent in the subject.

In middle school (in the early 1990's), Mirzakhani met someone who would become a lifelong friend and share her motivation and academic interests. Her name was Roya Beheshti (Mirzakhani, 2014). One afternoon, Mirzakhani and Beheshti came across some mathematics questions from the International Mathematical Olympiad (Klarreich, 2018). This was an annual competition for high school students, and the girls were excited to be able to solve some of these problems on their own. Mirzakhani and Beheshti knew that they could do even better if they received mathematics problem-solving instruction like the classes that were already offered to boys at other high schools. Mirzakhani and Beheshti convinced the principal of NODET to organize preparation courses for the International Mathematical Olympiad (Zorich, 2015). Mirzakhani has noted that the principal's positivity and support influenced her life tremendously. The principal responded by saying, "You can do it, even though you'll be the first one" (Klarreich, 2018).

Before this moment, the Iranian Mathematical Olympiad team had never included females (Zorich, 2015). Mirzakhani and Beheshti attended their preparation courses and, through their determination, achieved excellent results. At the 1994 Hong Kong International Mathematical Olympiad, Mirzakhani won a gold medal with a score of 41 out of 42. She was the first female Iranian student to ever win a gold medal at such a competition (Obasi, 2016). Mirzakhani competed again at the International Mathematical Olympiad in Canada in 1995, where she became the first Iranian student to achieve a perfect score (42 out of 42) and to win two gold medals (Obasi, 2016). By this point Mirzakhani's outstanding mathematical ability was clear to those around her, and she was being hailed as a genius. Although she did not like the pressure that the title "genius" brought, Mirzakhani began to consider pursuing a career in mathematics in her senior year of high school. Even though mathematics at that time was not an easy career choice for an Iranian woman, her early success made it possible (Rafi, 2017). The more time that Mirzakhani spent on mathematics, the more excited she became. As she got older Mirzakhani began to enjoy solving harder problems in mathematics, especially those involving geometric structures (Mehta, Mishra, & Henriksen, 2016).

Education and Professional Life

Mirzakhani attended the prestigious Sharif University of Technology in Tehran for her undergraduate studies. At Sharif University, Mirzakhani and her classmates attended problem-solving sessions and informal reading groups (O'Connor & Robertson, 2017). She attributed much of her mathematical interests to the friendships and support that she gained from attending Sharif. Mirzakhani published several academic papers as an undergraduate, including "Decomposition of Complete Tripartite graphs into 5-Cycles" (published in 1995 with E. S. Mahmoodian), "A Small Non-4-Choosable Planar Graph" (1996), and "A Simple Proof of a Theorem of Schur" (1998) (O'Connor & Robertson, 2017). In 1999, Mirzakhani obtained her Bachelor of Science in Mathematics from Sharif University (Obasi, 2016). Mirzakhani moved to the United States to pursue a Doctorate of Philosophy in Mathematics at Harvard University. As a student at Harvard, Mirzakhani was best known for her relentless questioning. The language barrier was a definite challenge—Mirzakhani asked her professors questions in English while taking notes in Farsi (Myers & Carey, 2017). Mirzakhani's mathematical background prior to her studies at Harvard was mostly in combinatorics and algebra (Mirzakhani, 2014). Although she always enjoyed complex analysis, it was not one of her areas of expertise. This sparked Mirzakhani to attend a seminar taught by 1998 Fields Medalist Curt McMullen. She was fascinated by how McMullen could explain concepts simply and elegantly (Mirzakhani, 2014).

Mirzakhani went on to earn her Ph.D. from Harvard in 2004 under the supervision of McMullen as her academic advisor (Series, 2017). Curtis McMullen described Mirzakhani as a student who "was filled with fearless ambition" (Myers & Carey, 2017). He also recalled that she "had a sort of daring imagination. She would formulate in her mind an imaginary picture of what must be going on, then come to my office and describe it. At the end, she would turn to me and say, 'Is it right?' I was always very flattered that she thought I would know" (Klarreich, 2018). At the same time, Mirzakhani felt that she learned an endless amount from McMullen, and because of him, she developed a long list of initial ideas that she wanted to explore (Mirzakhani, 2014).

Mirzakhani's dissertation (2004) was titled, "Simple Geodesics on Hyperbolic Surfaces and Volume of the Moduli Space of Curves." In her dissertation, Mirzakhani solved several deep problems about hyperbolic surfaces. She created a formula that shows how the number of simple geodesics of length L grows as L gets larger (Klarreich, 2018). Another brilliant result of her dissertation was a new proof of the Witten conjecture (Mirzakhani, 2004). Mirzakhani's impressive connection between two major mathematical ideas brought her widespread recognition as a mathematician. Solving just one of these major research questions would have been impressive for any mathematician, but to solve both was simply phenomenal. Alex Eskin, a mathematician at the University of Chicago, put it succinctly— "It's the kind of mathematics you immediately recognize belongs in a textbook" (Klarreich, 2018). Her dissertation work resulted in multiple papers, including three that were published in top mathematics journals: "Weil-Petersson Volumes and Intersection Theory on the Moduli Space of Curves" (2007); "Simple Geodesics and Weil-Petersson Volumes of Moduli Spaces of Bordered Riemann Surfaces" (2007); and "Growth of the Number of Simple Closed Geodesics on Hyperbolic Surfaces" (2008) (Riddle, 2017). After turning down a junior fellowship at Harvard, Mirzakhani accepted a 2004 Clay Mathematics Institute Research Fellowship at Princeton University (Rafi, 2017). This fellowship provided Mirzakhani with a generous salary, paid research expenses, and freedom to choose where she wanted to perform research.

Mirzakhani worked for several years at Princeton University where she rose to assistant professor in a short time (Obasi, 2016). In 2008, at the age of 31, Mirzakhani became a Full Professor of Mathematics at Stanford University in California (Zorich, 2015). One of Mirzakhani's colleagues at Stanford, Ralph L. Cohen, described her as a wonderful colleague (Myers & Carey, 2017). He explained, "She not only was a brilliant and fearless researcher, but she was also a great teacher and terrific Ph.D. adviser. Maryam embodied what being a mathematician or scientist is all about: the attempt to solve a problem that hadn't been solved before, or to understand something that hadn't been understood before. This is driven by a deep intellectual curiosity, and there is great joy and satisfaction with every bit of success" (Myers & Carey, 2017). At that time, Mirzakhani was considered a leader in the fields of hyperbolic geometry, topology, and dynamical systems (Rafi, 2017).

Mathematical Contributions

It is not difficult to draw parallels between how Mirzakhani created stories as an eight year old and how she approached life as an adult (O'Connor & Robertson, 2017). The only difference was that her made up stories were based on mathematical research about hyperbolic surfaces, moduli spaces, and dynamical systems. Mirzakhani did not have a specific method for developing new proofs. She related this obstacle to "being lost in a jungle and trying to use all the knowledge that you can gather to come up with some new tricks, and with some luck you might find a way out" (Myers & Carey, 2017).

Mirzakhani's major contribution in mathematics was the theory of moduli spaces of Riemann surfaces. Mirzakhani mainly worked on problems related to geometric structures on surfaces and their deformations (Mirzakhani, 2014). She was interested in understanding hyperbolic surfaces and found it fascinating to discover the many ways one could look at the same problem and approach it with different methods and perspectives. She studied the geometries behind moduli spaces and their applications to differential, hyperbolic, and algebraic geometry. In her early work, Mirzakhani found the volume of a moduli space with a given genus as a polynomial in the number of boundary components (Obasi, 2016). As mentioned, this resulted in Mirzakhani discovering a newfound proof of Witten's conjecture. Edward Witten and Maxim Kontsevich's formula was on the intersection numbers of tautological classes on moduli space (Obasi, 2016). Mirzakhani's formula now included characteristic classes for the moduli spaces of Riemann surfaces with marked points (Zorich, 2015). In 2006, Mirzakhani took on the problem of what happens to a hyperbolic surface when its geometry is deformed using a mechanism akin to a strike-slip earthquake (Klarreich, 2018). Most mathematicians felt that this problem was completely unapproachable before Mirzakhani solved it with only a one-line proof.

Mirzakhani's work was influential in topology, prime numbers, and cryptography (Myers & Carey, 2017). Her findings were highly theoretical in nature and had surprising impacts on ideas in theoretical physics of how the universe came to exist. Since her proofs inform quantum field theory, they are applicable to engineering and material science (Myers & Carey, 2017). They also provide connections with theoretical physics, topology, and combinatorics (Mirzakhani, 2014). Mirzakhani collaborated with Alex Eskin, Professor at the University of Chicago, and Amir Mohammadi, Associate Professor at the University of California, San Diego, to answer a mathematical challenge that physicists had struggled with for a century: the trajectory of a billiard ball around a polygonal table. This finding on the dynamic of abstract surfaces was described as "probably the theorem of the decade" (Klarreich, 2018). In 2011, Mirzakhani and Eskin published a paper on this work: "Counting Closed Geodesics in Moduli Space" (O'Connor & Robertson, 2017).

Fields Medal and Accomplishments

In 2006, Mirzakhani was named one of Popular Science's "Brilliant 10" extraordinary scientists (Riddle, 2017). Mirzakhani was awarded the International Mathematical Union's premier prize, the Fields Medal, on August 13, 2014 for "her outstanding contributions to the dynamics and geometry of Riemann surfaces and their moduli spaces" (O'Connor & Robertson, 2017). Every four years since 1936, the Fields Medal is awarded to mathematicians under the age of 40 to honor their existing outstanding mathematical work and for the promise of their future achievement (Sury, 2014). This is considered the highest mathematical achievement and has often been described as the "mathematician's Nobel Prize" (Mehta, Mishra, & Henriksen, 2016).

Mirzakhani was both the first woman and first Iranian to ever win the Fields Medal (Rafi, 2017). Hassan Rouhani, the Iranian president at the time, acknowledged her achievement by saying the "unprecedented brilliance of this creative scientist and modest human being, who made Iran's name resonate in the world's scientific forums, was a turning point in showing the great will of Iranian women and young people on the path towards reaching the peaks of glory…in various international arenas" (Myers & Carey, 2017). At 38 years old, Mirzakhani became an icon and made international headlines. Mirzakhani was highly influential in a male-dominated field. She also represented Iran's tradition of intellectualism.

Mirzakhani received many academic awards and acknowledgments during her lifetime. In addition to the Fields Medal, she won the 2009 American Mathematical Society (AMS) Blumenthal Award, the 2013 AMS Ruth Lyttle Satter Prize in Mathematics, and a 2014 Clay Research Award, to name a few (Riddle, 2017). She was a noteworthy guest speaker at many academic conferences and was elected to the 2015 Paris Academy of Sciences, the 2015 American Philosophical Society, the 2016 National Academy of Sciences, and the 2017 American Academy of Arts and Sciences (O'Connor & Robertson, 2017). Despite all this attention, Mirzakhani stayed modest, focused, and humble. In fact, when she was awarded the Fields Medal, she admitted that she thought notification letter was a mistake! "To be honest, I don't think I've had a very huge contribution," she replied (Klarreich, 2018).

Personal Life

Maryam Mirzakhani described herself as the "slow mathematician." She enjoyed pure mathematics because of the longevity of the questions it offered (Myers & Carey, 2017). Mirzakhani said that this sometimes felt as if she was torturing herself along the way, but her positive outlook would shine through with a follow-up statement of "but life isn't supposed to be easy" (Mehta, Mishra, & Henriksen, 2016).

On March 25th, 2005, Mirzakhani married her partner Jan Vondrák, a Czech man with a Ph.D. in Computer Science from the Charles University of Prague and a Ph.D. in Applied Mathematics from the Massachusetts Institute of Technology (O'Connor & Robertson, 2017). Vondrák was also a postdoctoral teaching fellow at Princeton University from 2006-2009. Following this, Vondrák worked as a computer scientist at IBM Almaden Research Center in San Jose, California. In 2016, Vondrák became an Associate Professor at Stanford University. Although Mirzakhani is remembered most for her mathematical mind, Vondrák described her as being athletic (Myers & Carey, 2017). She was a fast swimmer, and she had aspired to run a marathon someday. He also said that she "loved stories about people who were different" (Myers & Carey, 2017).

In 2011, Mirzakhani and Vondrák had a daughter named Anahita (O'Connor & Robertson, 2017). According to her sister Leila, Mirzakhani always set boundaries for her family's privacy (Myers & Carey, 2017). After winning the Fields Medal, Mirzakhani avoided the attention and focused on what she found most important —her family and mathematics. Mirzakhani was always working on her mathematics around the house, often doodling notes and making connections with papers laid out on the floor. Mirzakhani said that doodling helped her focus and that the process of drawing something helped her to stay connected to her research (Mehta, Mishra, & Henriksen, 2016). When Anahita was three years old, she would often shout, "Oh, Mommy is painting again!" when she would see Mirzakhani drawing (Klarreich, 2018).

Mirzakhani felt that the most rewarding part of mathematics was the enjoyment of understanding something new and the excitement of discovery. She described this as the "aha" moment (Mehta, Mishra, & Henriksen, 2016). She believed seeing the beauty of mathematics requires energy and effort, and that discussing mathematics with others with different perspectives is one of the most productive ways of making progress (Mirzakhani, 2014).

Mirzakhani continued to remain humble and grounded. At academic conferences, she could always be found talking with graduate students. She would generously share her ideas with the community, and she would always listen to the work of other mathematicians with excitement (Rafi, 2017). Mirzakhani's lectures were enthusiastic, optimistic, and inspiring. Students felt motivated to persevere through difficult problems and to appreciate the beauty of mathematics upon leaving her talks (Zorich, 2015).

