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CELEBRATING 10 Years OF CONTRIBUTIONS TO MATHEMATICS EDUCATION

JOURNAL OF MATHEMATICS EDUCATION AT TEACHERS COLLEGE

A Century of Leadership in Mathematics and Its Teaching

Promoting Equitable Practices in Mathematics Education

Published Semiannually by the Program in Mathematics and Education

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AIMS AND SCOPE

The Journal of Mathematics Education at Teachers College (JMETC) is a recreation of an earlier publication by the Program in Mathematics and Education at Teachers College, Columbia University. As a peer-reviewed, semiannual journal, it is intended to provide dissemination opportunities for writers of practice-based or research contributions to the general field of mathematics education. Although each of the past issues of *JMETC* focused on a theme, the journal accepts articles related to any current topic in mathematics education, from which pertinent themes for future issues may be developed.

JMETC readers are educators from pre-kindergarten through 12th grade teachers, principals, superintendents, professors of education, and other leaders in education. Articles appearing in the *JMETC* include research reports, commentaries on practice, historical analyses, and responses to issues and recommendations of professional interest.

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INTRODUCTION

A DECADE OF CONTRIBUTIONS TO THE FIELD

The Tenth Anniversary Issue of The Journal of Mathematics Education at Teachers College

By Brian Darrow, Jr. and Dyanne Baptiste

To begin this special issue, we wish to note its significance. Since the field of mathematics education was founded here at Teachers College, Columbia University (TC) by David Eugene Smith, the Department of Mathematics Education has been contributing to research and practice in the field for over one hundred years. This department sponsored the very first dissertations in mathematics education in 1906 and has since been the single largest producer of mathematics education doctoral graduates in the world. This department is also the oldest and largest graduate program in mathematics education in the country.

The Journal of Mathematics Education at Teachers College (JMETC) continues to honor our distinguished legacy of contributing leading research and practice to the field. Its predecessor, *Contributions to Mathematics Education (CME),* began this history. *CME* dates back as early as the founding of the field itself and focused on the publishing of doctoral dissertations. After its final issue in 1970, a publication in mathematics education at TC was not in print until the inaugural issue of *JMETC* in 2010. Since this "resurrection of *CME*" by Professor and Director of the Program in Mathematics Education, Bruce Vogeli (whose service of over fifty years to mathematics education and TC has been extensively recognized), and his colleagues, *JMETC* has continued with its modern focus on publishing leading research and practice-based articles in mathematics education.

This issue of *JMETC*, therefore, marks a decade of excellence in such publication. Since its inaugural issue, *JMETC* has contributed 20 issues of over 160 research and practice-based articles by, and for, pre-kindergarten through twelfth grade educators, students, administrators, professors, researchers, and other leaders in mathematics education. Our publication has featured such recognizable leaders in the field as Robert Berry, Erika Bullock, Ubiratan D'Ambrosio, Herbert Ginsburg, Carole Greenes, Jeremy Kilpatrick, Sandra Okita, Henry Pollak, Alfred Posamentier, Alan Schoenfeld, Edward Silver, Clarence La Mont Terry, Jamal Young, and Hung-Hsi Wu, to name a few. We have also featured individuals that deserve to be recognized as the future leaders in mathematics education. In this way, we continue to promote quality scholarship at all levels.

Our history serves as a beacon for our future. In honoring our past, we are inspired to proceed with a renewed sense of purpose. Our commitment to quality, excellence, service, and most importantly, our readers, is stronger than ever. We continue to value the support we receive from our readership, authors, editors, reviewers, and all those who value mathematics education. With this, our scholarship will persevere for decades to come.

PREFACE

Supporting Students at all Levels through Equitable Practices

This issue of the *Journal of Mathematics Education at Teachers College (JMETC)* features articles highlighting equitable practices that support student achievement in mathematics education. In general, we see such practices as those that help educators meet all students where they are, support their individual needs, and enable them to achieve their full potential. Each of the articles that follow provide specific examples of how practitioners and researchers can anticipate, recognize, and address opportunities to meet students' learning needs, regardless of their educational backgrounds. Readers will see that these efforts are not solely at the student-level, but also at the level of educators and larger educational systems.

To begin this issue of *JMETC,* Thomas and his colleagues explore how bias emerges in the way preservice teachers interpret students' mathematical thinking. The authors describe how non-academic factors—such as perceived notions of race—may influence preservice teachers' perceptions of children's mathematical abilities. This provides several implications for these teachers' future practice, including their equitable noticing of students' mathematical behavior and ability to be address potential bias in the classroom.

Meagher, Koca, and Edwards also focus on preservice teachers, by exploring their generation of mathematical conjectures. Through detailing a classroom episode, the authors discuss the importance of preservice mathematics teachers continuing to engage in mathematical conjecturing, even as experienced mathematics students themselves. By doing so, the authors argue that these experiences enable teacher educators to model best practices for nurturing the generation of mathematical conjectures in their future classrooms. Furthermore, teacher educators engaging in such practices will be better equipped to structure such activities for their students in the future, and authentically address difficulties that these students may encounter.

Osborn and Ma shift the focus to examining student behavior, particularly their help-seeking behaviors and how these relate to mathematics achievement. Through examining Programme for International Assessment (PISA) data, the authors found a significant, positive relationship between these behaviors and different outcome measures. This finding seems to transcend individual students, classrooms, and schools. The authors also highlight how educators can promote help-seeking behaviors in an equitable way for all students to become competent "doers" of mathematics.

Khan continues the examination of student behavior and investigates a specific support system for first semester calculus students in college. The author analyzes anecdotal and survey data to reveal the influence of

PREFACE (Continued)

supplemental instruction and online components on students' growth in metacognitive and study skills. The modest connections between these is detailed, which paves the way for future research on how technological and supplemental supports can influence student success in high-risk courses. Through incorporating such supports into these classes, educators may be able to address underlying differences in student achievement that are not evident in a traditional classroom.

Sole shifts the focus again, to that of equitable practices at the program level by investigating how policy and systemic procedures influence the educational paths of community college students. The author challenges traditional mathematics course placement methods, which she describes as reinforcing inequitable circumstances among students from under-represented populations and those deemed underprepared for college-level work. The experimental placement initiative focused on high school grades rather than high-stakes standardized tests to place students and included innovative academic support structures within the classroom. In her analysis of this, Sole discusses how such an initiative can address issues of equity inherent in current procedures and support a diverse population of students to be successful in science, technology, engineering and mathematics (STEM) and non-STEM educational pathways.

We conclude this issue with an invited piece in honor of our 10th anniversary publication from an alum who began the doctoral program 10 years ago, and who continues to advocate for humanizing mathematics education. Dickman, named a 2020 Early Career Award winner for Teachers College, writes in his article about two novel approaches to support teachers and students as they engage in problem posing. He begins with some of the history related to this important yet underemphasized component of mathematics education, and then focuses on providing examples of authentic practitioner materials for working towards educational environments that support participants in both feeling and being mathematically generative.

Collectively, these articles detail equitable practices at the level of the student, educator, teacher educator, and system. It is our sincerest hope that the research presented here inspires our readers to continue to learn and promote equitable practice in mathematics education. In doing this, we can support and value all students, and enable them to reach their highest potential in mathematics.

Dyanne Baptiste
Brian Darrow, Jr.

Guest Editors

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Investigating the Manifestations of Bias in Professional Noticing of Mathematical Thinking among Preservice Teachers

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Cindy Jong, *University of Kentucky*
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ABSTRACT This study examines potential bias with respect to perceived gender and ethnicity in preservice teachers' professional noticing of children's mathematical thinking. The goal of the study was to explore how, and to what extent bias emerges within pre-service teachers' professional noticing of children of differing perceived races and genders. Our findings suggest that bias tends to emerge in the interpreting phase of professional noticing; however, such emergence did not appear to vary in conjunction with the perceived ethnicity and gender of the student. Further, our findings suggest that the inclusion of visual imagery (i.e. photos) influence the manifestation of bias among preservice teachers to some degree when professionally noticing in the context of a written case.

KEYWORDS *equity, professional noticing, teacher noticing, numeracy, preservice teachers*

Equity concerns in mathematics have been pervasive for decades (Breslich, 1941; Diversity in Mathematics Education Center for Learning and Teaching [DiME], 2007; National Council of Teachers of Mathematics [NCTM], 2014), but there has been renewed attention to how mathematics classroom environments can support students from diverse backgrounds. Research has shown that students experience school differently because such experiences are informed by their racial and gender identities (Boaler, 1997; Gutierrez & Dixon-Roman, 2011). This is especially evident in STEM disciplines such as mathematics where students from non-dominant groups may have received implicit and explicit messages from an early age that they are not capable or do not belong (Goffney et al., 2018; Museus et al., 2011). In some cases, students have inequitable experiences because their teachers have lower expectations for them or do not consider their culture in their practice (Savage et al., 2011; Zavala, 2014).

We view teaching mathematics for equity as providing opportunities for all students "to learn rigorous mathematics in culturally specific, meaningful ways that seek to improve the economic and social conditions of marginalized individuals and groups, and that work toward reduc[ing] deficit-oriented beliefs about who is or is not 'good' at mathematics" (Leonard & Evans, 2012, p. 100). In order to make mathematically appropriate decisions in a classroom, the framework of *professional noticing of children's mathematical thinking* (hereafter, *professional noticing*) is used to guide teachers' understanding of children's knowledge (Jacobs et al., 2010). Teachers' beliefs about who is and is not "good" at mathematics represents an intersection of professional noticing and equity. The process of professional noticing describes teachers' *perceptions* of student thinking. There are, however, opportunities for manifestations of bias inherent in such a process. The purpose of this study was to explore how and to what extent bias emerges within pre-service teachers' professional noticing of children of differing perceived races and genders.