Death and Remembrance

Mirzakhani was diagnosed with breast cancer in 2013 (Rafi, 2017). Mirzakhani refused to take a long-term leave from her work at Stanford University for her illness. She continued her obligations as an editor of the Journal of the American Mathematical Society for as long as she could. Despite the troubles that Mirzakhani faced, she still spoke about mathematics and offered helpful insights to those who asked for it (Rafi, 2017). By 2016, the dreadful disease had spread to her liver and bones. At 40 years old, Mirzakhani died at Stanford Hospital on July 14, 2017 (Series, 2017). The mathematical community lost one of its most brilliant minds much too early. Mirzakhani was a calming force that "rose above the pressures of academia" (Rafi, 2017). Mirzakhani's death at such an early age provoked the mathematical community to wonder what she could have achieved if she had had more time to continue working. Mirzakhani is deeply mourned and survived by her husband and daughter (Series, 2017). However, even with cancer, Mirzakhani described herself as lucky. She felt that she was born with a good mind and into a loving family, and "there's too much trouble in the world for people to be crying for her." She asked for people to "cry for those who are close to you and who you can help" (Myers & Carey, 2017).

Marc Tessier-Lavigne, President of Stanford University, said, "Maryam is gone far too soon, but her impact will live on for the thousands of women she inspired to pursue math and science. Maryam was a brilliant mathematical theorist, and a humble person who accepted honors only with the hope that it might encourage others to follow her path. Her contributions as both a scholar and a role model are significant and enduring, and she will be dearly missed here at Stanford and around the world" (Myers & Carey, 2017).

Classroom Connections and Closing Remarks

There are many benefits to including history, or biographies of historical mathematicians, in mathematics education. Using history to teach mathematics can improve students' motivation, develop their mathematical thinking, and reveal the humanistic aspects of mathematical knowledge (Liu, 2003). Mathematics lessons in secondary school can be taught using Maryam Mirzakhani's story and philosophies. She was passionate about mathematics and creativity. Her work was visual and focused on patterns and ideas, rather than on numbers and algorithms. In addition, she applied a slow and steady approach to her mathematical work. If students are taught to follow her lead, they would likely feel comfortable taking time to work as mathematicians and may find it easier to find solutions to difficult problems. Teaching about Mirzakhani's positive attitude and her mathematical journey has the ability to inspire students to embrace new mathematical ideas and to approach mathematics the same way she did.

Maryam Mirzakhani truly saw mathematics everywhere. Her mathematical discoveries were influenced by her interests, from running and swimming to doodling in her mathematics notebooks. She had a mindset that was always open to new ideas and a willingness to try new and innovative things (Mehta, Mishra, & Henriksen, 2016). Mirzakhani was driven by wonder and creativity. Her raw talent was rare, as was her drive to tackle the most challenging problems (Rafi, 2017). Mirzakhani's mathematical theorems are just as impressive as her determination, imagination, and optimistic outlook on life. She will continue to inspire all of those who believe in seeing the beauty of mathematics and solving problems in different ways. Maryam Mirzakhani's mathematical legacy will be remembered and celebrated for many years to come.

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District-University Collaborations to Support Reform-Based Mathematics Curriculum Implementation

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ABSTRACT Curriculum change is inevitable in schooling. For content areas such as mathematics that are already under the national spotlight, transitioning to new curriculum materials while concurrently enacting instructional reform creates both a challenge and an opportunity. This paper discusses how partnerships between two state universities and respective neighboring school districts resulted in the creation and implementation of graduate courses for teachers targeted at curricular and instructional reform specific to each district. Common course components between both university-district partnerships were identified in the areas of mathematics research, practice, and leadership advocacy and found to be instrumental in supporting instructional reform and fostering sustained development. Implications for collaborative partnerships, curriculum implementation and research are discussed.

KEYWORDS *curriculum, mathematics, teachers, professional development, reform, collaboration*

Improving student achievement in mathematics has been a critical topic in the education community for decades. Teachers are the most important factor in student achievement (McCaffrey, Lockwood, Koretz, & Hamilton, 2003; RAND, 2012) and therefore, "substantial" teacher professional development based on research-informed instructional practice is of high importance (Yoon, Duncan, Lee, Scarloss, & Shapley, 2007). The Association of Mathematics Teacher Educators (AMTE) suggests "close, respectful, bidirectional relationships" (p. 166) between universities and PK-12 districts as one strategy to improve the effectiveness of current and future teachers of mathematics (AMTE, 2017). Curricular reform environments provide a prime opportunity to build these collaborations.

Many PK-12 districts adopt reform-oriented mathematics curriculum in an effort to improve educational outcomes. However, district leaders report curriculum implementation often looks different than intended (Olsen & Kirtman, 2002) because providing reform-oriented curricular materials does not ensure reform-oriented practices (Handal & Herrington, 2003). Lai (2015) challenges district leaders to reduce or close the gap between intended curriculum and implemented curriculum by addressing factors that may hinder change. In this paper we share how graduate courses for teachers were co-developed and implemented by two university-district collaborations to target the factors that impact curricular and instructional reform in elementary mathematics. We found key course components related to research, practice, and leadership instrumental in supporting instructional reform and fostering sustained development.

Background and Rationale

In 2017, two state public universities and respective local school districts developed two targeted graduate courses, both with a focus on research-informed best-practices in mathematics education during curriculum implementation. A mathematics university faculty member (authors Nebesniak and Gomez Johnson) and district curriculum leader (e.g., curriculum specialist or instructional coach) co-taught each course associated with the selected district curricula. Participants for each course were limited to only district elementary teachers. Instructors designed the graduate courses specifically for collaborating district teachers to attend to their unique systems and cultures for timely and practical application. Participating teachers received graduate credit for completing the district-funded professional development. The instructors aimed to establish stronger ties between PK-12 districts and higher education by connecting theory and practice.

Key Course Components

Although each graduate course focused on corresponding district needs and realities, we identified three key components of the two courses that were instrumental in curriculum and instructional reform in each district: research, practice, and leadership. We identified these three components as integral because they were central aspects of both courses and supported by research in professional development and teacher learning. First, each course incorporated research to define and analyze evidence-based mathematics teaching and learning practices. Through the research component, teachers not only learned the specific structures of their curriculum, but also a more global perspective of their role as teachers of mathematics. Without getting lost "in the weeds" of the new curriculum, the research grounded course content and activities in a common language and frame of reference for all participants.

Instructors also emphasized the translation of research to practice. This component included identifying and aligning theory to each district's newly adopted curriculum. Participating teachers had the opportunity to bridge their understanding of why certain curricular components were emphasized related to research while also examining where gaps might exist for supplementation.

The third component prompted teachers to put their learning into action. Teachers were empowered to use and share their learning with other stakeholders (e.g., administrators, fellow teachers, parents, students) to pave the way for sustainable change in their building and district.

Research: Define and analyze evidence-based mathematics teaching and learning practices

Research on effective professional development highlights the importance of discussing and analyzing effective instructional practices specific to the content being taught (American Educational Research Association, 2005; National Staff Development Council, 2001). Therefore, course instructors used the eight high-leverage, research-grounded mathematical instructional practices presented in the Principles to Actions: Ensuring Mathematics Success for All (PtA) framework (National Council of Teachers of Mathematics [NCTM], 2014) as the foundation of the course. "High-leverage" refers to "those practices at the heart of the work of teaching that are most likely to affect student learning" (Ball & Forzani, 2010, p. 45). The instructors engaged teachers in detailed study of the PtA mathematical practices and the supporting research behind each practice. Teachers read NCTM publications, conceptualized teacher and student actions aligned to each practice, and participated in course activities to better understand how the practices might be applied in their classrooms.

Practice: Identify and align theory to newly adopted curriculum

Since teachers are more likely to implement new instructional practices if those strategies are tied to their current curriculum and practice (Cohen & Hill, 2001; Putnam & Borko, 2000), the instructors designed opportunities for teachers to investigate their newly-adopted curriculum at a deeper level through a PtA lens. Direct ties between research-based practices and the district's newly adopted curriculum were emphasized in course activities and assignments. Instructors helped teachers identify how the reform-based curriculum leveraged the mathematical practices, which underscored the purpose and rationale of the curriculum. Based on their understanding of best practices in PtA, teachers also identified crucial components of curriculum lessons/units where mathematical practices may have been lacking and then collaborated to supplement those lessons/units with support materials.

Leadership: Empower teachers to lead sustainable change

According to Loucks-Horsley, Stiles, Mundry, Love, & Hewson (2010), quality professional development occurs when teachers have the opportunity to serve in leadership roles and connect their newly gained information to other district initiatives. In these courses, instructors sought to empower teachers by emphasizing the role of reflection in practice, connecting elements of the new curriculum to previously established district structures, and building teachers' confidence to share their new learning with others. In addition, the instructors highlighted the role and importance of teachers as leaders and advocates of change. Course assignments included teacher-created district support materials (e.g., pacing guides, formative assessments), communication plans for principals, and elevator speeches and infographics as a way to share fundamental concepts of PtA and their curriculum with other stakeholders. The instructors intended for these courses to be a springboard for further leadership opportunities for enrolled teachers and also a catalyst of sustainable change for participating school districts.

Discussion & Implications

Reactions from teachers, district leaders, and university faculty have been favorable to this collaborative approach. In particular, teachers shared how the attention to research, practice and immediate application were valuable to their overall learning. One teacher noted, "Before taking this course I wasn't always sure how to make myself a better educator. Now I can continue to use the things I learned through this course to impact my class this year and for years to come."

A district principal added,

> My teachers have also become stronger reflective practitioners. Reflecting on their students' demonstration of mathematical skills and the impact of their instructional practices has become a part of their daily routine. I believe this is due to their own deeper understanding of best practices in the area of mathematics. All of which had been developed during their time as a student in this graduate course.

Although our study of teachers' changing beliefs and instructional reform is ongoing, both school district and university participants have expressed value from the collaborative effort and courses. The districts have noted increased teacher support of the newly adopted curriculum, as well as more discussions on effective mathematics teaching strategies among teachers. Also, enrolled teachers have taken on increased leadership responsibilities within their districts as curriculum "experts." The collaboration furthered the university faculty's understanding of current trends in curriculum and practice, as well as provided research opportunities and graduate program exposure. Furthermore, unanticipated collaborative opportunities have arisen between the district and universities including ongoing professional development, instructional coaching, Q and A sessions with building staff, and presentations at local, state, and national conferences. The continued interactions between research and practice are evidence of successful collaboration and provide momentum for future projects and research.

The National Council of Teachers of Mathematics challenges a variety of mathematics education stakeholders, including PK-12 and post-secondary educators, to initiate critical conversations to improve learning experiences and mathematical outcomes (NCTM, 2018). This collaboration during curriculum reform serves as a model for creating customized teacher professional development to meet local needs while also maintaining a global perspective. The coordination of district and university collaborations like this requires an investment of time and resources from all parties, yet the value-added aspects of social ties, learning, and engaged scholarship outweigh the costs.

Admittedly, the task of enacting change in mathematics instruction is a challenging one (NCTM, 2014). Solving complex problems requires innovative ideas and unified efforts. We believe our course model serves as a mutually beneficial option for local districts and universities to support each other in this national conversation regarding mathematics education. These district-university collaborations evolved from conversations on how to best support teachers through change. Each course and district-university collaboration had its own flavor, yet the core components of research, practice, and leadership defined the vision and overall outcomes of the courses. We hope our model inspires other collaborations where diverse perspectives can leverage research, practice, and leadership advocacy concurrently to create sustainable change and progress for the benefit of teacher and student growth.

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A Qualitative Metasynthesis of Culturally Relevant Pedagogy & Culturally Responsive Teaching: Unpacking Mathematics Teaching Practices

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ABSTRACT This article uses Culturally Relevant Pedagogy (CRP) and Culturally Responsive Teaching (CRT) as the theoretical frameworks and qualitative metasynthesis as the methodological framework to synthesize qualitative research published between 1994 and February of 2016. Initial searches produced 1,224 articles, but through a process of appraisals, 12 articles were synthesized to understand how researchers interpret mathematics teaching practices that support CRP and CRT in pre-kindergarten through 12th grade. There were five findings focused on teacher practices, classroom interactions, and student experiences with CRP and CRT within mathematics education, including: caring, context, cultural competency, high expectations, and mathematics instruction.

KEYWORDS *qualitative metasynthesis, culturally relevant pedagogy, culturally responsive teaching, mathematics teaching practices*

Mathematics education has benefited from teaching and research using the tenets of Culturally Relevant Pedagogy (CRP) and Culturally Responsive Teaching (CRT), yet there is little understanding about the impact of these tenets on mathematics teaching practices. Much of the research focused on CRP and CRT employs qualitative methodologies to examine the intersections of mathematics teaching with CRP and CRT frameworks. This research has yet to be synthesized, analyzed, and interpreted to provide the field of mathematics education with deeper insights and broader perspectives of teaching practices within the frameworks of CRP and CRT as evidence-based practices. CRP and CRT are frameworks that respond to traditional mathematics teaching practices by empowering learners to see the multiple purposes for learning mathematics, helping learners appreciate why mathematics is important in their lives, and allowing learners to believe they can succeed in mathematics.

Within CRP and CRT, mathematics is experienced as problem-solving and ways to critique and understand the world (Gutstein, 2009). The ways in which students experience mathematics significantly impact the ways in which they identify themselves as doers of mathematics. CRP and CRT are frameworks that recognize that learners' identities in mathematics are highly contextualized and mediated by environments; consequently, these frameworks consider the contexts of learners' lives, experiences, and backgrounds. Mathematics teaching varies across context and is challenging to generalize because teaching is dependent on contextual, cultural, and social factors. While it is challenging to generalize across varying context, we can learn a lot from unpacking research focused on mathematics teaching that considers contextual, cultural, and community factors. Significant research centralizes the experiences and contexts of marginalized[1] learners. Mukhopadhyay, Powell, and Frankenstein's (2009) work

[1] When we use the term marginalized learners, we are not ascribing a sweeping set of attributes to the collectives of Black, Latinx, Indigenous, and poor peoples; we recognize that collapsing these groups into one group does not acknowledge the intersectionality within these collectives. Martin (2015) argued that one dominant discourse in mathematics education research focuses on a fixed set of cultural and cognitive explanations for negative outcomes, including cultural differences or deficits, limited mathematical knowledge and problem-solving skills, family background and socioeconomic status, and oppositional orientations to schooling. Although there are differences among the collectives, they share legacies of being positioned as deficient in research and they also share values and beliefs that prioritize community and family, a respect for spirituality, and interconnectedness with the natural world (Barnhardt, 2001; Berry, 2008; Gutiérrez, 2013).

acknowledged that mathematics teaching must consider the practices of all peoples. Lampert (2001) found that mathematics teaching should include building relationships with all students so that diverse ideas can be examined and understood. Building relationships and considering the practices of all peoples are described by many researchers as building on students' "funds of knowledge." Funds of knowledge assume a broad range of elements in peoples' lives including cultural experiences, artifacts, values, feelings, language and identity (Moll & Gonzalez, 2004). Bonner (2014) described three teachers using identity, language, and culture in their teaching of mathematics. Civil and Khan (2001) unpacked teaching practices to connect students' families' experiences with teaching counting, measurement, perimeter, and area. The common thread through these works challenges the notion that mathematics teaching is culturally neutral and that there are universal truths regarding teaching practices. These studies situate mathematics teaching as eliciting shared frames of references to make meaningful connections between teaching and the cultures, lives, and experiences of learners.

Frameworks

Theoretical Frameworks: CRP & CRT

This research used Gloria Ladson-Billings' (1994) and Geneva Gay's (2000) frameworks to unpack and understand mathematics teaching practices embedded in classrooms as sites for social change and social justice. These frameworks connect cultural framing to academic skills and concepts, build cultural competence through teaching, and use teaching as a way to critique power discourses and representations.

Gloria Ladson-Billings (1994) defined CRP as pedagogy "that empowers students intellectually, socially, emotionally, and politically using cultural referents to impart knowledge, skills, and attitudes" (pp. 17-18). Teachers must develop both sociocultural consciousness and a holistic view of caring before they can truly engage in CRP (Morrison, Robbins, & Rose, 2008; Ladson-Billings, 1995; Ladson-Billings, 2006). The three tenets of CRP are:

- *Academic achievement* refers to helping learners realize that they have the potential to attain high levels of achievement. Teaching practices associated with this tenet include setting high expectations for learners, providing support mechanisms, assisting learners in determining long-term goals, and helping learners advocate for their own well-being.
- *Cultural competence* refers to ways in which teachers keep the cultures of their children in the forefront of their minds and honor and respect the learners' home culture within daily interactions and instruction (Ladson-Billings, 1994). Teaching practices related to cultural competence include providing supports for learners in navigating dominant cultural capital to attain academic achievement while simultaneously helping learners to honor their own cultural identity.
- *Sociopolitical consciousness* is developed within historically marginalized youth when teachers help their students "to understand the world as it is and equip them to change it for the better" (Ladson-Billings, 1994, p. 139). Teaching practices linked to sociopolitical consciousness create structures to help learners recognize, understand, and critique current and social inequalities.

Geneva Gay (2010) defined CRT as "…using the cultural knowledge, prior experiences, frames of reference, and performance styles of ethnically diverse students to make learning encounters more relevant to and effective for them" (p. 31). CRT is the behavioral expression of knowledge, beliefs, and values that recognizes the importance of racial and cultural diversity in learning. Gay (2010) outlines six dimensions of CRT:

- CRT *validates* children's cultural heritages to "build bridges of meaningfulness between home and school experiences as well as between academic extractions and lived sociocultural realities" (Gay, 2010, p. 31). Teaching practices validate learners' cultural heritage by incorporating instructional strategies and multicultural resources and curricula.
- Culturally responsive teachers develop intellectual, social, emotional, and political *comprehensive* learning opportunities to teach the whole child (Gay, 2010). Teaching practices related to comprehensive learning opportunities create structures where learning is communal and supports helping learners maintain their cultural identities as members of their communities.
- CRT is *multidimensional* because it "encompasses curriculum content, learning context, classroom climate, student-teacher relationships, instructional techniques, classroom management, and performance assessments" (Gay, 2010, p. 33). Teaching practices have to engage extensively with cultural knowledge, experiences, contributions, and perspectives.
- CRT leads to *self-determination and empowerment.* Self-determination and empowerment help learners believe that achievement is within their reach. Teaching practices linked to self-determination and empowerment support learners, holding them to high expectations both academically and socially.

- CRT is *transformative* because it defies traditional educational practices and cultural hegemony and develops social consciousness, intellectual critique, and political and personal efficacy. Teaching practices that are transformative create structures to help learners combat prejudices, racism, and other forms of oppression and exploitation.
- CRT is *emancipatory and liberating* because it "lifts the veil of presumed absolute authority from conceptions of scholarly truth typically taught in schools" (Gay, 2010, p. 38). Teaching practices associated with being emancipatory and liberating challenge the notion of universal truths and the belief that knowledge is permanent.

Aronson and Laughter (2016) collectively examined the work of both Ladson-Billings and Gay and defined culturally relevant education (CRE). They identified four markers of CRE: a) academic skills and concepts, b) critical reflection, c) cultural competence, and d) critique discourse of power. Aronson and Laughter (2016) stated that their findings were supported by a sufficient body of research. We examined their study critically because their literature search produced "more than 286 results" across all subject areas (p. 16). This qualitative metasynthesis produced 1,224 articles just in the discipline of mathematics education. In the end, Aronson and Laughter (2016) synthesized eight studies in mathematics from 1995 to 2013 while we synthesized 12 studies focused on teaching practices that support CRP and CRT in prekindergarten (Pre-K) through 12th grade.

Methodological Framework: Qualitative Metasynthesis

Qualitative metasynthesis is a methodological process to integrate a large body of related research literature. While reviews of literature and meta-analyses synthesize research, a qualitative metasynthesis is distinct because of its methodological framing. A review of literature summarizes the strengths and weaknesses of previous research for the purpose of establishing previous findings and claims that are relevant to the current focus of inquiry. During a review of literature, researchers locate their original inquiry within the context of what has previously been studied so as to convince the reader that this additional study is justifiable and that the results of the study will have relevance to some aspect of advancing the body of literature (Thorne, Jensen, Kearney, Noblit, & Sandelowski, 2004). A qualitative metasynthesis is not a review of literature; it is an analysis and interpretation of the findings from selected studies. Researchers conducting qualitative metasynthesis use a deliberate process of selecting studies with the emphasis on synthesizing, analyzing, and interpreting findings across the selected studies. The process of selecting, synthesizing, analyzing, and interpreting findings across studies differentiates qualitative metasynthesis from a review of literature (Thorne et al., 2004).

Synthesizing a collective body of qualitative research in education provides us with deeper insights and makes for a greater contribution to understanding how a collective body of research contributes to our understanding of a particular topic within the field. In this milieu of evidence-based support, qualitative metasynthesis broaden the perspectives on evidence-based research, practice, and policy by expanding how knowledge can be generated and used. In an effort to connect research to practice, qualitative metasynthesis move from knowledge generation to knowledge application by helping researchers make sense of a collective body of research for practice (Erwin, Brotherson, & Summers, 2011; Berry & Thunder, 2012).

Six discrete steps were followed for this qualitative metasynthesis: 1) identify a specific research metaquestion; 2) conduct a comprehensive search; 3) select initial relevant studies; 4) appraise the quality of initially selected studies; 5) synthesize findings of selected studies; and 6) present findings across the studies.

The formulation of a research question for a qualitative metasynthesis is similar to the formulation of a research question for a qualitative research study. A qualitative research question encapsulates the purpose of a qualitative study and identifies the central phenomenon to be studied. A qualitative metasynthesis research question is referred to as a metaquestion—a question that has already been studied qualitatively. The research metaquestion for this study is: How do researchers interpret mathematics teaching practices that support Culturally Relevant Pedagogy (CRP) and Culturally Responsive Teaching (CRT) in pre-kindergarten through 12th grade?

The purpose of this study was to synthesize papers that demonstrated CRP and/or CRT in mathematics education, which has yet to be done within the methodological framework of a qualitative metasynthesis. For the purpose of this work, we will specifically look at our findings as they relate to unpacking mathematics teaching practices that support CRP and CRT in pre-kindergarten through 12th grade.

Design/Methods

Researcher Positionality

We take the position that researchers conducting qualitative research should acknowledge their influence in the study by describing their experiences and assumptions with which the researchers enter the research (Foote & Bartell, 2011). Both our experiences as mathematics teachers and the equity lens that we bring to this study shape the ways we position teaching mathematics. As former secondary mathematics teachers, we reflect upon ways to improve teaching practices to make mathematics more accessible, equitable, and empowering for all learners, especially those who have been historically marginalized. We do not discount the fact that race, gender, social class, and political views affected the research process. Acknowledging the roles that race, gender, and power play in the research process, the co-authors identify themselves as a White woman and a Black man; and as doctoral student and doctoral advisor.

Data Collection

Published peer reviewed research papers between 1994 and 2016 using qualitative methodologies focused on CRP and CRT were sought for this qualitative metasynthesis. Prior to conducting database searches, inclusion and exclusion criteria were developed based on four parameters: topical, population, methodological, and temporal (as seen in Table 1). All papers used CRP and/or CRT as frameworks (topical) and the research focused on mathematics teaching and learning in Pre-K-12 contexts in the United States (population). Qualitative research was the methodological framework for all papers; however, mixed methods research studies were included if the qualitative findings were distinguishable.

Subject term searches were conducted using EBSCO to simultaneously search five databases for peer reviewed journal articles. The five databases included: Academic Search Complete, Education Full Text (H.W. Wilson), Education Research Complete, ERIC, and Psychology and Behavioral Sciences Collection. The search terms were *culturally responsive teaching* or *culturally relevant pedagogy,* and math*. Math* was used to encompass all articles which may have used math and/or mathematics as keywords. Additional criteria were selected to generate articles which were peer reviewed and fell within the source type as academic journals and journals within the time frame specified. Book reviews, reports, chapters, and dissertations are examples of items that were excluded. Figure 1 shows the flowchart of inclusion and appraisal to determine articles for the qualitative metasynthesis.

Table 1
Inclusion and Exclusion Criteria

Inclusion Criteria	Exclusion Criteria
Topical Culturally Responsive Teaching/Culturally Relevant Pedagogy	**Topical** Not Culturally Responsive Teaching/Culturally Relevant Pedagogy
Population • Pre-K-12 students and educators • Preservice Teachers • Only within the United States • Only Mathematics: with strong mathematics focus • Teachers/Students: with strong focus on teaching and learning practices	**Population** • Not within the United states • Doesn’t focus solely on Mathematics
Methodological • Qualitative Research • Mixed Methods if it clearly distinguishes qualitative data from quantitative	**Methodological** • Quantitative Research • Qualitative data with no student/teacher interactions • Mixed Methods that doesn’t distinguish qualitative data from quantitative
Temporal 1994 – February 2016	
Additional Inclusion Criteria Peer reviewed and refereed journal articles	**Additional Exclusion Criteria** • Newspaper Articles/Journalistic • Dissertations, non-peer reviewed articles, and book chapters

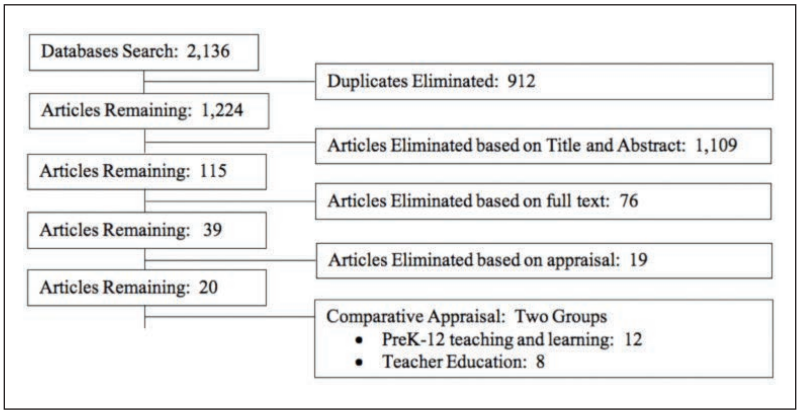

Figure 1. Flowchart of Inclusion and Appraisal

The initial EBSCO search produced 1,224 articles. Following our initial search, we worked through a validation process by looking at the titles, abstracts, subject terms, and full text for published peer reviewed journal articles. This process left further 39 articles fitting the inclusion criteria. We then performed individual appraisals for each article, appraising the quality of the research methodologies using the rubric published by Thunder and Berry (2016) as seen in Table 2. Following their appraisal process, 20 articles

Table 2

Appraisal Rubric

Criteria	Possible Appraisal Points	Appraisal Points Given
1. Research Problem, Purpose, and/or Question a. Problem is stated clearly and related to the research literature b. There is a clear statement of research purpose and/or question	2	
2. Method: Data Collection and Analysis a. Study is methodologically qualitative i. Sample plan and data collection are appropriate to the question ii. Data analysis plan is consistent with design and purpose b. Described the participants of the study and how they were selected c. Researcher showed an awareness of their influence on the study and its participants (describe experiences and/or assumptions with which the researcher entered the research) d. Data collection procedures are fully described e. Steps/process of the data analysis are clear with examples f. Techniques for credibility and trustworthiness are described and used correctly	6	
3. Findings a. Interpretations of data are plausible and/or substantiated with data b. Overall findings address the purpose of the study c. Ideas (themes, categories, concepts, etc.) are precise, well developed, and linked to each other d. Results offer new information about or insights into the targeted phenomenon e. Quotes provide support/evidence for each theme/concept presented	5	
4. Discussion and Implications a. Return to the research questions/purpose proposed at the beginning and discuss interpretations and significant findings b. Recommendations for intended audience and future research issues	2	
Total Points	15	
High overall standards of quality and credibility = 11-15 points. Moderate overall standards of quality and credibility = 6-10 points. Low overall standards of quality and credibility = 0-5 points.		

were identified. Further, we did a comparative appraisal, dividing the articles into two groups: 1) Pre-K-12 teaching and learning; and 2) teacher education. This qualitative metasynthesis treats the findings from 12 articles focused on Pre-K-12 teaching and learning as informants (the 12 articles are marked with an * in the reference section). Dedoose, a data analysis software, was used to support data analysis and initial codes were developed and defined. Six initial codes with eight child codes were used to code the data; we re-read, re-coded, and unpacked the data to synthesize and interpret for reporting.