Professional Noticing of Children's Mathematical Thinking

The professional noticing framework used in this study incorporates three interrelated components: attending, interpreting, and deciding (Jacobs et al., 2010). The first component of professional noticing, *attending,* is to observe and identify children's words and actions when engaging in mathematical activity. Next, *interpreting,* is to relate what is attended to, with what is known about development in mathematics knowledge in order to determine what the child understands. Finally, *deciding,* is decision making based on what is interpreted in order to ensure a student is learning in a way that best fits their current understanding of mathematics.

Professional noticing and related practices (e.g., teacher noticing, professional vision) have captured the attention of mathematics education researchers for decades (Goodwin, 1994; Mason, 2002; Schack et al., 2017; Sherin et al., 2011). Researchers in the past have considered forms of observation and interpretation such as *professional vision* (Goodwin, 1994), *teacher noticing* (Sherin et al., 2011), or simply the *discipline of noticing* (Mason, 2002). Building upon these conceptions, Jacobs and colleagues (2011) conjectured a third, interrelated component, *deciding,* thus creating the phrase *professional noticing* (of children's mathematical thinking). While professional noticing may seem somewhat intuitive, Jacobs et. al (2010) determined that focused practice (rather than simply years of teaching experience) was predictive with respect to noticing quality.

In their explorations of teacher noticing, Sherin and van Es (2009) used teacher video clubs as a way to analyze student thinking. Participants provided videos of their own classrooms and analyzed the videos through a noticing lens during the professional learning sessions. They found that focusing on noticing children's thinking impacted the teachers' instructional practices in a positive way.

When authentic classroom experiences are not available for instructing preservice teachers, classroom videos are commonly used for those instructional purposes. We used video vignettes to improve the professional noticing skills of elementary preservice teachers and found positive changes within one semester of instruction focused on professional noticing (Fisher et al., 2018). Using a wider lens for analysis, Stockero et al. (2017) used classroom video to gain practice in identifying key opportunities for pedagogical action within a mathematics lesson. Referred to as Mathematically Significant Pedagogical Opportunities to Build on Student Thinking (MOST), these incidences may be thought of as teachable moments (Stockero et al., 2017). In this instance, professional noticing may be thought of as a narrowing practice aimed at filtering instruction to find key points of leverage. Conversely, other portrayals of professional noticing describe the practice more as a net aimed at gathering all pertinent information within a moment and leveraging such information for interpretations and decisions (Schack et al., 2013).

Equity

Examination of professional noticing through the lens of equitable teaching practice has been the focus of recent studies (Jong, 2017). Specifically, professional noticing has been interwoven with equity constructs and frameworks to better understand how teachers' activity in the moment influences students' participation and positioning (broadly construed) in the mathematics classroom. Examining the relationship between professional noticing and equity concerns, Louie (2018) makes a case that strict cognitive orientations of professional noticing can often miss important cultural and ideological dimensions of children's mathematical activity. Louie describes a teacher who challenges historical/traditional views of intelligence in mathematical contexts. By positioning students from underrepresented cultures/races as mathematically capable, these students are able to assume more positive mathematics identities. This type of positioning also espouses an asset-based perspective where students' backgrounds and their contributions are valued. Similarly, Harper (2010) presents an anti-deficit framework of research on students of color in STEM by shifting questions to focus on assets. For example, rather than asking why there are so few African American females who succeed in mathematics, the issue is reframed such that teachers focus on promoting mathematical success among Black females. While subtle, the reframing of questions, challenges, and issues in mathematics education is consistent with asset-oriented perspectives regarding particular research initiatives.

Further, Jackson et al. (2018) integrated professional noticing with four dimensions of equity (i.e., access, achievement, identity, power) put forth by Gutierrez and Dixon-Roman (2011) to create an equitable noticing framework for the investigation of equitable presence within each component of noticing. Jackson et al. (2018), write:

> Classroom episodes are complex; it is inevitable that individuals choose, consciously or subconsciously, what they notice, or attend to, and use the interpretations of these events to make instructional decisions. . . . Teachers may attend to equitable issues in the classroom as a process or a product. . . . Seeing equity as a process means treating all students equally without regard to race, ethnicity, or economic background. On the other hand, seeing equity as a product means differentiating instruction based on students' needs; implementing equitable approaches that are respectful of students' ethnic, racial, and economic background and promoting equal learning outcomes. (pp. 266–267)

The application of an equity lens to the practice of professional noticing provides space to consider how, and to what extent, mathematics teachers are considering and responding to such difference in the moment (e.g., racial, ethnic, gender, socio-economic, etc.) among students.

Kalinec-Craig (2017) also provides another example of the intersection between professional noticing and equity concerns. To examine the role of race in professional noticing, she examined how three Mexican American preservice teachers considered their own, in-the-moment teaching practices. Interestingly, two of the three subjects engaged in more detailed professional noticing of status and participation when the background of their students differed from their own. This suggests that culture, experiences, and biases influence, to some extent, the manner in which teachers use professional noticing in the mathematics classroom. In summary, intersections between noticing and equity concerns are of rising prominence in the mathematics education literature.

Methodology

Survey Design

To examine the emergence of bias (i.e., asset/deficit perspectives), an electronic survey was constructed. The primary element of this survey was an adaptation of a video-based professional noticing measure from a study of preservice teachers' (PSTs) professional noticing capabilities (Schack et al., 2013). Specifically, rather than using a video-recording as the anchor for professional noticing measurement, we substituted a transcription of the video recording. Prior iterations of this study only included a transcription of the video with various names of perceived ethnicities and genders. In this study, there was an additional feature added to the survey—pictures attached of students which appear to match the race and gender of the child's name.

Similar to Schack et al. (2013), PSTs were asked to respond to three prompts; however, in the current study, the picture was also visible. Each prompt aligned with a particular component skill of professional noticing:

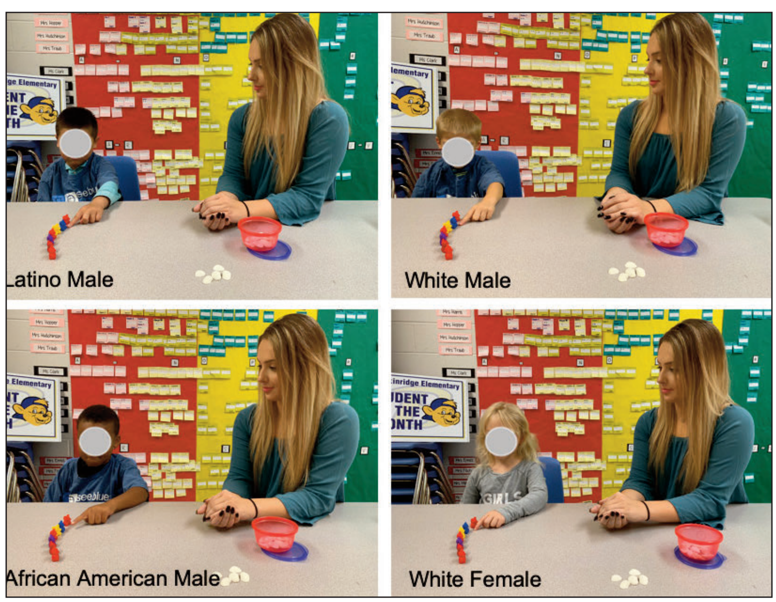
1. Please describe in detail what [Student Name] did in response to the problem. (attending)
2. Please explain what you learned about [Student Name]'s understanding of mathematics. (interpreting)
3. Pretend that you are [Student Name]'s teacher. What problems or questions might you pose next? Provide a rationale for your answer. (deciding)

Additionally, PSTs were prompted to provide some basic demographic data (i.e., gender, ethnicity, age, home state) as well as their familiarity with professional noticing. However, demographic questions were asked at the end of the survey to alleviate any priming effects. Specifically, posing demographic questions (i.e., race, gender) at the onset may prime participants to more consciously consider such constructs in subsequent items thus distorting measurement of unconscious forms of bias (Lavrakas, 2008).

The affordance of using a transcript rather than a video recording was that it allowed us to easily modify the perceived gender and race of the student in question. As such, we generated transcripts featuring the names and pictures of four different students with the aim of each student eliciting different perceptions of gender and/or race. The transcript case names were *Margaret* (perceived white female), *William* (perceived white male), *Shaquan* (perceived African American male), and *Miguel* (perceived Latino male). We acknowledge that the previous iteration of this study assumes that participants perceived the intended race solely based on names, which is why we eliminated the limitations of assumption based solely on the name by adding a visual representation paired with the stereotypical names (see Figure 1). Note, we staged these case photos such that student and

Figure 1

Case Visuals



teacher poses were as close to identical as possible, and the clothing was similar (i.e., child wearing the same t-shirt) in three of the photos. We limited the situations to these four cases since we wanted to maximize opportunities to examine differences across gender (i.e., male/female—William/Margaret) and race (i.e., African American/Latino/white—Shaquan/Miguel/William). While more cases would have allowed for additional comparisons (e.g., Latino Female/Latino Male), they would also have necessitated a much larger data set to ensure that each case had an adequate number of survey respondents.

Participants

The electronic survey was fielded nationwide among PSTs who were in various stages of their respective teacher education programs at their institutions of higher learning. To increase the probability of PST response rates, we leveraged professional connections to mathematics teacher educators as the mechanism for fielding this survey. We sent the survey (along with some brief recruitment text) to 31 teacher educators across 18 states. These individuals were within the professional networks of the authoring group. Our reasoning for these invitations was that these individuals would be more likely to field the survey given such professional connection. These individuals were then asked to forward the instrument to PSTs in their mathematics and/or mathematics methods courses. The survey had 315 total respondents; however, 145 of the respondents only answered preliminary questions regarding familiarity with professional noticing and then exited the survey without completing the remainder of the questions focused on responding to the transcript as well as the demographic questions at the end of the survey. The incomplete surveys were manually discarded during evaluation of the data which left a total of 170 completed responses. Among the 170 participants, as one might expect of a preservice teacher sample (McFarland et al., 2019), the largest gender and ethnic demographic was 18–24-year-old white females. Additionally, we used an electronic apportioning tool to ensure a relatively equivalent distribution of cases across respondents. That is, the survey tool selected a particular case (Margaret, Miguel, etc.) for a given respondent and ensured that each pool of respondents by case was relatively similar in number. Note, though, that incomplete responses rendered this distribution marginally unequal. See Table 1 for response apportionment across cases.

A prior iteration of this study involved a fielded survey that was identical in every way but *did not include a photo.* That prior survey was sent to 30 teacher educators across 17 states, and these individuals were asked to forward the instrument to PSTs in their mathematics and/or mathematics methods courses (Thomas et al., 2019). The survey had 214 total respondents; however, 63 of the respondents only answered demographic questions and then exited the survey without completing the remainder of the questions focused on responding to the transcript. The incomplete surveys were manually discarded during evaluation of the data which left a total of 151 completed responses.

Table 1

Response Apportionment Across Survey Types and Cases

Case	# Respondents (Photo)	# Respondents (No Photo)
Margaret	44	38
Miguel	39	45
Shaquan	45	32
William	42	36
Total	**170**	**151**

Analysis of Professional Noticing Skills

Each response was scored for quality of responses using the same flow-process tool (American Society of Mechanical Engineers [ASME], 1947) developed for the professional noticing study upon which this inquiry is based (Schack et al., 2013; 2015). The flow-process tool featured a series of yes/no choice-points for raters regarding the perceived quality of the three components of professional noticing. In order to ensure there was no bias within the raters regarding the child's perceived race or gender, data were blinded and combined into one list per component. Each component (attending, interpreting, deciding) was scored with individual scoring tools by two raters. Based on the previous studies (Schack et al., 2013; 2015), benchmarks were established for the ranked responses for each component resulting in four ranks for attending (Score 1-4), three ranks for interpreting (Score 1-3), and three ranks for deciding (Score 1-3). The *attending* component warranted an additional rank as the researchers agreed that there were mathematical actions beyond the key components of the mathematical activity that merited an additional rank. After scoring, the raters combined data and negotiated any discrepancies in scoring. This resulted in interrater reliability (i.e., rate of agreement) above 70% for each pair of raters before the negotiation of discrepancies.

Analysis of Asset/Deficit in Professional Noticing

The asset/deficit perspectives of the participant responses to the three questions were evaluated using a different flow-process tool (AMSE, 1947). Rather than score for the quality of the response as in the previous study, the scoring tool for this part of the study scored the presence or absence of asset-oriented or deficit-oriented language describing the child. Each response was ultimately ascribed one of four different codes—asset, deficit, both [asset and deficit], and neutral. We refer to these codes as *bias categories* in subsequent sections. Note, neutral responses contained no asset/deficit-oriented descriptions of the child's thinking/activity. See Figure 2 for example responses, by category, in the interpreting component of professional noticing.

Figure 2

Bias Category Example Responses—Interpreting

Asset	Deficit
"He understands the context of the problem and the idea that addition and subtraction are related"	"Student didn't understand the method of counting up on his fingers"
Both (asset & deficit)	**Neutral**
"The student understands cardinality and grouping but may not have mastered basic equations yet to make the mental leap of 11 minus 7 equals 4"	"The student used a counting strategy"

Two raters used the flow process tool to calibrate with sample data, from a previous data set, until an 80% interrater reliability was achieved. The data from the current study were again blinded and combined into one list and scored independently by the two raters. Per previous studies of professional noticing, rating discrepancies were resolved via discussion (Jacobs et al., 2010; Krupa et al., 2017).

Findings and Results

Measuring Bias in Professional Noticing

A previous study measuring equity in professional noticing, scored using the same asset/deficit scale, without a picture of a child that matches the perceived ethnicity and gender of the name, showed that bias tends to manifest significantly in only the interpreting stage of professional noticing (Thomas et al., 2019). Adding the feature of a picture of a child resulted in the same finding. The percentage of responses across perspectives with and without picture can be found in Table 2.

Table 2

Percentage of Responses Across Perspectives

		% With Picture (*n*=170)	% Without Picture (*n*=151)
Attending	Asset	19	6
	Deficit	4	5
	Neutral	75	87
	Both	2	2
Interpreting	Asset	38	31
	Deficit	13	27
	Neutral	24	12
	Both	25	30
Deciding	Asset	2	1
	Deficit	2	3
	Neutral	96	95
	Both	0	1

As evident in the table, there PSTs' bias manifestation occurs most prevalently within the interpreting component of professional noticing. Attending and deciding show that more than the majority of the pre-service teachers responding to questions in these stages tend to be neutral, meaning no bias is shown.

Results of Bias in Each Case

Bias manifesting in the interpreting stage of professional noticing is evident in the results from responses to both survey types (photo, no photo) of the children with perceived gender and ethnicities. As a result, it is necessary to show each student's responses with and without a picture to compare how having a picture may change pre-service teachers' biases (Table 3). Adding a photo to the survey and readministering the instrument with a different group of PSTs resulted in an increase in asset (positive) responses in all cases besides William (perceived white male). Deficit (negative) responses decreased with a picture in all cases.

We conducted chi-square tests to determine whether there are any relationships between case, survey type (picture, no picture), and bias categorizations (asset, deficit, neutral, both) (see Tables 4 and 5). Specifically, for each of the four cases (Margaret, William, Miguel, Shaquan), chi-square tests for independence were performed to test whether the different survey types were

Table 3

Percentages of Responses by Perceived Race/Gender

	Margaret With Picture	Margaret Without Picture	Shaquan With Picture	Shaquan Without Picture	Miguel With Picture	Miguel Without Picture	William With Picture	William Without Picture
Asset	33	26	47	41	53	29	22	31
Deficit	10	24	11	19	19	24	10	39
Neutral	35	39	13	31	13	29	35	22
Both	19	11	29	9	16	18	37	8

Table 4

Chi-square Independence for Each Survey Type Between Case and Noticing Component

Survey Type	Case vs. Attending	Case vs. Interpreting	Case vs. Deciding
No Picture	13.508 (9)	8.061 (9)	7.628 (9)
Picture	7.890 (9)	20.575 (9)*	5.719 (6)

Note. Results are reported as χ^2 (df).

* $p < .05$, ** $p < .01$, *** $p < .001$.

Table 5

Chi-square Independence for Each Case Between Survey Type and Noticing Component

Case	Survey Type vs. Attending	Survey Type vs. Interpreting	Survey Type vs. Deciding
Margaret	7.168 (2)*	9.025 (3)*	3.606 (3)
Miguel	0.935 (3)	4.826 (3)	0.781 (2)
Shaquan	2.430 (3)	1.204 (3)	1.506 (3)
William	7.294 (3)	14.629 (3)**	N/A

Note. Results are reported as χ^2 (df).A test of independence could not be calculated for the William case in the deciding component because all responses for this condition were categorized as "neutral."

* $p < .05$, ** $p < .01$, *** $p < .001$.

associated with a different distribution of attending, interpreting, or deciding bias categories. Furthermore, for each of the two survey types (picture, no picture), chi-square tests for independence were performed for each noticing facet (attending, interpreting, deciding) to test whether each case was associated with bias categorization. All chi-square tests with statistically significant results were further analyzed with post-hoc tests.

For the survey that contained a picture of student and teacher, the results suggest a significant association between case (Margaret, William, Shaquan, Miguel) and bias categorization within the professional noticing component of interpreting. Further, the Margaret case showed a significant association between survey type and the attending bias categorization, and both the Margaret and William cases showed significant association between survey type and the interpreting bias categorization.

A review of the adjusted standardized residuals within the interpreting component revealed significant differences in specific bias categorizations for Margaret, Miguel, and William across the two survey types (see Table 6). We note a design limitation in this comparison as each survey had a different group of respondents. The first survey (no-picture) was fielded in Spring 2018 while the second survey (picture) was fielded in Spring 2019. While both respondent groups were demographically similar (i.e., predominately white females age 18-24), comparisons across surveys should be considered with caution. Nevertheless, adjusted standardized residuals (of Chi-square testing) provide some insight regarding the nature of differences across the survey types and bias categories.

When comparing bias categorization across survey types for Miguel, we see a significant increase in "asset"

Table 6

Interpreting Component Adjusted Residual Z-Scores by Bias Category

Case (Survey Type)	Asset Adjusted Residual Z-Score	Deficit Adjusted Residual Z-Score	Neutral Adjusted Residual Z-Score	Both Adjusted Residual Z-Score
Margaret (No Photo)	-.5	.9	-2.5**	2.1*
Margaret (Photo)	.5	-.9	2.5**	-2.1*
Miguel (No Photo)	-2.1*	.4	.6	1.5
Miguel (Photo)	2.1*	-.4	-.6-	1.5
Shaquan (No Photo)	-.5	.9	-.5	.2
Shaquan (Photo)	.5	-.9	.5	-.2
William (No Photo)	.9	3.1***	-2.7**	-1.3
William (Photo)	-.9	-3.1***	2.7**	1.3

* $p < .05$, ** $p < .01$, *** $p < .001$ for Z-score conversions.

categorizations when a picture is included in the survey. For William, we see a significant decrease in "deficit" categorizations and a significant increase in "neutral" categorizations when a picture is included in the survey. For Margaret, we see significant increase in "neutral" categorizations and a significant decrease in "both" (i.e., responses that contain both asset and deficit perspectives) categorizations when a picture is included in the survey.