Data Analysis

Throughout every step within this process the two authors initially worked separately. We then came together to negotiate the retention of articles and our findings, documented within an audit trail. For instance, once we determined our search terms in EBSCO, we separately mined through the 1,224 articles, prior to collectively deciding which articles met our criteria from those which we had both selected. Once we determined the 12 articles that would be treated as informants for this qualitative metasynthesis, all 12 articles were read and re-read by each researcher to note emerging themes. We met to negotiate the themes and to identify initial codes. Our initial codes lacked specificity (especially the one noted as mathematics instruction), and so our definitions were revisited and articles were re-coded. We periodically determined two to three articles to double-code on Dedoose, and we later met to negotiate the codes from each article in its entirety to ensure credibility; all articles were coded in this way. Following the coding process, we examined the excerpts identified for each code across the 12 articles to unpack our findings and to determine mathematics teaching practices that support CRP and CRT in pre-kindergarten through 12th grade.

Findings

Twelve articles were synthesized to understand how researchers interpret mathematics teaching practices that support CRP and CRT in pre-kindergarten through 12th grade. There were five findings: a) caring; b) knowledge of contexts and teaching practices using contexts; c) knowledge of cultural competency and teaching practices using cultural competency; d) high expectations; and e) mathematics instruction/teacher efficacy and beliefs. The five findings focus on teacher practices, classroom interactions, and student experiences with CRP and CRT within mathematics education.

Caring

Caring is a continuous cycle of working to establish a rapport, using knowledge gained from that rapport to inform teaching practices, and then, reflecting upon teaching and learning to understand learners' mathematical knowledge. Caring was demonstrated in the ways in which teachers created positive learning environments where learners saw themselves as participatory; teachers took an active role in seeking out knowledge about learners and communities; and teachers supported learners emotionally and academically by making mathematics content accessible and empowering learners mathematically. In the following excerpt, we see the significance of teacher-student relationships and how that translates into mathematics instruction.

> When establishing relationships, teachers cannot merely go through the motions because students know when teachers are genuine and really care about them. African American students must relate to the teacher and the teacher must relate to them. The teachers realize they must have a relationship before they can make mathematics lessons relevant to the students. They take the opportunity to know their students and discover their motivations and interests. They tailor their instruction with this knowledge. (Jackson, 2013, p. 7)

Although caring is not specifically noted as a tenet of CRP or CRT, it is clearly evident within the dialogue surrounding the tenets. For instance, Gay claims that CRT is multidimensional, for which a key dimension includes fostering positive student-teacher relationships. Likewise, CRP places emphasis on the teachers having respect for learners' culture. "Respect" was noted as one of our vocabulary terms which indicated a caring rapport.

Context

In addition to developing rapports with learners, context played a crucial role in making mathematics relevant and accessible. *Context* incorporated two dimensions as seen in *knowledge of context* and *teaching practices and strategies that use context.* Knowledge of context is related to space and place in the ways teachers gained knowledge of their students' home-life, communities, and neighborhoods. In the following excerpt, we see how Ms. Finley gained knowledge of context.

> Ms. Finley often "walk[ed] the neighborhood," taking time out in the evenings to visit with students and their families. She knew that this type of connection with the community was important, and she was able to weave the knowledge that she

> gained through these interactions into the mathematical content that was the basis for her lessons. (Bonner & Adams, 2012, p. 30)

After the teachers sought out knowledge, they integrated mathematics instruction and knowledge of context by making meaning of the mathematics curricula and tasks. Teachers were actively engaged in communities to work with learners' parents and families for mathematizing contexts, creating and adapting mathematical problems, utilizing questioning strategies to elicit learners' local knowledge, requiring explanation and justification as it relates to context knowledge, and creating project-based opportunities incorporating funds of knowledge. Gay states that CRT is *validating* and should build bridges between school and learners' homes; essentially, the presence of this bridge is how we have defined *context* and the findings that support the presence of *context.*

Cultural Competency

Cultural competency was found in the ways teachers developed knowledge and skills associated with various forms of communication and funds of knowledge. Further, the teachers acted on this knowledge of cultural practices by incorporating such knowledge into their teaching practices. Teachers promoted engagement by incorporating nonverbal communication through proximity and by integrating music and movement into teaching practices. The teaching practices and strategies primarily focused on classroom discourse including storytelling, utilizing call and response, and dynamic forms of interactions. Teachers made mathematics accessible by unpacking and connecting cultural artifacts.

In the following excerpt, we see how Inga engaged in interviews with her learners to develop an informed understanding of her learners' cultural practices and funds of knowledge as it relates to shopping and currency.

> …From this, Inga learned about her students in ways she did not expect, finding that those students who shopped with their families were able to quickly solve problems regarding currency. These students demonstrated a remarkable facility with these transactions that suggest they had powerful strategies for dealing with the situation. Although Inga learned much about her students' interaction with money when outside of school, she could have taken this further by exploring the specific strategies they used. The strategies children use with money are often non-routine, and this might have offered an opportunity to gain a deeper knowledge of students' understanding. (Wager, 2012, p. 16)

As previously mentioned, the findings for cultural competency strongly aligned with the ways in which Ladson-Billings unpacked cultural competency with CRP. Additionally, this finding also ties into CRT and how it *validates* learners' cultural heritages in such ways that teachers build cultural practices into classroom instruction.

High Expectations

Teachers must have *high expectations* both for their learners and for themselves. Teachers made necessary teaching revisions based on their learners' needs, interests, and understandings as they relate to mathematics. There was a level of flexibility and impromptu teaching that was evident with the teachers who were most capable of reaching their learners. Furthermore, teachers were warm-demanders who established learning environments in which learners were held accountable and empowered by taking an active role in their own learning; we see these practices within the context of Ms. Bradley's classroom.

> …Ms. Bradley's classroom was highly structured and disciplined, focusing on high expectations and success through "tough love." When a student did not have his or her homework, for example, Ms. Bradley would take the student in the hallway to call his or her parent or guardian…Ms. Bradley explained that this type of discipline is "what they get at home from their mama or grandmamma—you can't mess around." Furthermore, she indicated that this type of culturally connected communication and maintenance of high expectations allowed students to develop racially and culturally "so that they don't have to give up what they are used to for the sake of passing class...they have to do this in other classes and I'm not going to teach them to be White." (Bonner, 2014, p. 395)

This excerpt specifically demonstrates how the finding is not just about having high expectations for learners, but rather how those expectations are culturally connected to learners' lived experiences. The conceptualization of high expectations is seen both in CRP by focusing on academic achievement and in CRT by focusing on the comprehensive achievement of the whole child. Additionally, both frameworks advocate for teaching practices that support students in realizing that achievement is within their reach, translating into student empowerment and self-determination.

Mathematics Instruction

Mathematics instruction highly correlates with teaching practices and strategies for both context and cultural competency. The findings are specific to mathematics teaching practices and incorporate aspects which are indicative of high-quality mathematics instruction. For instance, teachers utilized technology, incorporated tools and manipulatives in their instruction, and engaged in modeling their thinking for learners. It is important to keep in mind that we are not claiming that when one practices high-quality mathematics instruction that he or she is engaging in CRP and/or CRT; rather, when a teacher has high confidence in teaching mathematics and high self-efficacy, believing that mathematics instruction should be student-centered, open-ended, inquiry-based, highly interactive, and impromptu, based on learners' needs and interests, CRP and CRT are more likely to occur. When teachers felt confident with mathematics, they were more likely to create opportunities for their learners in which they were able to take ownership of their own learning and make personal connections to the content. In the following except, we see Chela make relevant connections to everyday classroom experiences and mathematics.

> …Chela loved math. Chela turned this passion for math into a professional strength—she took advantage of all math professional development opportunities and she made mathematics a central part of her practice. Unlike many of her peers, Chela didn't have a math center or a math time—that seemed silly to her, as math was everywhere. Weaving math into daily activities was what Chela did best. As she designed different games or visual supports she looked for the math hook. For example, Chela used 10 frames in attendance… a typical opportunity for name recognition and counting; extending the activity in several ways that deepened learning opportunities. (Graue, Whyte, & Delaney, 2014, p. 308)

Within the excerpt, the mathematics instruction is explicit as we see Chela using mathematical tools such as ten frames, which are two-by-five arrays often used to help students learn to subitize, to connect the mathematics instruction to everyday activities and practices like student attendance. Though this finding is specific to mathematics teaching and learning, it does relate to the theoretical framework for CRT in that it calls for transformative education that defies traditional educational practices.

Discussion & Implications

As with any synthesis of literature, this piece is time sensitive. This work specifically examines articles that were on the EBSCO database up until February of 2016. Thus, since data collection, surely more papers have been published which would fit our inclusion criteria, but performing a qualitative metasynthesis is simply a laborious process that demands an extensive amount of time to appropriately analyze the data. Such process requires at least two researchers who have some knowledge of literature and who understand the nuances necessary to make decisions throughout the process. In our case, we made decisions to focus on peer reviewed articles, negotiated codes, and negotiated the appraisal process. Because we focused only on peer reviewed articles, researchers can build from the work to examine book chapters, dissertations, and non-peer reviewed works. Our contribution to the field of mathematics education is providing one frame from which qualitative metasynthesis can be conducted.

There is a dearth of research focused on unpacking mathematics teacher actions focusing on CRP and CRT. While Ladson-Billings and Gay provide frameworks for CRP and CRT, there appears to be inconsistent ways in which these frameworks are interpreted in mathematics education research. There are inconsistent interpretations on whether mathematics or culture should be centralizing agents. There were examples in which the research documented teaching practices of simply changing the context of mathematics tasks to reflect the cultures of learners. There were examples in which the research documented teaching practices mathematizing elements of contexts and communities to highlight social justice issues. A critique of the body of work is that very little research documented sociopolitical consciousness and critical consciousness. It is not clear whether critical consciousness is central in mathematic teaching using these frameworks.

More work is needed in the field to unpack teaching practices that promote access, equity, and empowerment. The findings of this research suggest that teachers who incorporate CRP and CRT know their learners and the communities of their learners. More work is needed to unpack the continuous cycle teachers use to develop rapport with learners and communities. It is not clear in what ways contexts and support within schools and communities are central elements in CRP and CRT. That is, what are the kinds of supports teachers needs to draw on to incorporate elements for funds of knowledge and communal aspects? The findings from this work suggest that mathematical knowledge for teaching positively impacted teachers' lens for CRP and CRT; more work is needed to understand and unpack the interactions of teachers' knowledge of context and culture with knowledge of mathematics and teaching mathematics.

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Theory of Professional Competence in Teaching of Mathematics: Development and Explication through Cross-cultural Examination of Teaching Practices in India and the United States

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ABSTRACT This paper describes the theory of professional competence in teaching of mathematics developed through a cross-cultural examination of teaching practices of mathematics teachers recommended as competent by their principals in two selected high-achieving high schools of India and the United States. A detailed study of teacher cases from both of the research settings yielded a rich conceptualization of the relationship between teachers' professional knowledge base and professional competence in the teaching of mathematics. The substantive theory explains the processes of both the development and the display of professional competence and enables predictions of the ways teachers would most likely utilize to meet the performance expectations of their work environments. The study makes a unique contribution to the field of teacher education and views professional competence as a dynamic interplay of various components of teachers' professional knowledge base activated in actual teaching situations in the context of the classroom, school, and wider social culture.