Quality of Professional Noticing

Of the 170 respondents to the (picture) survey, 31% of them reported that they were familiar with professional noticing. A breakdown of each component of professional noticing shows that the quality of professional noticing when a photo is introduced tends to be lower than measurement of such noticing sans photo (See Table 7). When comparing the means of the component processes (i.e., attending, interpreting, deciding) between the two studies, it reveals that all three components decreased when the picture was added to the transcript. However, a Wilcoxon Signed Ranks Test, a non-parametric hypothesis test used to compare two related samples, was conducted to determine if any of the decreases were statistically significant. That test showed no statistical significance in the decreases.

These results show that adding a picture does not significantly impact the overall quality of professional noticing skills. However, the purpose of the study was to determine if perceived gender and/or ethnicity impacts bias (as measured through the professional noticing framework), thus a Wilcoxon Signed Ranks Test was conducted on the data from the picture survey by case and component to determine if and where bias occurred (see Table 8).

The asset and deficit responses were calculated through the interpreting component as that was the component where bias was present. When comparing the interpreting scores between cases, there is only a significant difference between the PSTs' scores who received the Shaquan and William cases. Consequently, Shaquan and William are the two cases with the greatest difference between asset perspectives in interpreting (47% with asset perspective on Shaquan and 22% with asset perspective on William). While interpreting was the only component where asset and deficit perspectives

Table 7

Descriptive Statistics and Wilcoxon Signed Ranks Test by Component

	Without Picture Mean	With Picture Mean	Z-Score	*p*
Attending (Range 1-4)	2.03	1.96	-.956	.339
Interpreting (Range 1-3)	1.41	1.35	-.823	.410
Deciding (Range 1-3)	1.58	1.44	-1.940	.052
Sum (Range 3-10)	5.03	4.74	-1.896	.058

Table 8

Wilcoxon Signed Ranks Test Comparing Cases and Components

		Miguel	Shaquan	William
Margaret	Attending	Z = -1.048 p = .295	Z = -.948 p = .338	Z = -.843 p = .399
	Interpreting	Z = -1.237 p = .216	Z = -.557 p = .577	Z = -1.864 p = .062
	Deciding	Z = -2.077 p = .038*	Z = -.158 p = .874	Z = -2.168 p = .030*
	Sum	Z = -1.849 p = .064	Z = -.096 p = .924	Z = -2.009 p = .045*
Miguel	Attending		Z = -.129 p = .897	Z = -.123 p = .902
	Interpreting		Z = -1.758 p = .079	Z = -.922 p = .356
	Deciding		Z = -2.101 p = .036*	Z = -.206 p = .837
	Sum		Z = -1.742 p = .081	Z = -.253 p = .800
Shaquan	Attending			Z = -.464 p = .643
	Interpreting			Z = -2.015 p = .044*
	Deciding			Z = -2.646 p = .008*
	Sum			Z = -1.743 p = .081

* p < .05

were prevalent, when comparing the attending, deciding, and sum scores between the cases, there are other statistically significant results to note. In particular, the deciding component revealed that the PSTs who received the Shaquan and Margaret cases constructed responses which scored significantly higher than those receiving the Miguel and William cases. Additionally, when comparing the sum of the professional noticing components, only those PSTs' receiving the Margaret case scored significantly higher than those with the William case.

Finally, a comparison of professional noticing quality across the component processes was made between survey responses sans photo and the current responses from surveys that included a photo. The results of that comparison (Table 9) show that there is no statistically significant difference between the results of the two studies.

Table 9

Wilcoxon Signed Ranks Comparing Component and Case Between Survey types

	Margaret	Miguel	Shaquan	William
Attending	Z = -.442 p = .658	Z = -.195 p = .845	Z = -.163 p = .870	Z = -0.23 p = .982
Interpreting	Z = -.662 p = .508	Z = -.906 p = .365	Z = -.784 p = .433	Z = -.851 p = .395
Deciding	Z = -.672 p = .502	Z = -.863 p = .388	Z = -1.496 p = .135	Z = -.663 p = .507
Sum	Z = -.355 p = .722	Z = -1.919 p = .055	Z = -1.444 p = .149	Z = -1.176 p = .240

Discussion

The purpose of this study was to explore how and to what extent bias emerges within pre-service teachers' professional noticing of children of differing perceived races and genders. First and foremost, we find that manifestations of bias predominately occur within the interpreting component of professional noticing. Further, our findings suggest that the inclusion of visual imagery (i.e. photos) influence the manifestation of such bias among PSTs to some degree when professionally noticing a written case.

Turning to the professional noticing of cases that do not involve imagery, but rather just rely on a student's name, there is a significant literature base suggesting that individual names may provoke individual biases. Indeed, such biases may be manifested across a range of professional decision-making processes (e.g., resume screening/hiring, apartment rental decisions, etc.), and perceptions of ethnicity tend to provoke more negative decisions or outcomes (Bertrand & Mullainathan, 2004; Hanson et al., 2016). For example, Bertrand and Mullainathan found that "white sounding names (such as Emily Walsh or Greg Baker)" resulted in 50% more employer callbacks than "African American sounding names (such as Lakisha Washington or Jamal Jones)" when such names were randomly assigned to identical fictitious resumes. Thus, we concur with Gaddis (2017) that, "the research base clearly shows that race can be signaled through names and that using names as a signal of race can successfully capture some version of racial discrimination" (p. 470).

However, from our findings, the mere changing of names (e.g., Shaquan, William, Margaret, Miguel) does not appear to provoke the directly negative biases observed in other studies. For example, William's percentage of deficit responses (39%) on the no-picture survey is far higher than that of the other students while Shaquan's percentage of deficit responses (19%) is the lowest. We note, however, that this finding, while somewhat counter to extant literature, could indicate a pattern of *bias inversion* where respondents elevated their expectations for William and lowered their expectations for Shaquan. Another plausible explanation, given the relatively balanced bias (i.e., asset, deficit, neutral, both) percentages among all four cases is that merely changing the student names on identical cases was not sufficient to provoke measurable response bias among the responding preservice teachers. This is intriguing as it opens the possibility that noticing, as a practice, may mediate biases that emerge quite consistently in other professional contexts with this level of provocation. A less plausible but possible explanation is that the PSTs sampled for this study were, in some manner, less prone to exhibit bias in the context of approximated professional noticing.

Regarding the manner in which such biases may impact teaching practice, Rudman (2004) describes the relationship between individuals' implicit and explicit biases as connected but somewhat distant. One's explicit biases exist downstream of one's implicit biases in that unconscious bias flows toward and informs conscious biases. As such, conscious biases are inherently more malleable by the individual given their conscious awareness of said bias or belief. For example, "even when people are truthful, self-reports can only reflect what they believe about their orientations, whereas implicit measures [e.g., the Implicit Associations Test, etc.] bypass this limitation" (Rudman, 2004, p.133). Nevertheless, Rudman continues, "judgments and behavior may be influenced by implicit orientations without intention or awareness. That is, the *application* of implicit biases may be nonconscious" (p.134). With respect to teacher noticing, implicit biases may (and likely do) influence teacher decision-making (i.e., deciding) and the genesis of such biases appears to be within the interpreting component. As such, when approaching professional noticing with PSTs, discussions of equity concerns and issues of bias may be naturally joined with interpreting.

For this discussion, we primarily focus manifestations of bias rather than professional noticing quality. From our results, the only bias manifestations within the interpreting component of professional noticing were statistically significant via the chi square test. Interestingly enough, with the inclusion of a photo, the asset perspective increased in every single demographic, except for the white male student. Instead, William's asset perspective decreased when a photo was included with the survey. From existing literature, one would anticipate William's asset response to be elevated with respect to other cases. However, it may be that this decrease is related to an increase in William's "both" bias category (See Table 6). It is also plausible that some manner of respondent compensation is manifesting here. As equity concerns become more prominent in the field of teacher education (AMTE, 2015), it is conceivable that one's awareness of bias rises and conscious desire to consider and counteract such bias rise in concert. This sort of *bias compensation* (i.e., conscious consideration of one's biases which influences subsequent activity), we argue, would signify a positive step for the field.

Regarding directions for future research, investigations of PST demographics (e.g., race, gender, age etc.) with respect to manifestations of bias would further illuminate the varied enactment of professional noticing. Is, for example, the practice of professional noticing—and the explicit focus on the mathematical thinking of children—a space that mediates, in some manner, one's biases or is such noticing a mere channel or conduit for one's biases? Given that focused experiences engaging in professional noticing results in more sophisticated practice (Jacobs et al., 2010), might such manifestations of bias change as teachers become more adept at such noticing? More broadly, probing conjectures of bias compensation and bias inversion would also likely be fruitful avenues for study. Indeed, we ponder whether or not an increase in experience of professionally noticing children's mathematical thinking translates to professionally noticing of ethnicity and gender. These are but a few areas where future research may provide a better understanding of interplay between noticing and bias.

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Nurturing the Generation and Exploration of Mathematical Conjectures with Preservice Teachers: An Example with a Perimeters Task

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ABSTRACT Generating mathematical conjectures is an important mathematical habit of mind for preservice teachers to develop so that they can, in turn, help their students to develop this skill. In this paper we present a classroom episode in which preservice mathematics teachers experience conjecturing in the context of a rich, open-ended task. They also reflect, throughout the task, on how they might nurture the generation and exploration of mathematical conjectures with their own students.

KEYWORDS *mathematical conjectures, preservice teachers*

Introduction

Doing mathematics involves discovery. Conjecture—that is, informed guessing—is a major pathway to discovery (NCTM, 2000, p. 57).

The National Council of Teachers of Mathematics (NCTM, 2000) defines conjecturing, an important aspect of learning about proof, as informed guessing and highlights it as one of the major components of mathematical reasoning. Furthermore, the NCTM states that "reasoning mathematically is a habit of mind, and like all habits, it must be developed through consistent use in many contexts" (p. 56). In a review of the literature on mathematics teachers' conceptions of proof, Ko (2016) reported evidence of large-scale issues with teacher knowledge regarding both their own understanding of, and their work with students on, mathematical proof. Therefore, it is important to think about how we work on conjecturing with preservice mathematics teachers throughout their period of induction so that they, in turn, can support their students in the conjecturing process.

One approach to this is by modeling conjecturing experiences for preservice teachers thereby placing them in the position of learners as they explore a task without knowing the result. In attempting to do this, we face the challenge of finding suitable tasks and scaffolding them in a way that maximizes opportunities to learn. This means that we should aim to scaffold enough to make a task tractable for students but not so much that they are simply following step-by-step instructions without opportunities to make mathematical decisions and discoveries for themselves. An important consideration is how much time a teacher should allow for the early stages of conjecturing, especially based on a single case—how many cases are enough? How can we help students relate those multiple cases? Do those new cases cause reformulation of the conjecture? How do we keep track of the evolution of a conjecture? In this paper we present an example, through a classroom episode, of one encounter with these considerations and describe how the classroom episode provided preservice teachers both with the experience of developing conjectures and with an opportunity to reflect on the process from a teacher's perspective.

Relationship to Literature

Astawa et al. (2018) state that "constructing mathematical conjecture involves a lot of [complex] processes of cognition" (p. 17) and emphasize the problem-solving aspect of conjecturing. Ponte et al. (1998) list the main three stages of conjecturing as "(i) proposing questions and establishing conjectures, (ii) testing and refining the conjectures, and (iii) arguing and proving the conjectures," (p. 4). Others, for example, Cañadas et al. (2007) and Morseli (2006) also highlight its multi-stage nature and, while stages suggest a hierarchy, we argue that conjecturing is cyclic. After a conjecture is formulated, it is often reformulated (perhaps multiple times) and refined until validated or proven. Moreover, many researchers (e.g. Astawa et al., 2018; NCTM, 2000) emphasize the intertwined relationship between exploration, discovery, and conjecturing.

Cañadas et al. (2007) identify five distinct types of conjecturing: Type 1: Empirical Induction from a Finite Number of Discrete Cases, Type 2: Empirical Induction from Dynamic Cases, Type 3: Analogy (to something already known), Type 4: Abduction (conjecturing on the basis of a single event), and Type 5: Perceptually Based Conjecturing (made from a visual representation of a problem). The first of these is the most common type in school mathematics with Type 2 becoming more common as dynamic geometry environments gain traction.

In this paper, we will share a classroom episode in which preservice teachers generate conjectures and attempt to validate those conjectures using a Type 4 and then a Type 1 (Cañadas et al., 2007) approach. We follow Driscoll's (2010) suggestion and track the formulation and reformulation of conjectures since, "Studies show that opportunities to explore and conjecture...followed by challenges to explain why conjectures seem true, hold promise for helping students become more proficient at constructing deductive proofs" (Driscoll, 2010, p. 25). The episode provides an example where an appropriate amount of scaffolding provided mathematical space for preservice teachers to work as mathematicians in their conjecturing. Finally, the episode describes how reflection on the process from a teacher perspective can be built into the enactment of the task in the classroom.

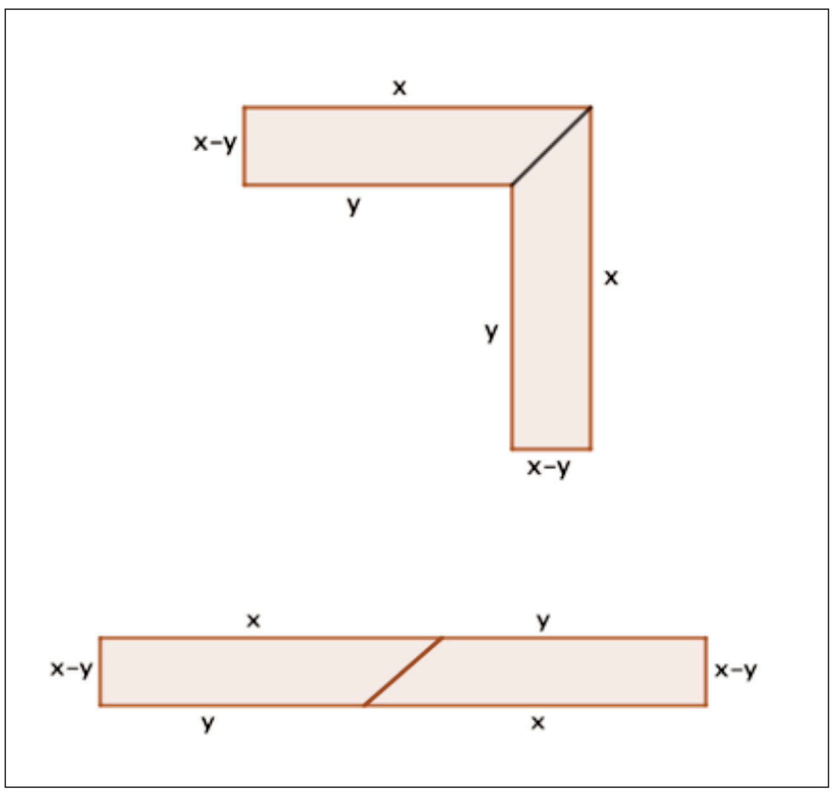
Classroom Episode

In this classroom episode, preservice teachers began working on a task that was designed to provide them with an example of visualizing algebraic relationships. The original task was modified with differences leading to a rich, extended exploration in which preservice teachers could develop conjectures without aiming to establish a known result, i.e., they were exploring as mathematicians in a true sense. The content of the task, perimeters of shapes, was especially useful because, while the content itself was accessible, the mathematical results that would arise were not known in advance.

Brumbaugh and Rock (2012) include a mathematical activity designed to provide a visualization of the difference of two squares identity, $x^2 - y^2 = (x - y)(x + y)$. One of the authors has used this activity regularly in his class: preservice teachers are led through the task starting with a large square and cutting out a smaller square to get an L shape (see Figure 1 below). The larger square has area x^2 and smaller square has y^2 so the area of the L shape is $x^2 - y^2$. But the L shape can also be cut along the diagonal and the resulting two pieces arranged to make a rectangle of dimensions $x + y$ by $x - y$ (see lower portion of Figure 1). Hence, we have a visual demonstration of the difference of two squares identity, $x^2 - y^2 = (x - y)(x + y)$.

Figure 1

Visual Demonstration of Difference of Two Squares Identity



In past iterations of the course, preservice teachers were given the task and led through all steps, including cutting the L, rearranging into a rectangle, and labeling. However, on this occasion, the task was paused when students had generated the L shape on the left (seen in Figure 1) and, rather than being led through the subsequent steps, were simply asked to explore further. A number of the preservice teachers went to the result in the original task, i.e., cutting the L, calculating areas, and finding the difference of two squares result. As we describe in the classroom episode below, sometimes scaffolding less is beneficial and can lead to unexpected (and genuinely interesting) results.

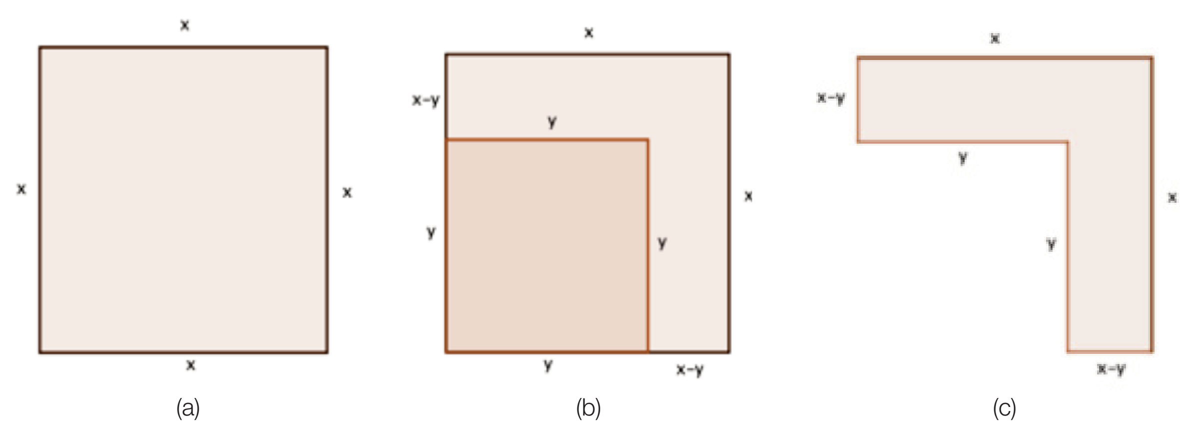
In the original task the focus is on calculating areas. On this occasion, not having been directed to examine the areas of the shapes, other preservice teachers took the path of examining perimeters of the shapes. As can be seen in the sample work in Figure 2, they quickly concluded that the L-shape and the large square had the same perimeter. Thus, per Cañadas et al., (2007) for the purposes of making conjectures, we had just one case to consider (Type 4: Abduction).