KEYWORDS *grounded theory, cross-cultural, professional competence, mathematics teaching, high school*

Background

The widespread concern about mathematics achievement in the United States has drawn considerable research attention to the quality of mathematics instruction and cross-cultural research in mathematics education. Related to this, mathematics teachers' professional knowledge and competencies for effective instruction are also under scrutiny. Schools' performance and student outcomes are increasingly quantified, measured, and compared on a global scale. Cross-cultural research has the potential to inform policy, practice, and to assist in the development of theory at a level appropriate to regional, national or global priorities (Clarke, Emanuelsson, Jablonka, & Mok, 2006). This paper deals with only a part of the findings of the cross-cultural study of mathematics teachers recommended as competent in selected high-achieving high schools of India and the United States (US). The overall purpose of the study was to develop a theory of professional competence that characterizes effective mathematics teaching in the two research sites. The paper describes the emergent theory of professional competence in the teaching of mathematics.

The notion of professional competence as a theoretical perspective has been developed in the field of management, industry, and clinical practice, and it is currently being emphasized in higher education. Mathematics teaching and learning is increasingly influenced by globalization, internationalization, and the demands of the corporate world. Fueled by the demands of the changing economy and technical demands of many professions, proficiency in the teaching of mathematics is being emphasized due to the increasing understanding of the influence of teacher competence on students' achievement (Darling-Hammond, 2000; Wenglinsky, 2002). This warrants the development of a sound theoretical basis for the notion of professional competence in the teaching of mathematics. The differences and similarities that characterized effective mathematics teaching and learning in the two settings assisted in developing the models of professional competence for each teacher and led to an understanding of how the quality of mathematics teaching and learning is influenced by the context in which this learning occurs. The educational significance of the study lies in the generation of a theory that has potential to enhance the understanding of effective mathematics teaching and learning and has implications for teacher education. To achieve the purpose, this study explored instructional processes, experiences of teachers, their conceptions about mathematics and its teaching, teachers' behaviors during instruction, and contextual factors that characterize effective mathematics teaching in two schools through classroom observations and in-depth semi-structured interviews before and after the observation period.

Methodology and Research Procedures

This study's methodology combined elements of the naturalistic case study with the grounded theory approach to gain an in-depth understanding of each of the teacher participants in the two schools, which led to the characterization of each teacher's practice. Grounded theory method (GT) calls for early data collection, analysis, further theoretical sampling, and category saturation. This process of data collection is "controlled" by the emerging theory (Glaser, 1978, p. 36). A particular strength of grounded theory is that it seeks to explain the process and interactions among the various parts of the model. The complexity of classroom life and the multi-dimensional nature of the transactional processes going on in the classrooms meant that far more was taking place than could be recorded or analyzed. Thus, the theoretical sensitivity of the researcher played an important role in data collection and analysis. As the data analysis progressed, the relevant literature was examined and was used as an additional lens through which to examine the data. The literature, data collection, and data analysis were systematically linked "to generate" theory (Glaser, 1992, p. 16). This study employed the Glaserian version of GT primarily to represent the emergent model, but ideas from other grounded theorists such as Gibson (2007), and Strauss and Corbin (1998) were also incorporated to understand the procedures.

Data Sources and Analysis

Two schools were selected from each of the research settings (criteria and teacher profiles explained elsewhere). "Purposeful sampling," which seeks information-rich cases that can be studied in depth (Bogdan & Biklen, 1992, p. 71), and criterion-based selection (LeCompte & Preissle, 1993) were used in the selection of the teacher participants. In addition to student achievement, this study used recommendations of the principal along with the selection criteria recommended by Palmer, Stough, Burdenski, and Gonzales (2005). The authors recommend that researchers should use the following indicators while selecting the sample of expert teachers:

(a) Teachers should have three to five years of experience in a specific teaching content area and with a particular population of students,
(b) teacher knowledge reflected in relevant certification and degrees that correspond to the field in which these teachers are currently teaching,
(c) recognition as an exemplary teacher by fellow teachers, administrator…based on recent… indicators of teaching effectiveness to include teacher knowledge and skills, and
(d) should be confirmed with documented evidence of teacher impact on student performance (p. 23).

Many scholars agree that the complexity of a teacher's professional knowledge cannot be captured by a single instrument. Kagan (1990) recommends the use of multimethod approaches which not only provide triangulation but also capture the complex and multifaceted aspects of teaching and learning. To achieve the purpose, this study used a concept map activity (Raymond, 1997) in combination with teacher interviews, class observations, and intra-site spreading to capture the complexity and complete picture of mathematics teaching. The classroom observations focused on the teachers' presentation of the topic, questioning skills, answers to students' questions, management of class time, quality and rigor of the mathematics taught, and approaches to dealing with students' difficulties. The analysis of lessons focused on the quality of mathematical content, organization and planning of lessons, patterns of classroom interaction, and the interplay among the following teacher attributes: teachers' content knowledge, conceptions, and instructional practices. It also looked at contextual factors such as school culture and the nature of mathematics as the subject matter context. Data corpus consisted of transcripts from pre and post-observation interviews, video recordings, field notes, and classroom artifacts (algebra, geometry, and calculus). A total of 43 lessons were observed in the classrooms of the nine teacher participants, and a sequence of at least three lessons was videotaped in the classrooms of the five teacher participants who agreed for video recording. The researcher acknowledges that an inductive approach to identifying teacher competencies during instruction may lead to omission of some competencies. Thus, the researcher employed 25 indicators of expertise and competencies related to effective teaching of mathematics determined from the review of extant literature to identify and describe competent mathematics teaching during classroom observations (Ahuja, 2018b). Two more competencies that emerged from the analysis were added to the list. The professional profiles of the participants are presented in Table 1.

Data Analysis

Data analysis in GT includes finding a core category, causal conditions that influence the process, the effect of context and intervening conditions, strategies used by the subjects to carry out the process, and finally the consequences of the behavior for the participants (Creswell, 1998; Glaser, 1978). Audio recordings and video clips of the lessons were listened to and viewed repeatedly, and transcripts were read and reread in an attempt to gain a deeper understanding of recurring themes and concepts. The data were coded relevant to the research questions by hand using the read, theme, re-read, and re-theme model espoused by Glaser and Strauss (1967). Care was taken to constantly return to the data to examine categories and properties in order to ground the emerging hypotheses and generate the theory. New data were coded and compared against previously collected and analyzed data until saturation. Finally, the selective coding resulted in an overarching core category. Through continuous data collection, theoretical sampling, and analysis, a substantive theory was developed to illustrate professional competence in mathematics teaching in the two research contexts.

The GT approach to research assumes that ongoing reading of the literature will be an integral part of data collection, analysis, and writing up of research (Glaser, 2005). The data analysis for this study included comparisons of the relevant literature with emergent themes and findings. A systematic literature review was done during data analysis to find relevant terms used in the literature to capture, examine teacher actions, and describe teaching behaviors observed. Thus, a "dynamic, reflexive, and integrative approach" was used for the literature review (Hussein, Kennedy, & Oliver, 2017, p. 1201; Stebbins, 2001) in order to generate new understanding from the existing literature (Torraco, 2005). Therefore, literature and data were integrated to build a portrait of professional competence that has potential to inform practice in other substantive areas. The researcher's professional experience, preparation in mathematics, and education informed the study and the data at the conceptual level. The bias arising out of her own work experience was checked by the constant comparative analysis, member reflections, triangulation of data sources, and peer debriefing.

The Grounded Theory of Professional Competence in Teaching of Mathematics

According to Glaser (1978), two key questions are asked during grounded theory research: (1) *what is the main concern or Basic Psychological Problem (BPP) of the subjects?* and (2) *what is the Basic Psychological Social Process (BPSP) that continually resolves the concern?* The BPSP refers to

Table 1

Professional Profile of the Participant Teachers

Name (Pseudonym)	Gender	Education	Teaching Exp. (Total, current location, middle, high)
Sumit	M	B.Sc. (Phy, Chem, Math), M.Sc(Math), B.Ed	11, 5, 0, 11
Suman	F	M.Sc. (Math), M.Phil (Math), B.Ed.	12, 10, 0, 12
Komal	F	M.Sc. (Math), Ph.D. (Applied Math)	14, 11, 0, 14
Lena	F	B.S. Accounting, M.S. (Sec. Education), Provisional certificate	5, 3, 1, 4
Sam	M	B.A. (Philosophy), Certified (Sec. Math)	23, 3, 0, 23

the processes such as becoming and personalizing (e.g., becoming a teacher, a nurse, a leader)—in this study: *becoming a professionally competent mathematics teacher*. Through coding, analysis, and reflection a BPSP emerged as the core category. The use of diagrams enabled the researcher to move from the descriptive to the analytical level of analysis. The grounded theory of professional competence in teaching of mathematics and the comprehensive diagram of the substantive model of professional competence following the procedures suggested by Glaser and Strauss (1967) are described in the next section.

The Main Concern

The main concern of the participants was to perform at their maximum potential in order to teach mathematics effectively in the context of the high-achieving status of the schools. The concern may not be clearly articulated by the participants, but the concern was displayed by their classroom behaviors and through their utterances during the interviews. For these teachers, professional competence was teaching in such a way that developed students' mathematical habits of mind, showed good results on assessments, and were well prepared for the next mathematics course.

BPSP: Becoming a Professionally Competent Mathematics Teacher

The purpose of data analysis in GT is to develop a core category that explains and characterizes these processes. The model to be developed is thus a model that explicates the core category. An explication of the core category in this study involved identifying the conditions influencing teachers' engagement with the development and display of professional competence as well as the *contextual factors,* and *context consequences* (the outcomes of the actions and strategies taken during the engagement with BPSP). These elements thus formed the framework of the theory that was developed. The theory enables predictions of the way teachers would most likely engage in order to meet the performance expectations of their work environments. By constantly comparing emergent concepts across the teacher cases and all elements of the data corpus for similarities and differences, themes and categories were linked to the phases of BPSP. *The five phases, six conditions, six contextual factors, and five context consequences* comprise elements of the emergent theory of professional competence (see Figure 1), and are explained in the ensuing section in terms of the emergent themes and subthemes. Some vignettes are also presented from the data.

Core Category: Interplay of cognition, Conception, and the Context

The core category of the study was interplay among conceptions, cognition, and the context. This was determined by identifying and describing existing teacher professional competencies that emerged during the course of data collection and analysis. During analysis, the researcher asked questions to herself and of the data: (1) What teacher competencies were derived from the teachers' content knowledge and pedagogical content knowledge according to their educational level? (2) What teacher competencies were related to their conceptions about teaching and learning of mathematics and the nature of mathematics? (3) Which competencies were specific to the school settings? (4) What competencies did they create in response to performance expectations from the department heads and recent changes in the curriculum or reforms? (5) What competencies did they develop as a result of their experience? Asking questions and making comparisons informed and guided analysis and assisted theorizing. The interplay of conceptions, cognition, and context explains most of the teaching behavior observed. By constantly comparing emergent concepts across the teacher cases and all elements of the data corpus for similarities and differences, themes and categories were linked to the phases of BPSP.

Progression through the Phases of BPSP

Participants' display of professional competence and their reflections on their own practice suggest that there were *five phases* in the development of professional competence: (1) developing a personal epistemology of teaching mathematics based on past and current experiences, (2) seeking knowledge to modify practice and perform at their maximum potential, (3) understanding of contextual factors, (4) maintaining balance between competing priorities, and (5) attaining good balance between emotional and interpersonal intelligence.

Developing a personal epistemology of teaching mathematics

The participants spoke of their experiences of teaching mathematics in ways suggesting a progression through five phases of the teachers' engagement with the development of professional competence. This progression was influenced by their lived experiences: experiences of learning mathematics, experiences of teaching mathematics, and experiences as members of the societal and cultural structures in their countries. These experiences affected their decision-making processes, and led to the

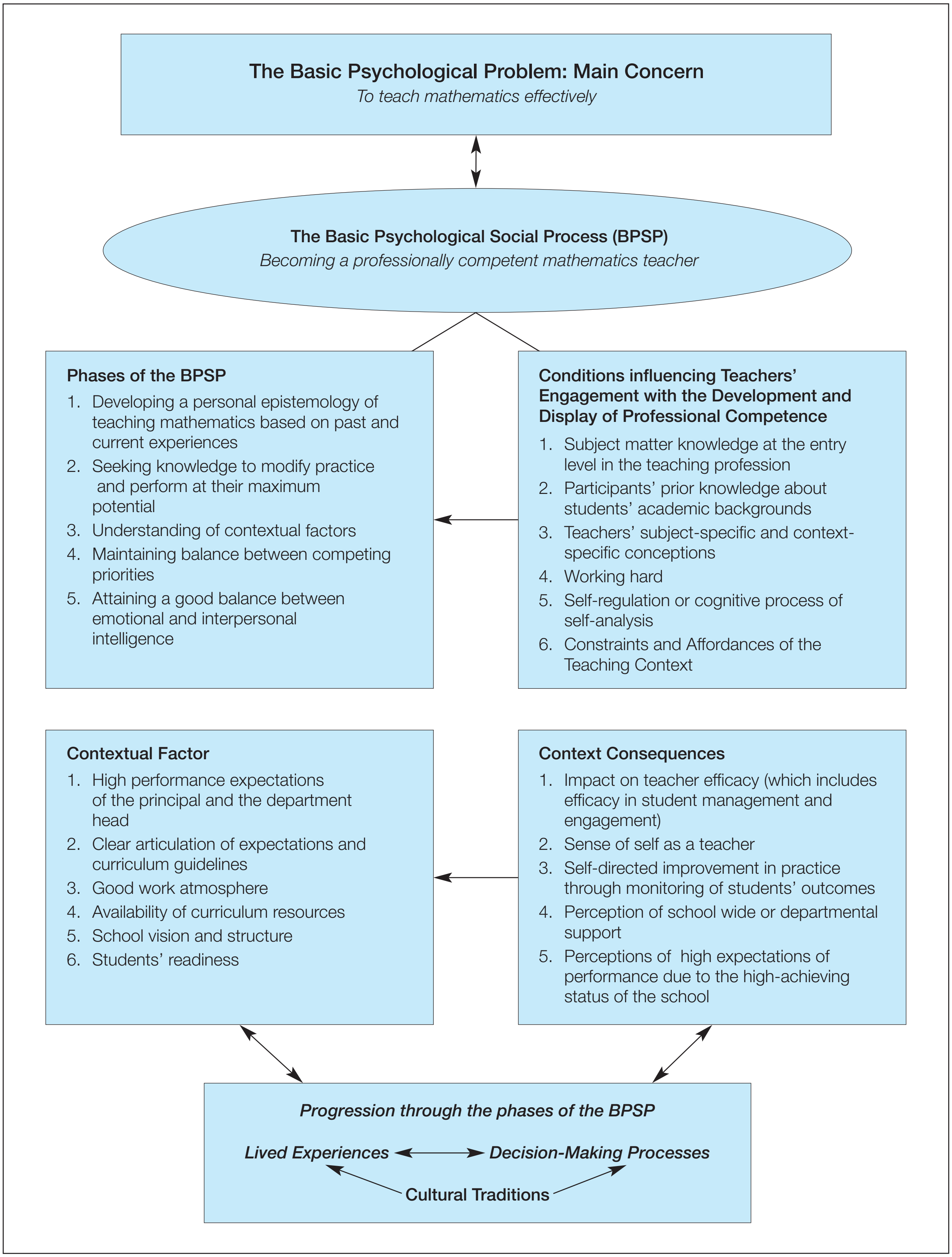

Figure 1. Ahuja Model of Substantive Theory of Professional Competence in Teaching of Mathematics.