As is often the case in mathematics, a calculation leads to the question "Did we actually need to do the work of that calculation or could we have reasoned our way to the result?" Indeed, in this case, it was argued that 'y' parts of the original perimeter were moved to a "different place" but still contributed to the perimeter of the resultant shape, so the work of the calculation was not necessary. Moreover, this particular result prompted the realization by the preservice teachers that, of course, the length of the smaller square is arbitrary and that any square cut from a corner of the larger square would result in a shape with the same perimeter. Thus, per Type 4 of Cañadas et al. (2007), a single example resulted in a conjecture and a general argument without the need to generate more examples. In addition, the process prompted several different directions for the generation of more cases as preservice teachers looked to uncover patterns, thus adopting a Type 1 approach per Cañadas et al, to wit, "a conjecture can be made based on the observation of a finite number of discrete cases, in which a consistent pattern is observed" (p. 58). Of course, it was not clear which aspect of the task should be explored so that further cases could be generated. In the moment in that class, our initial observation and some brainstorming prompted three avenues for further exploration, namely: (a) Could we cut any regular polygon from a large regular polygon of the same kind and preserve the perimeter? (b) Did it matter from where the small square was cut, i.e., did it have to be from the corner? and, (c) Did it matter that the original shape was a square, i.e., could we cut out a different shape?

Here, we highlight for preservice teachers the importance of not over scaffolding, specifically, not providing too much explicit direction. The previous, highly scaffolded, area task, was designed to direct students towards a particular result (a visual demonstration of the difference of two squares identity). Allowing some time for undirected exploration of the task allowed preservice teachers to find new avenues of exploration. The instructor's crucial role during such times of exploration is to

Figure 2

Perimeters of Original Square (a) and L-Frame (c)



Note. Original square perimeter (a): $x + x + x + x = 4x$. Frame perimeter (c): $y + y + (x - y) + x + x + (x - y) = 4x$.

monitor groups and see which, if any, pose interesting questions warranting further exploration. The instructor also must be mindful of bringing the group back together for the kind of brainstorming that produced the three questions above. This has multiple positive effects: (i) it allows the whole class to decide on a common focus for further exploration; (ii) it allows those groups who have started productive lines of inquiry to share their suggestions; and (iii) it provides a productive line of work for those students who struggled in the undirected exploration time.

This is also a demonstration of the power of open-ended exploration: an observation might be followed with multiple conjectures to be tested. Interestingly, in this particular case, the exploration of the perimeters, rather than the areas, meant that the task had become a new one for the instructor, who was then faced with the decision of taking time to explore avenues which may or may not be productive. So, what should an instructor do at this moment? We all know that instructional time is precious, and too few teachers afford students the opportunity to follow their conjectures. Moreover, an instructor might feel uneasy to continue with further exploration, especially not knowing exactly how these avenues would play out. Much preservice mathematics teacher education is concerned with modeling good practice since, as in any classroom, instructor moves give tacit messages to students about what matters in a classroom. Tasks such as the one described in this classroom episode show how student reasoning can be at the center of the class and that time spent exploring different avenues of inquiry is time well spent. Since this was a class of preservice teachers, this decision also became a point of discussion in the class. In the discussion, important points were raised about the Mathematical Knowledge for Teaching (MKT) (Hill et al., 2005) that teachers need in order both to guide the brainstorming for ideas and to be able to judge the productive potential of the ideas. In this case, with some ideas rejected, the instructor decided that each of the three ideas had potential and that different groups would pursue different conjectures.

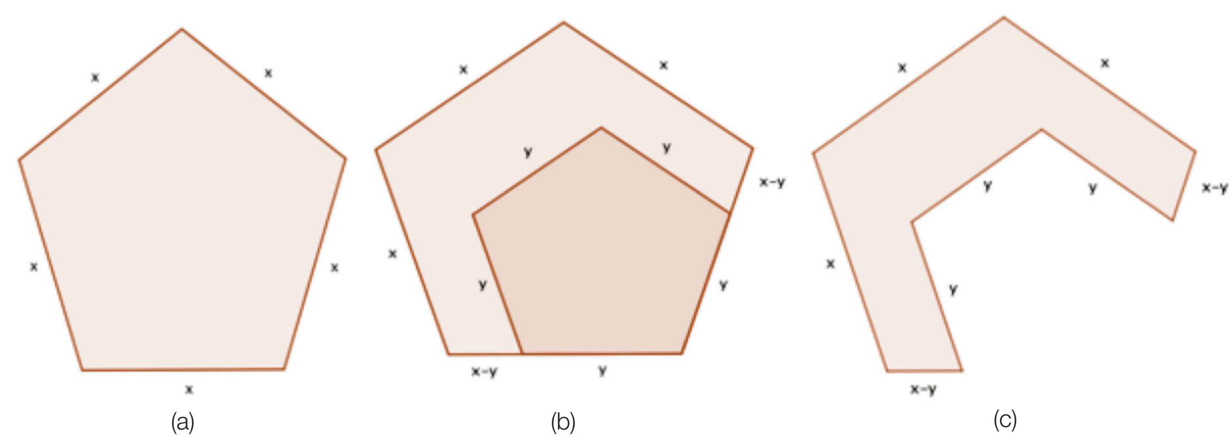
Other regular shapes:

One group of preservice teachers took a Type I (Cañadas et al., 2007) approach as in "observing cases, organizing cases, searching for and predicting patterns, and formulating a conjecture" (p. 63). They began to explore cutting small regular shapes from the corner of a larger version of the same shape to generate several cases and look for a pattern. Figures 3 and 4 highlight work with a regular pentagon and regular hexagon, respectively.

Looking at the regular pentagon and hexagon constructions, it can be seen that, similarly to the case of the square, parts of the original perimeter are "moved"; although in the new bases seen in Figures 3 and 4, new pieces of perimeter appear. Studying these constructions, and considering the square from earlier, a pattern emerged, particularly in regard to how many new pieces appear—no new pieces for the square, one new piece for the pentagon, two new pieces for the hexagon—leading to the formulation of a conjecture: *"When a small regular n-gon of side y is cut from the corner of a large version of the*

Figure 3

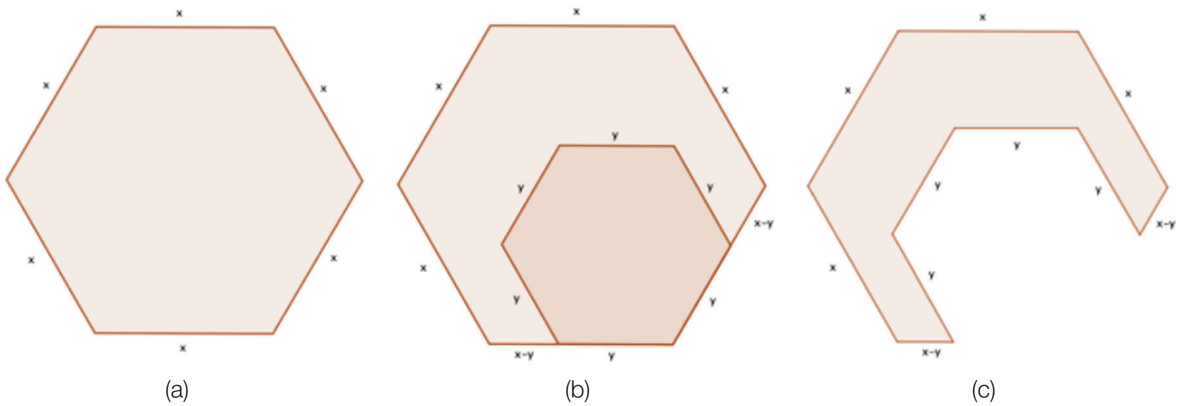
Perimeters of a Regular Pentagon (a) and Frame (c)



Note. Original pentagon perimeter (a): $x + x + x + x + x = 5x$.
Frame perimeter (c): $x + x + x + x + x - y + y + y + y + x - y = 5x + y$.

Figure 4

Perimeters of a Regular Hexagon (a) and Frame (c)



Note. Original pentagon perimeter (a): $x + x + x + x + x + x = 6x$.
Frame perimeter (c): $x + x + x + x + x - y + y + y + y + y + x - y = 6x + 2y$.

regular n-gon of side x, the resulting perimeter is increased by $(n-4)y$, i.e., the perimeter increases from nx to $nx + (n-4)y$." The result worked for the original shape of the square where the perimeter stayed the same. In that case, $n = 4$ and so the new perimeter is $4x + (4-4)y = 4x$.

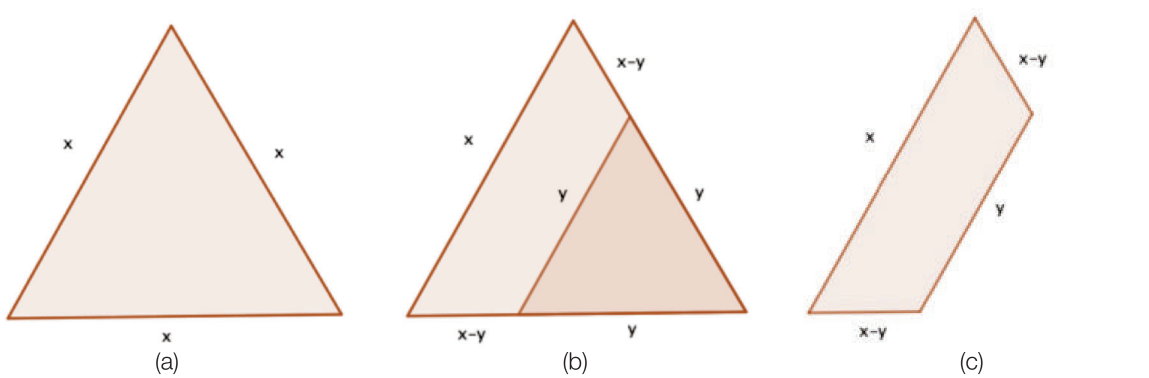
The preservice teachers agreed that the result was reasonable since when a regular shape is cut, two edges of the smaller polygon are "lost" from the original perimeter but are "moved" to the interior. That accounts for 4 edges and the remaining edges ($n - 4$) are added to the perimeter of the new shape. One preservice teacher pointed out that we had not tested the conjecture for triangles and that we would run into trouble since a triangle had fewer than 4 sides and thus $n - 4$ would be negative. However, the diagram provided in Figure 5 confirmed that the conjecture held, although this is the only case where the perimeter decreases. The preservice teachers were satisfied that the result held in all cases, although they were not entirely clear on what a rigorous proof of the result might look like.

Location of the cut:

Other preservice teachers explored the question of the location of the cut by cutting a square from a place on one side rather than from a corner. As Figure 6 illustrates, they soon discovered that the length, 'y', of the smaller square that lay on the perimeter is "moved up," with two new 'y' lengths added, yielding a new perimeter of $4x + 2y$.

Figure 5

Testing Conjecture for Equilateral Triangles



Note. Original equilateral triangle perimeter (a): $x + x + x = 3x$. Frame perimeter (c): $x + x - y + y + x - y = 3x - y$
[Also $nx + (n - 4)y$ when $n = 3$ is $3x + (3 - 4)y = 3x + (-1)y = 3x - y$.]

Figure 6

Cutting a Square from a Non-Corner

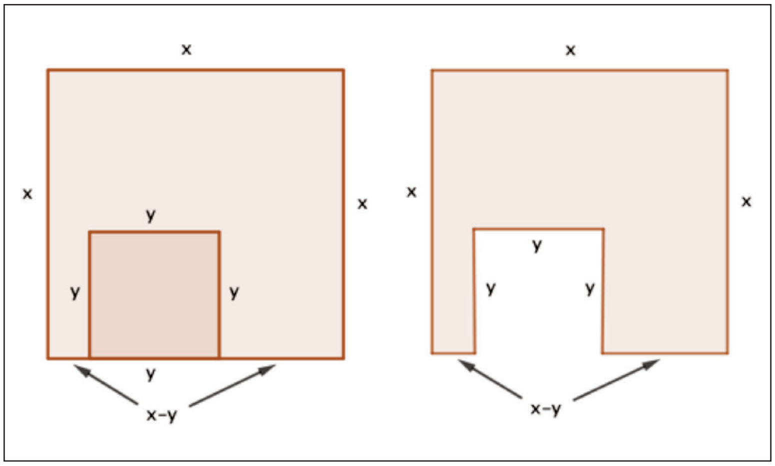
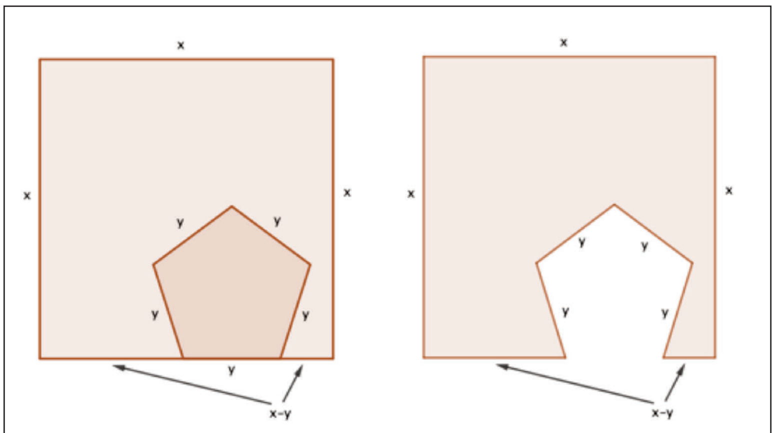


Figure 7

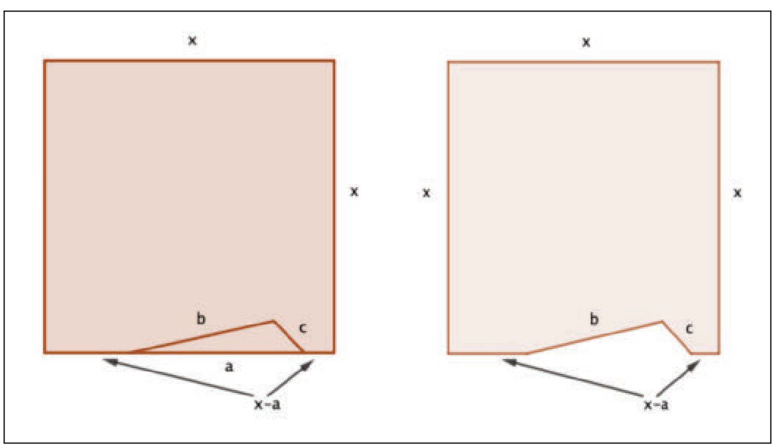
Cutting a Regular Pentagon from a Non-Corner



Note. Perimeter of original square (left): $x + x + x + x = 4x$. Frame Perimeter (right): $x + x + x + x - y + y + y + y + y + x - y = 4x + 3y$.

Figure 8

Cutting a Triangle from a Non-Corner



This prompted a new conjecture: *"If a small square is cut from an edge (rather than a corner) of a large square, the perimeter is increased by exactly half the perimeter of the small square."* Next, students sought to generalize the conjecture by cutting a regular pentagon from the original square, as shown in Figure 7. As groups shared their work, this was identified as a slight variation of an earlier conjecture with only one 'y' "lost," yielding a perimeter of $nx + (n - 2)y$. This was confirmed for the case of the square. When $n = 4$, the perimeter is $4x + (4 - 2)y = 4x - 2y$.

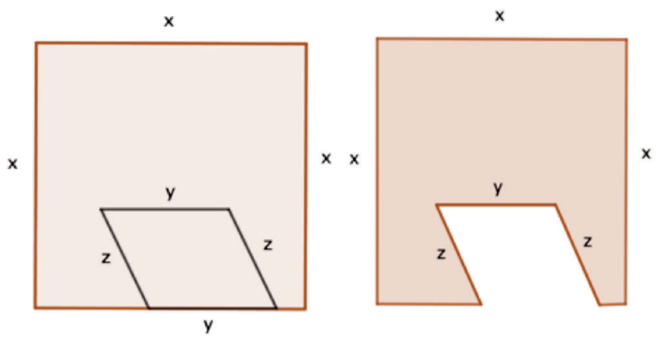
Considering other shapes:

Perhaps the most interesting moment in the activity happened when a pair of preservice teachers exclaimed, "This is an application of the triangle inequality theorem!" The students recalled that "the triangle inequality theorem says that the sum of those two interior sides must be greater than the third side so the perimeter must have increased." Still considering the original question of whether the perimeter is preserved when a shape is cut from it, they declared: "If you cut a triangle from a side of the square there will be one side of the triangle missing from the edge and two new sides on the interior." This is illustrated in Figure 8.

Preservice teachers explored a wide variety of different shapes that could be cut from a corner and from an edge. They quickly asserted and demonstrated that cutting a rectangle from the square leads to the same two conjectures as before. This is illustrated in Figure 9, with the modification that, for the second conjecture, the extra perimeter will come from the two sides of the rectangle—neither of which overlapped with the edge of the original square. Noting that you cannot cut a parallelogram from the corner, the second theorem held for a parallelogram cut from an edge, as shown in Figure 9. As preservice teachers cut other shapes, similar patterns of

Figure 9

Cutting a Parallelogram from a Non-Corner



Note. Perimeter of original square (left): $x + x + x + x = 4x$.
Frame perimeter (right): $x + x + x + x - y + y + z + z = 4x + 2z$

"lost" sides being "regained" in the interior, and other side lengths emerged.

The three lines of inquiry—regular shapes, the location of the cut, non-regular shapes—all led to interesting mathematical results aided by the instructor taking an active role in circulating among the groups and monitoring progress. At the conclusion, a discussion of how the activity was managed by the instructor and the main pedagogical choices made (not over scaffolding, time for free exploration, consolidation of ideas from free exploration, focusing the groups on agreed lines of inquiry) gave the preservice teachers an opportunity to reflect on the role they could take with their own students going forward.

Concluding Thoughts

Having the opportunity to explore mathematical tasks and to develop conjectures is an important part of developing mathematical skills and developing maturity as a mathematician. As Ko (2010) has reported in a review of the literature, preservice teachers would benefit from continuous and ongoing opportunities to engage in such activity. Preservice teachers, therefore, need tasks which provide useful avenues for exploration but that are not overdetermined with too much scaffolding so that they can both develop skills in exploration for themselves and reflect on how they would engage their own students in this kind of work. The task discussed in the classroom episode above provides an example of such a task. The open-ended nature of the task gave preservice teachers several avenues of exploration, engaged the preservice teachers in several types of conjecturing (Cañadas et al., 2007) and led to some unexpected results such as an application of the triangle inequality theorem. The task also provided the instructor an opportunity to demonstrate how to orchestrate various groups of students exploring various conjectures. This in turn showed preservice teachers how to help their future students value each others' conjectures and recognize how they are interrelated, inform one another, and can lead to a deeper understanding of the concepts at hand. Finally, the setting of a preservice teacher classroom allowed for "meta" discussion about how the task was being conducted by the instructor and the pedagogical choices that were being made.