development of their personal epistemologies (progression in the development of one's conceptions about knowledge and knowing) of teaching mathematics. Their personal epistemologies of teaching mathematics based on past and current experiences influenced the goals they set for themselves and their students in relation to the performance expectations of their work contexts. Sam (a geometry and AP calculus teacher from the US) during the interview said:

> Well, we have a curriculum to cover. Sometimes the curriculum is more or less (pause) laid out to you. By and large, you know what you have to cover for AP courses and exams. For other courses you know what's on the semester exam or on the final exam and you teach accordingly. Sometimes you find yourself doing what is necessary to get that final goal. By experience, you learn to do that.

This suggests that through experience teachers develop heuristics to handle classroom challenges. All the teacher participants reported that from their experience they know some of the concepts students may not grasp, and only through experience did they become familiar with some of the mistakes and misconceptions students have regarding particular topics. Rhodes and Bellamy (1999) assert that for teachers, although a curriculum may be set for them, it inevitably becomes shaped by them to reflect their own belief systems, their thoughts, and feelings about both the content of their instruction and their learners. It is, therefore, very important to achieve a balance between the structure and the flexibility in the teaching and learning of mathematics. Sam, when talking about planning lessons, expressed the following:

> Students from one year to the other grasp the topic poorly or easily. I have to add additional examples, worksheets, and explanations depending on their difficulty level so sometimes I am not able to finish what I plan. This is what I mean by being flexible. No pressure of state assessments so can afford to go at a slower pace to clear their doubts. Thus, I go with the flow of the individual class. Thus, I modify my lessons dependent on what students are grasping rather than what's convenient for me.

Sumit (precalculus and calculus teacher from India) also pointed out during the interview that he has been teaching at the school for 11 years, and through his experience he has learned to plan curriculum as per the examination-oriented system. He shared his thoughts:

> When you are going to the classroom, there are all sorts of students. You have to make sure that you are not aiming at one category of students… Though normally I try to go as per the average class. Sometime, I feel that students who are too good should not feel neglected. Similarly, students who are weak should not feel that this class is no good to them. I try my best to come down to the level of students and at the same time make sure that I am taking care of good students as well. I try to maintain balance between these three types of students and by experience you learn to handle all this.

The findings suggest that the participants did not demonstrate all the 25 professional competencies in teaching of mathematics, and these competencies were categorized into three broad domains: (a) competencies related to content knowledge and pedagogical content knowledge (a dimension of teachers' cognitions), (b) competencies related to conceptions about nature of mathematics, and teaching and learning of mathematics, and (c) competencies pertaining to the teaching context (see Ahuja, 2018b). These competencies are constituents of teachers' professional knowledge base (Ahuja, 2013). The data provided sufficient evidence that these competencies develop from the interplay of prior school experiences, immediate classroom situations, and social teaching norms at the school.

Seeking knowledge to modify practice and perform at their maximum potential

The teacher participants indicated during the interviews that they modify their lessons based on their past experiences of teaching the topics, and their understanding of mistakes generally made by the students, changes in the curriculum, and types of problems expected on the standardized assessments/examinations. The participants reported during the interviews that they update knowledge by reading extra books, journals, and information available on the internet. Thus, seeking knowledge to modify practice and to perform at their maximum potential represents a phase of the emergent BPSP. It is clear that success in this phase requires an understanding of contextual factors developed through careful analysis of the teaching context. The next section of the paper presents some vignettes from the data to illustrate the emergent themes.

One of the teacher participants from the Indian setting, Sumit (pseudonym), stated that his beliefs about mathematics and its teaching were influenced by the type of students in his classes and by the school culture. Sumit, when asked what influences his teaching practice and what made him stay at the school, remarked that the students' attitudes, his coworkers' attitudes, and the principal's expectations influenced his practice the most. Sumit remarked that he liked working at the school, saying that "A good atmosphere, a good name and fame, and very good level of students; due to which, you have to be the best to be here, and keep proving that you are worth staying here" (Interview transcript). During the interview, he responded that he feels very confident teaching at the 12th-grade level, as evident from the following excerpt:

> Very comfortable. Because the content and the curriculum are the same as it was 10 years ago, they just add some more topics and change the marking scheme. I have been teaching for 11 years. I keep upgrading my knowledge through new books and internet. That is what I do. The only thing I have to do is to arrange the questions as per the difficulty level and that changes from class to class and year to year.

This also suggests Sumit's ability to self-regulate his learning to modify his practice by *seeking knowledge* to keep up with the changing needs of the students and assessments, and the changes in the curriculum. He has engaged himself in the *cognitive process of analysis of self-efficacy*. The researcher defined this construct as the self-awareness of gaps in the knowledge, reflecting about one's own process of problem-solving for better job performance, and achieving means to address this gap in knowledge. This involves reflection on actions (Schon, 1987) and outcomes. Komal, a teacher from India, stated "All students are capable of learning and should be given the opportunity to learn." When asked about the main factors affecting her preparation, she responded:

> Time, resources, data as to where my students are at the start and where I need to get them. Students are learning in new ways all the time, so if I do not change with them, then I will lose them.

This suggests she is engaged in the cognitive process of analysis of self-efficacy and her demonstrated efforts to seek knowledge and means to modify her practice by acknowledgment of the changing needs of the students. Lena, a teacher from the US, shared that her beliefs about mathematics were influenced by the way she was taught mathematics, by the curriculum, and by the current issues presented in mathematics education journals. Komal, when asked what mathematics teaching means to her, stated, "Mathematics teaching to me is an approach to problem-solving in different circumstances. Creating curiosity is very important; what is madam going to do today in class?" She further remarked, "These days students and their parents focus on getting 100% on the exams, and parents' expectations from the school and its status influence how they structure the curriculum and the instruction." Komal's responses suggest the influence of school culture and cultural factors on her competency to enculturate her students mathematically. Sam expressed similar views when asked about the goals for his students. He remarked:

> I think many math teachers and department heads think that we should prepare mathematicians. Whereas, we have to realize that very few or any of the students in a given year major or minor in math or even major in something math intensive such as physics. We have to gear our instruction to the vast majority of the students who will use math tangentially. They might use statistics, psychology or might use geometry in art or engineering. So my vision for my students is that they are able to do everyday math, understand enough math so that they are able to apply math in the area they are willing to pursue; and also socially they feel better about themselves in social situations. They feel comfortable about it. Their parents say all the time when we meet them, "I was never good at math." I don't want them to say that, to their children. Ultimately, they should not feel intimidated with math.

Sam's responses suggested his vision of making his students function socially and mathematically, and his vision seems to be aligned with a sociocultural perspective of teaching mathematics. Similar thoughts were expressed by other participants. Participants' responses suggested their concern for developing students' ability to live competently and confidently in a world that relies heavily on proficiency in mathematics and their *sensitivity* to the teaching context.

Suman, another teacher from India, reported during the interview that she tries to make sure that students are able to understand the concept, and that they develop an appreciation of the subject. To achieve this goal, Suman reported that she makes them feel relaxed in the class. One example of this is that instead of calling on students randomly to answer her questions, she asks them one by one so that every student is able to participate. Suman further explained that she tries to involve students in group discussions to encourage them to discuss different ways of solving the problems and exploring the concepts. From a sociocultural perspective, it is well recognized that fear inhibits learning and learning is more effective when it involves a student's emotions as well as intellect. Suman also remarked that her teaching is influenced by the work culture at the school: "During the last ten years, I have worked with different supervisors; they all have been very organized. The principal's talks have always been very inspiring. She expects us to give 100%." Suman, when asked what mathematics teaching means to her, stated:

> My teaching is not influenced by the way, I was taught. It was chalk and talk method followed by drill and practice. Students do not learn that way these days. We never questioned adults. Students have become more vocal these days. You have to connect the content with their lives…I involve them in games and solving puzzles. Our system depends on mental calculations and estimation. They have to be very thorough with basic facts. Sometimes, I have to remark: a vegetable vendor calculates faster than you because he has become habitual. For him, calculation is a habit of mind. You need to focus and take mathematics as a life tool. You have to connect to these kids and respect their individual differences.

Suman revealed her awareness of sociocultural aspects of teaching when she pointed out during the interview that the multifarious nature of Indian society teaches them to respect the differences that exist in the society and the world, and thus making them ready to be a global citizen. According to Suman, a good teacher has to be a global citizen so that he/she can inculcate in his/her students respect for all cultures and teach them to embrace the differences between them. Her awareness about sociocultural aspects of teaching and learning as well as her acknowledgment of the need to give due consideration to the affective domain was also clear from her statement:

> If one comes to Indian schools, one sees students of different religions sitting together, studying together, celebrating festivals together, and sharing their problems. They realize that despite their superficial differences, their hopes, dreams, and fears are the same. One also sees teachers coping with large numbers of students with relative ease. A good balance between strict discipline and complete freedom is maintained by the teachers.

This emphasis on sociocultural aspects of learning has become the focus of much research in the context of the achievement gap in mathematics and science between various student populations in the U.S. Researchers (Ladson-Billings, 2001) advocate for inculcating multicultural competence as a part of educators' repertoire in the wake of diverse student populations in the schools.

Understanding of contextual factors

The participants' responses, planning of their instruction, and classroom behaviors suggest that analysis of their teaching context (a dimension of teacher's cognition) is an interplay among cognitive process of analysis of self-efficacy, diagnosing and understanding students' abilities and academic backgrounds, and reflection of their actions on student outcomes. Thus, this competency encompasses developing an understanding and awareness of various contextual factors: (a) understanding of the curriculum; (b) understanding of students' academic and cultural backgrounds (teachers in the Indian setting demonstrated an awareness of students' cultural backgrounds); (c) school culture characterized by predetermined vision and expectations of standards of mathematics teaching and performance; and (d) their reflection on students' outcomes. These teachers added experiential knowledge to their existing knowledge base to improve their context-specific conceptions, and then used this knowledge in decision-making.

Maintaining a balance between competing priorities

Data analysis revealed that, based on their prior experiences (as students, in teacher preparation programs and in prior teaching experiences) and due to the constraints posed by the contextual factors, teachers prioritized their teaching tasks and developed heuristics to analyze their teaching contexts and regulate their teaching. Additionally, principal and department heads as part of the immediate teaching context influenced this balance of competing priorities, leading to the creation of social norms that dictate performance expectations from the students as well as teachers. It is argued that the extent to which this balance is maintained and nurtured by the teacher determines the patterns of interaction and the mathematics learned in the class. Attaining a good balance between emotional intelligence and interpersonal intelligence plays an important role in this phase as research in Neurology suggests that emotions are central to all judgment and decision making (Demasio, 1994).

Attaining a good balance between emotional and interpersonal intelligence

During the pre- and post-observation interviews, all the teacher participants expressed their satisfaction with their work environments by expressing positive emotions using words and phrases in their statements during. (e.g., "happy ever since," "like working here," "feedback I get from all around," "principal has confidence in my teaching," etc.). Satisfaction with the work environment generates positive emotions thereby affecting teachers' attitudes toward their work. Thus, acquiring emotional intelligence in the context of their teaching is a part of a teacher's cognition in addition to interpersonal intelligence (Gardner, 1993). Responses from the participants suggest that this knowledge base grows with teachers' reflection and observation of students' actions in the class. Some of the behaviors associated with the phases of the BPSP were common across more than one phase suggesting *the interdependence* and *cyclical nature* of these phases.

Conditions influencing Teachers' Engagement with the Development and Display of Professional Competence

Six conditions were determined to influence the progression through the phases of BPSP. These conditions were (1) subject matter knowledge at the entry level in the teaching profession, (2) participants' prior knowledge about students' academic backgrounds (students' abilities and misconceptions about various topics), (3) teachers' subject-specific and context-specific conceptions, (4) self-regulation or cognitive process of analysis of self-efficacy, (5) working hard, and (6) constraints and affordances provided by the teaching contexts. The first two conditions are constituent of teachers' cognition. Teachers with undergraduate majors in mathematics and advanced degrees in mathematics were observed to involve students in mathematically rich classroom discourse. The students in the classrooms of these teachers were observed to emulate their teachers. Participants with strong content knowledge provided more powerful conceptual explanations and demonstrated rich subject-specific conceptions. Competence in quality questioning and the ability to orchestrate mathematical discourse was demonstrated by the participants to varying degrees. All the participants asked a mix of higher order and lower order questions during their lessons. An excerpt from a teaching episode from Sam's (see Figure 2a) and Suman's (see Figure 2b) classes given below suggests their competency in quality questioning and scaffolding.

Detailed discussion of vignettes from the lessons observed is beyond the scope of this paper. Teacher participants demonstrated a mix of teacher-centered and student-centered pedagogy tailored to students' needs, and performance expectations of their work environments. Participants' responses revealed good knowledge of their students' abilities and academic needs.

The findings of this study suggest that teachers' knowledge of mathematics affects their *planning* and decision-making and influences their ability to diagnose and handle students' misconceptions and mistakes. These teachers' professed articulation of their goals of successful student outcomes and strategies used to realize those outcomes, perceptions of school-wide support, and high expectations of performance are suggestive of their engagement in the cognitive process of analysis of self-efficacy. Significant differences were noticed in participants' awareness of gaps in their knowledge (a dimension of cognitive process of analysis of self-efficacy). This dimension of teacher cognition was revealed during class observations and likely influenced teachers' handling of students' mistakes and sequential planning of lessons. *Planning* provides teachers with mechanisms to maintain the balance between competing priorities and allows them to focus on students' needs, students' engagement, and classroom management.

Working hard is the process whereby teachers put in extra effort as a way of displaying their satisfaction with their work environment. The process of working hard is suggestive of participants' display of positive emotions that guided their goal-directed behavior of performing to their maximum potential to realize student outcomes. The participants' responses to interview questions indicated the constraints and affordances of the teaching context influencing their planning and instruction.

Excerpts from Sam's Class

Sam, after correcting the problem on the drill, moved to the back of the classroom to check the homework problem based on SAS postulate. A student posted a column proof for the problem. In the proof, statements were correct but reasons for two of the steps were not correct. Sam asked:

Sam: What is segment bisector?

(A Student gave an incomplete definition. Sam wrote the correct definition on the board).

Sam: Ok, what did we not mention when we talk about this definition here?

Student: Midpoint.

Sam: You are close, that's one thing, and we need different reason. What else did we not mention?

Student: Congruent (response came from the same student who posted answer for the problems on the drill and homework during the lessons observed).

Sam: Excellent; that means what? We can't prove segments congruent by definition of segment bisector. You have to add a step before this one.

Student: B is the midpoint.

Sam: You never just say that B is the midpoint. Say B is the midpoint of what?

Student: Segments JL and HC. (Sam writing the step that B is the midpoint of JL & HC on the board)

Sam: Now we have the definition. What else is needed to say that they are congruent?

Student: Midpoint Theorem

Sam: Exactly, so we use the definition of midpoint and then take another step to say that segments are congruent by using the theorem.

An example of lower order questions from Sam's Class:

- **Sam:** Is K the midpoint of GK? (See the screen shot of the clip on the right)
- (A student wrote K as midpoint of GK and MR)
- **Student:** No, only MR.
- **SAM:** K is the midpoint of the segment bisected and not the segment bisecting.
- **Sam:** So far, how many methods do we have for proving triangles congruent?
- **Students in chorus:** Three
- **Sam:** Name them.

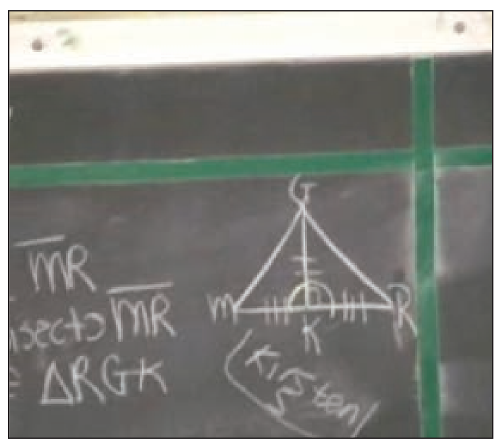

Figure 2a. Excerpt from Sam's geometry class.

Excerpts from Suman's Class—First Day

Suman, while going over the postulates and theorems for proving triangles similar, asked one of the students:

Suman: Do you need three angles to prove triangles similar? What if you have two angles given congruent? Would you be able to prove that triangles are similar?

Student 1: Yes, two angles will be sufficient to prove that triangles are similar because of the angle sum property of a triangle.

Suman: Good. (Pointing to another student) What is the angle sum property of a triangle?

Student 2: No response.

Suman: What is the sum of three angles of a triangle? Is it 90° [pause], 100°?

Student 2: 180°

Third Day

Suman, after spending another ten minutes on review of the proof of the Pythagorean Theorem done in the last class, moved to the applications of the theorem. The problem was taken from the textbook: Prove that in a rhombus, the sum of the squares of its sides is equal to the sum of the squares of its diagonals. After drawing the figure and labeling it as ABCD, she asked one of the students:

Suman: What is special about a rhombus?

Student 1: It has four congruent sides.

Suman: What about its diagonals?

Student 2: They are perpendicular.

Suman: What is given?

Student 3: A rhombus ABCD

Suman: What needs to be proved?

Student 4: $AB^2 + BC^2 + CD^2 + AD^2 = AC^2 + BD^2$

Suman: Anybody want to try the proof? Any idea how you will prove the result?

Student 2: Since diagonals are perpendicular, we can apply Pythagoras theorem.

Suman: How? Give me the first step.

Student gave the step.

(The proof was developed and completed with the help of the students. One of the students was called to the board to go over the proof with the class).

Figure 2b. Excerpt from Suman's geometry class

Constraints and affordances of the teaching context

All the participants cited time constraints and excellent teaching expectations from the administration. During the post-observation interview, Lena remarked, "We have to finish the syllabus for the state exam, and some students struggle with the basic mathematics, and they don't show up for the coach class." Lena further stated, "In addition, periods are only 47 minutes long, so group activities are not possible most of the time. But I try to facilitate peer tutoring." She added, "Sometimes, I have to deviate from my beliefs when it is required to follow the district curriculum guidelines or when some students' plans require me to deviate."

Sam also pointed out the time constraints to finish the syllabus and get the students ready for the AP exams. He noted that it is up to the student to come to coach class:

> It is not easy during the class to get to the students individually and recognize their difficulties. In my old school, classes were 90 minutes long. Thus, there was ample time to observe students while they were practicing. That was better in a lot of ways. I wish, I could do more, but after 47 minutes, they have to go to the next class.

Sam also shared:

> In mathematics we can pretty much say that your ability level is here; you probably will not succeed here. Learning mathematics is more of ability than effort. Unfortunately, when the administration comes to scheduling students, they often ignore such past experiences and recommendations from teachers. They just put students where it is convenient for them or for students. Some students are struggling because of that.

Here, Sam hints at the *constraints and affordances of the teaching context* and the conflict arising out of the interplay between students' academic backgrounds in mathematics and administrative routines.

Teachers in the Indian setting also pointed out the time constraints to finish the prescribed curriculum by the set deadlines to allow for sufficient review and practice for the board examinations. Suman remarked that the emphasis of the mathematics program at the school was to cover more content and challenge students by doing various types of problems expected to be on the exams. Given the short periods, she encouraged students to come before and after school for coaching.

Teachers in this study used their *understanding of the teaching context* (students as context, the subject matter as context, and school structure and culture as context) to shape their interactions with the students. Participants described student readiness and school-wide support as important contextual factors influencing their practice and competence. The influence of *school as context* was seen on teacher's competence (which includes competence in student management and engagement), sense of self as a teacher, self-directed improvement in practice through monitoring of students' outcomes, perception of school-wide or departmental support, and perceptions of high expectations of performance due to the high-achieving status of the school. The influence of *student as context* was seen in teacher participants' professional competence as alertness to students' behaviors and needs during the class. The *constraints and affordances of the teaching context* lead teachers to develop *"coping strategies"* (Hargreaves, 1984, p. 75) *to maintain a balance between competing priorities* (e.g., students' needs, curriculum deadlines, assessment requirements, etc.). As a result of data analysis and reflection, the coping strategies used by the participants were revealed. These strategies were *working hard, seeking knowledge to modify practice, using interpersonal and emotional intelligence* to handle social interactions (with parents, colleagues, students, and administrators) and cognitive conflicts arising in their classroom and work environments, *reflecting* on student outcomes and reflection-in-action and on action (Schon, 1987), and *collaborating* with the department heads and the supervisors.

An Interpretive Summary of the Emergent Themes Relevant to Observed Teaching Practices

In order to resolve the main concern of teaching mathematics effectively to realize successful student outcomes, participants progressed through *five phases*. These phases, along with the conditions # 1-5, represent dimensions of cognition (see Figure 1). The data analysis revealed that teachers' cognitions and conceptions were influenced by *six contextual factors:* high performance expectations of the department head and the principal, clear articulation of expectations and curriculum guidelines, good work atmosphere, availability of curriculum resources, school vision and structure, and students' readiness by and large. The *context consequences* were the outcomes of the actions and strategies used by the teachers during their engagement with the process of developing professional competence. The five consequences were: (1) impact on teacher efficacy (which includes efficacy in student management and engagement), (2) sense of self as a teacher, (3) self-directed improvement in practice through monitoring of students' outcomes, (4) perception of school-wide or departmental support, and (5) perceptions of high expectations of performance due to the high-achieving status of the school. The following section provides an interpretive summary of the emergent themes relevant to observed teaching practices and illuminates the interplay of various components of the professional knowledge base of a competent mathematics teacher as interplay of teachers' conceptions, cognition, and the context in which they enacted their practice.

Teachers' Conceptions

Teachers' conceptions of the nature of mathematics are defined as their "conscious or subconscious beliefs, concepts, meanings, rules, mental images, and preferences concerning the discipline" (Thompson, 1992, p. 132). Research in the field of cognitive neuroscience suggests that cognition is tightly linked to perception and action. Many researchers argue that beliefs are instrumental in defining tasks and selecting the cognitive tools to execute such tasks; hence they play an important role in defining behavior and organizing knowledge (Kagan, 1992; Nespor, 1987). Pintrich (1990) asserts that both "knowledge and beliefs…influence a wide variety of cognitive processes including memory, comprehension, deduction and induction, problem representation, and problem solution" (p. 836). This study provides additional support for the assertion from previous research (Pepin, 1999; Thompson, 1992) that epistemological conceptions about mathematics and its teaching influence the way mathematics is represented in the class, and that teachers' conceptions influence, and are influenced by, teachers' cognitions. For example, Sumit saw mathematics as a hierarchical and logical subject, and emphasized conceptual understanding during his instruction. He purposefully selected relevant examples in a logical sequence. Sumit also demonstrated powerful representations of mathematical concepts indicating strong underpinnings of the content. Thus, professional competence in the teaching of mathematics can be seen as an interplay between teachers' cognition and conceptions.

The data suggested a high degree of consistency observed in professed and enacted conceptions about (a) social norms influencing participants' teaching practice, and (b) nature of mathematics and its teaching for most of the participants. The inconsistency between professed conceptions and actions of some of the teachers seem to arise from the differences in the nature of teachers' cognitions. The factors that might have caused the inconsistency seems to be (a) the depth of conceptual and procedural knowledge of mathematics, (b) ability of the teacher to reflect and analyze the teaching context, (c) the constraints and affordances of the teaching context: students as context (their academic backgrounds and dispositions towards mathematics), (d) teachers' subject-specific and context-specific conceptions, (e) school structure and culture, and (f) teachers' personal epistemology of teaching mathematics developed as a result of their lived experiences shaped by the above factors (Ahuja, 2018a). The findings of this study suggest that teachers enter the teaching profession with some formal knowledge, and then gain knowledge from the teaching contexts based on the school culture and structure. Since teachers' acquire their subject-specific and context-specific conceptions through experience (Calderhead, 1996), the process of acquiring conceptions evolves based on teachers' lived experiences. The culturalist view of cognition emphasizes the construction of knowledge as a cultural process derived from the cultural context of the people. Thus, it is appropriate to conclude that teachers' cognitions and conceptions are informed by cultural traditions.

Since teachers' conceptions are manifested in teacher behaviors and conceptions are culturally determined, it is appropriate to view the effect of culture as a context for professional competence in any cross-cultural investigation of teaching. Once it is recognized that teachers' lived experiences are situated in social settings, it is easy to understand that professional competence is social and developmental, and progression through various phases of development, and display of professional competence is an interplay among teachers' lived experiences, decision-making processes, and cultural traditions existent in teaching contexts. Thus, inter- and intra-cultural differences in the professional competence of teachers can be attributed to the utilization and access to intellectual and cultural resources, and the meanings teachers derive based on their discipline-specific conceptions. Professional competence is intimately related to the personal styles of the teacher. That is, a particular teacher will always bring to their professional practice their own personal characteristics, which arise from and relate to their past and current experiences and cultural traditions in a unique way. Due to the dynamic nature of the culture and evolving lived experiences, it is argued that this process of evolution of teachers' conceptions is also constructive, integrative, and fallible.

Teachers' Cognition

This study suggests that teachers' cognitions are the driving force behind their actions and pedagogical articulation. The data analyses revealed that teachers' cognition is everything that teachers think, experience, and acquire in order to teach effectively. It includes dimensions such as knowledge about the content, planning of lessons, understanding of students, using tacit knowledge and learning from personal experience, updating existing knowledge, asking questions to self, understanding context (student, teaching, and subject), handling cognitive conflict, setting goals, thinking through lesson-specific and/or teaching situation-specific situations, assessing students, interacting with students and school personnel, problem-solving, analyzing teaching tasks and contexts, understanding emotional intelligence (a combination of interpersonal and social intelligence), and communicating effectively. Affective dimensions of cognition include responsibility, patience, professional ethics, and flexibility. For this study, the definition of conceptions accords with that of Thompson (1992), teachers' cognition also includes teachers' intuitions, skills, values, and feelings as well as attitude towards their work. Reflection, a cognitive skill is the ability to give thoughtful consideration to the knowledge used when one spontaneously responds to a situation (Schon, 1987). Reflection can be either reflection-in-action (Schon, 1987) or reflection-after-action with a sole purpose of improving one's practice by highlighting the deficiencies in one's knowledge and skill repertoire.

The process of developing self-awareness of gaps in one's knowledge and making statements about one's own process of problem-solving and achieving means to address this gap in knowledge was revealed during participants' responses during the interviews. This cognitive process was called "analysis of self-efficacy" and was reported as one of the competencies demonstrated by the participants during instruction. This study found that the development of professional competence is characterized by constant engagement in reflection and in responding to challenges of the work context, thereby engaging in the kind of learning that extends one's competence. Thus, professional competence *entails the process of activating* various components of professional knowledge base in a variety of problematic situations. The emergent definition of cognition extends the existent definition of cognition in the literature. The various dimensions of teachers' cognition are experience based and are context dependent. Furthermore, this process is amenable to analysis and modification. The foregoing discussion suggests that due to the cyclical nature of various phases of the process of becoming professionally competent, cognition is a constructive, integrative, and fallible process. The foregoing discussion again suggests that professional competence is an interplay among teachers' conceptions, cognition, and the context.

Teaching Context

Davis and Sumara (1997) view cognition as a process of organizing and reorganizing one's own subjective world of experience which involves revision, reinterpretation of past and present, and projected actions and conceptions. This view underscores the situatedness and context-dependent nature of cognition. This view of cognition in the context of mathematics teaching and learning is referred to as enacted cognition or embodied cognition (Nunez et al., 1999) and is in tandem with the sociocognitive view of mathematics learning. It suggests that both mathematics and competence in its teaching are products shaped by human brains, societies, and culture. Schools, as institutional cultures, aim to create ideal futures for students and teachers (Grossman, Smagorinsky, & Valencia, 1999). Students and teachers aspire to culturally defined futures that motivate their activities, and the ways in which they mediate one another's progress toward those goals (Joseph, Bravmann, Windschitl, Mikel, & Green, 2000). Social teaching norms and immediate classroom situation play key roles in influencing mathematics teaching practice (Raymond, 1997; Yackel & Cobb, 1996). Thus, context is crucial in understanding the experiences of both the students and teachers in mathematics classrooms. McLeod (2001) views teaching as "a social act defined as much by its sociocultural, biographical, and historical contexts as by its structural context" (p.1).

The professional competency: *understanding and the analysis of teaching context* as a constituent of teacher cognition has already been explained in the foregoing discussion. An important common feature of mathematics teaching as observed in the two research settings is the existence of a supportive learning environment characterized by student engagement and cognitively demanding conceptual mathematical tasks. The study found that the contextual factors that facilitated teachers to develop personal mathematical epistemologies and a sense of professional competence were high expectations of the department head and the principal, good work atmosphere, clear articulation of curriculum guidelines, availability of curriculum resources, and high levels of academic press. Thus, *a high level of student engagement* was both a *product of and a stimulus to* the teacher's professional competence. McLaughlin (1993) argued that students help to define the context of teacher work and teachers' perceptions of their students influenced how teachers approached their work. Additionally, a school's mission, organizational structure, policies, procedures, and "patterns of communication" (McLaughlin, 1993, p. 89) also shape teachers' goals of teaching leading to change in their cognitions and conceptions. Hence, the *context* constantly constrains teachers' cognitions and conceptions.

Thus, the core category which appears to account for all of the processes and conditions through which professional competence was displayed is the notion of the *interplay among conceptions, cognition, and the context.* The emergent theory predicts the ways teachers would operate in order to resolve their main concern of professional competence in the teaching of mathematics by activating all the resources available in their professional knowledge base to handle the actual teaching situations which at times can be problematic, and this process is an ongoing pursuit. Having said that, it is appropriate to conclude that professional competence is embedded in practice and is transformed through deliberate goal-directed behavior. This leads to the conclusion that the nature of professional competence is constructive, integrative, and fallible.

Evaluation of GT

The theory is a good and trustworthy theory if it meets the four criteria: fit (validity), work (generality), relevance (understanding), and modifiability (control).

Fit

Since categories are generated directly from the data, the criterion of fit is automatically met (Glaser, 1978). The procedures followed for data analysis were documented. Description and conceptualization of the theory have been explained earlier in this paper. Additionally, to provide a "true description of a given reality" (Janesick, 1998, p. 119) direct quotes from the interviews and class observations along with shots of video clips were presented in the portrayal of mathematics teaching from the two research settings. The compliance with the criterion of fit was ensured by closely following the procedures such as selective coding, theoretical sampling, constant comparison across all the data sources, and memo sorting.

Work

A grounded theory works when it explains the major variations in the way participants respond when dealing with their main concerns. By explaining how the teachers' main concern was resolved and by presenting the interpretation of the emergent theory and its contrast with the extant literature, the researcher has tried to show that it works.

Relevance

"If it fits and works the grounded theory has achieved relevance" (Glaser 1992, p. 15). Relevance is ensuring that the theory "deals with the main concern of the participants" (Glaser, 2001, p. 18). This means that it will offer explanations of the main concern in the substantive area. This study attempted to develop an understanding of how quality in teaching and learning of mathematics is influenced by the context in which this learning occurs. During the data collection and analysis phase, the researcher questioned herself: What is really going on that is important to the participants in the substantive area and in the context of their work that has an impact on their actions.? The BPSP of interplay among conceptions, cognition, and the context has specific relevance to the participants operating within the context of the high-achieving school environment where teacher efforts were supplemented by school-wide efforts. The school culture characterized by set vision and expectations of standards of mathematics teaching and performance, and their reflection on students' outcomes constantly motivated teachers to enhance their professional competence. The framework as envisaged by the theory of professional competence enables predictions of the way teachers would most likely engage in order to meet the performance standards of their teaching contexts. Thus, by staying close to the data, avoiding preconceived theories and generating concepts that were germane to the participants' basic psychological problem of teaching mathematics effectively, it was ensured that the grounded theory of professional competence: interplay of conceptions, cognition, and context ensures compliance with the criterion of relevance.

Modifiability

Within the context of this study, the theory was constantly modified through constant comparison across all the elements of data corpus. The technique of constant comparison is both inductive and deductive. Care was taken to return to the data to substantiate the emerging themes and categories, and this process continued until theoretical saturation.

Summary

The Theory of Professional Competence: Interplay of Conceptions, Cognition, and the Context

The substantive theory that emerged from the data suggests that *professional competence in the teaching of mathematics is a complex interplay of teacher's conceptions, cognition, and the context* (see Figure 3). These three components chase each other in a cyclical process through intervening conditional and consequential variables as evident from figures 1 and 3. The study found that pro-

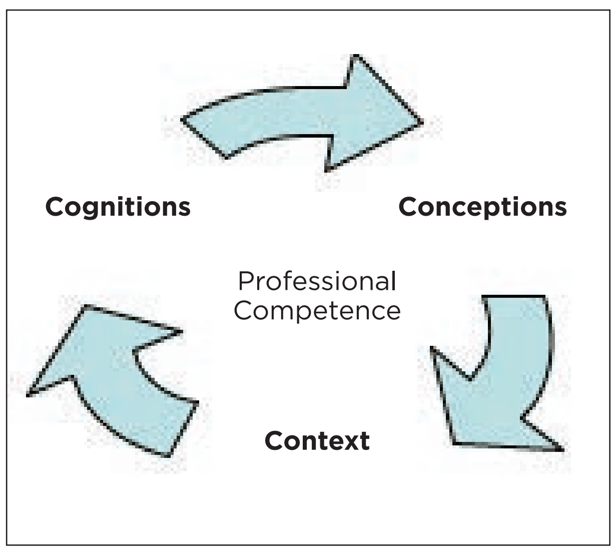

Figure 3. Ahuja Theory of Professional Competence: The Interplay of Conceptions, Cognition, and the Context.

fessional competence can be defined in terms of the competencies and/or qualities, and as a process associated with the individual and the context. An important conclusion that can be drawn from this study is that professional competence is the possession of a set of components of the professional knowledge base and the process of activation of these components in the context of actual teaching situations in response to contextual factors. These components chase each other in a cyclical manner emphasizing the role of cultural, historical, and political influences on the professional competencies of teachers as these influences are deeply rooted in the cultural traditions of countries. The content knowledge combined with experiential knowledge, responsibility, and flexibility emerged as important factors influencing teacher's competence in mathematical discourse, organization of instruction, representation of content, persistence, and the management of class time. The findings of this study suggest that due to the constructive, integrative, and fallible nature of teacher cognition, conceptions, and the context, the nature of professional competence is dynamic, constructive, integrative, and fallible. The idea of professional competence as the result of an interplay among these three components suggests that rather than being a static feature, professional competence is a dynamic one manifesting itself in a variety of behaviors even in the same teacher. It may thus be defined as the possession of a set of components of professional knowledge base and the activation of these components in the context of actual teaching situations.

Limitations

This study is limited to the analysis of mathematics teaching practices in the selected two high-achieving high schools located in Delhi, India, and Maryland in the United States. Limited resources and funding restricted sophisticated analyses of videotaped lessons. The teacher participants selected from the two schools were not representative of the national sample. A small sample with quality interviews and in-depth analysis can produce significant findings (Charmaz, 2014). Findings may be applied in similar contexts. Hence, the goal of this study was not to formulate generalizations, but rather to generate a substantive theory using categories, the relationships between them, and their properties.

Concluding Statement

The author believes that different observers may understand and interpret data differently based on their professional knowledge and experience. Thus, rich portrayals of settings and mathematics teaching (which included direct quotes from the interviews and photos from the videos, examples of teaching episodes, mathematical discourse, student-teacher, student-student interactions) were presented in the detailed description of the findings (discussed elsewhere) to enable the reader to evaluate the authenticity of the findings thereby reducing the effect of researcher bias. In addition, multiple methods of data collection, the constant comparative technique of data analysis, and peer debriefing were employed to ensure trustworthiness of the findings. The teacher participants interpreted their conception of self in the context of the high-achieving status of the school. A learning culture characterized by high performance expectations existed in both the settings where teacher efforts were supplemented by school-wide effort. The knowledge domain of professionally competent mathematics teachers' cognition is vast and is a complex combination of various components of teacher's professional knowledge base (Ahuja, 2013). The teacher's classroom practices can then be seen, in part, as an attempt to find a balance between competing priorities involving curriculum, content, teaching goals, teaching context, social and pedagogical climate, student-teacher relationships, instructional strategies, cognitive conflicts arising out of classroom situations due to misconceptions and prior dispositions held by the students about mathematics, time constraints, and performance. Professional competence lies in responding to these challenges intuitively. The data collected from the participants suggests that competent teachers respond to these challenges spontaneously and this spontaneity develops with experience and is founded on a strong grasp of subject matter knowledge and mathematics-specific pedagogical knowledge.

Teachers' conceptions, cognitions, and the constraints and affordances of the teaching context in the work environments of the teachers are so intertwined that it is almost impossible to separate one from the other while studying and developing the notion of a construct of professional competence in teaching of mathematics, as complex as the discipline of mathematics itself. It is reiterated that the purpose of data analysis in grounded theory research is to develop a core category that explains and characterizes the process in which the participants engage to resolve their main concern. The goal is conceptualization and abstraction rather than description (Glaser, 2001). The theory of professional competence developed because of this study offers a tool to teachers, educators, administrators, and researchers to evaluate and understand the interaction between various parts of the emergent theory. The theory can be tested across various teaching contexts. A point to note here is that certain sub-concepts and subcategories in the theory are not unique to the substantive area of mathematics education.

The findings of this study suggest that teacher's cognitions and conceptions are contextualized. Due to the context-dependent nature of professional competence, holistic assessment (Eraut, 1994) of teachers' competence should be the norm. This holistic assessment should be emphasized during the training of administrators, supervisors, and department heads. Realizing the fact that it is a teacher's responsibility to enculturate his/her students mathematically, it is imperative for other stakeholders in public education to facilitate this process of enculturation by providing a favorable context, climate, and resources. Glaser (1992) states, "the theory provides a conceptual approach to action, changes, and accesses into the substantive area" (p. 15). Due to limited resources, the sample was delimited. The study may be replicated at the elementary and middle school levels. The emergent theory of professional competence has the potential to rise to a formal theory by carrying out comparisons of emergent theories from diverse teaching contexts and other fields.

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ACKNOWLEDGEMENT OF REVIEWERS

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