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Effects of Student Help-Seeking Behaviors on Student Mathematics Achievement

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ABSTRACT Using the nationally representative sample from the United States in the 2012 Programme for International Student Assessment ($N = 7{,}429$ students from 240 schools), the present study examined the effects of student help-seeking behaviors on eight measures of student mathematics achievement. To account for the multilevel structure of the data with students nested within schools (i.e., students attending different schools), a two-level hierarchical linear model was used in the data analysis. Student help-seeking behaviors showed statistically significant positive effects on all eight measures of mathematics achievement, even after controlling for student characteristics and school characteristics. Furthermore, for all eight measures of mathematics achievement, the positive effects of student help-seeking behaviors on student mathematics achievement are independent of which schools students attend.

KEYWORDS *help-seeking behaviors, mathematics achievement, Programme for International Student Assessment (PISA)*

Despite the vast amounts of educational resources that have been invested into K-12 education in the United States (U.S.), students in the U.S. are continually outperformed by their international peers in mathematics achievement (Duncan, 2013). According to the 2012 Programme for International Student Assessment (PISA), even though the U.S. spends more money per student than all but four of the 34 Organisation for Economic Co-operation and Development (OECD) countries, it ranks only 27th in mathematics (OECD, 2013). Numerous educational reforms have been implemented with the aim of improving the performance of U.S. students in mathematics (see Dossey et al., 2016). We joined these national efforts by examining the relationship between student help-seeking behaviors and student mathematics achievement, as a way to improve mathematics instruction.

Review of Literature

Student Help-Seeking Behaviors in General

Regardless of their knowledge or ability, students encounter problems too difficult for them to solve on their own. Being able to recognize when external assistance is needed and seeking the most appropriate assistance out of the available options are integral components of the learning process and fundamental aspects of cognitive development (Kominsky et al., 2018; Vredenburgh & Kushnir, 2015). Student help-seeking behaviors are defined as the ways in which students search for and make use of external resources when attempting to solve problems (Ogan et al., 2015). These behaviors include such actions as asking someone for help, reading a textbook, and searching online for example problems or instructional videos. Unlike other skills involved in the process of cognitive development such as goal-setting and time management, help-seeking is unique in that it is not only a self-regulation strategy but typically involves social interaction as well. This social dimension has the added benefit of improving the cognitive and social abilities of both helpers and seekers (Mueller & Kamdar, 2011). There is some evidence that constructive help-seeking behaviors have positive effects on learning outcomes (Schenke et al., 2015). Overall, help-seeking behaviors aimed at learning and not just producing correct answers are appropriate because they lead to the cultivation of transferrable knowledge and skills that can be applied to future problems (Roll et al., 2011).

Researchers recognize the resistance among some students to seek help. Low-achieving students, who would benefit the most from seeking help, tend to be the least likely to actually do so (Amemiya & Wang, 2017). Although, by early adolescence, students have developed the cognitive skills required for determining when and which help-seeking behaviors are necessary, they often choose to avoid help-seeking due to personal autonomy concerns, perceptions of cognitive and social incompetence, lack of competent helpers, and an unwelcoming classroom environment (Kiefer & Shim, 2016). In some cases, students are advised to avoid help-seeking behaviors because they tend to produce dependency on external resources and, as a result, are viewed as impediments to the learning process (Amemiya & Wang, 2017). Some students struggle to determine whether the reward for solving a problem outweighs the risk of appearing incompetent in front of others (White & Bembenutty, 2013). A direct consequence of deriving self-worth primarily from the opinion of others is for students to avoid help-seeking even when they realize the need for it (Reeves & Sperling, 2015). Some students choose not to seek external assistance because they believe no one available is capable of providing the help they need (Kiefer & Shim, 2016).

A distinction has been made between executive and instrumental help-seeking behaviors (Inbar-Furst & Gumpel, 2015). Executive help-seeking behaviors lead to correct answers as quickly as possible with little to no regard as to whether learning actually happens. Conversely, instrumental help-seeking behaviors aim at learning and not just producing correct answers. Classroom goal structure plays a role in student help-seeking behaviors (Schenke et al., 2015). A classroom with a mastery-goal orientation emphasizes the intrinsic value to learning and leads students to recognize the need to understand the knowledge and skills (Amemiya & Wang, 2017). Because the focus is on gaining understanding and not just getting correct answers, students often engage in instrumental help-seeking behaviors (White & Bembenutty, 2013). A classroom with a performance-goal orientation creates an atmosphere of competition by emphasizing individual performance relative to peer performance (Ryan et al., 2005). The need to constantly prove they are smarter than their peers encourages students to engage in executive help-seeking behaviors (White & Bembenutty, 2013).

Student Help-Seeking Behaviors in Mathematics

Since mathematics can be one of the most difficult subjects for some students to learn (Fleming, 2019), help-seeking should naturally be a critical component of mathematics learning and teaching. Unfortunately, few studies have examined the relationship between student help-seeking behaviors and student mathematics achievement. Of these, some do not directly relate student help-seeking behaviors to any outcome measures in mathematics. Ryan and Shim (2012) concluded that students are more likely to seek help from the teacher than classmates; further, they are more likely to seek help from the teacher if they perceive the teacher as being encouraging of asking questions. Mehdizadeh et al. (2013) reported that invoking cooperative learning in mathematics classrooms increases students' willingness to seek help.

Other studies have attempted to identify some direct correlations between student help-seeking behaviors and student mathematics achievement. Students who have low prior mathematics achievement are more likely to avoid seeking help (Newman, 2002; Ryan & Pintrich, 1997). In contrast, Beal et al. (2008) revealed that in the context of geometry problem solving, students with low prior mathematics achievement are just as likely as students with high prior mathematics achievement to engage in appropriate help-seeking behaviors; in fact, both groups are more likely to seek help than the group of students with average prior mathematics achievement. This inconsistency in findings may be a result of differences in resources of help: computer software in Beal et al. (2008) versus teacher and peers in both Ryan and Pintrich (1997) and Newman (2002).

After controlling for prior mathematics achievement, Ryan et al. (2005) found that students who engage in instrumental help-seeking behaviors score higher than students who either engage in executive help-seeking behaviors or avoid help-seeking altogether. Schenke et al. (2015) also reported that students who engage in instrumental help-seeking behaviors experience larger gains in mathematics achievement over the course of a year than students who engage in executive help-seeking behaviors. The most "dramatic" finding comes from Ogan et al. (2015). After examining students in Costa Rica, the Philippines, and the U.S., the researchers found that student help-seeking behaviors (with a computer-based tutoring system as the resource of help in this case) are even better than student mathematics pretest scores for predicting student mathematics posttest scores, regardless of geographical location.

Method

To explore the relationship between student help-seeking behaviors and student mathematics achievement, we sought to address the following research questions:

1. Do student help-seeking behaviors have any effects on student mathematics achievement, with control over student and school characteristics?
2. Do the effects of student help-seeking behaviors on student mathematics achievement vary across schools? If yes, what school characteristics contribute to this variation?

PISA 2012 provided sufficient data that could be used to answer these research questions.

Data

PISA is an international assessment of 15-year-old students conducted every three years by OECD. While seeking to measure students' mathematics, reading, and science literacy, the intent of PISA is not as focused on assessing students' knowledge of content-related facts as it is on how well students can apply their content knowledge to real-world problem-solving situations (OECD, 2014a). PISA implemented a two-stage, stratified, random probability sampling procedure. At the first stage, a random sample of schools was selected in proportion to school enrollment size from all public and private schools containing 15-year-old students in grade 7 or higher. At the second stage, within each sampled school, a random sample of students was selected from a list of all eligible students. In addition to the standardized paper-and-pencil achievement tests, students and their school principals completed questionnaires to provide information about student and school background characteristics. For the present study, we utilized the national sample of the U.S. with 7,429 students from 240 schools from PISA 2012 because it is the latest cycle with a focus on mathematics.[1]

Dependent Variables

PISA 2012 contained 85 items in its assessment of mathematics measuring student mathematical literacy (OECD, 2013). These items measured four mathematical literacy areas: change and relationship; space and shape; uncertainty and data; and quantity. Change and relationship involves using equations, inequalities, functions, and graphs to model changes that occur over time, as well as how one object changing affects another object. Space and shape involves using geometry and measurement to understand the visual and physical world. Uncertainty and data involves using probability and statistics to produce models, interpret data, and make inferences in situations involving uncertainty, chance, and variation. Quantity involves applying knowledge of numbers and number operations, along with quantitative reasoning, to a broad range of real-world scenarios.

In addition, these items also tapped into three mathematical processes: *formulating*, *employing*, and *interpreting*. Formulating involves identifying real-world problems that can be solved using mathematics and then developing mathematical structures that can be used to determine solutions. Employing involves applying mathematical reasoning and concepts to produce solutions to mathematically-formulated problems. Interpreting involves reflecting upon mathematical solutions and then interpreting them in view of the context of the real-world problems. To have a comprehensive examination of the relationship between student help-seeking behaviors and student mathematics achievement, we selected as dependent variables scores on (a) the overall mathematical literacy (a combination of the four literacy areas), (b) the four mathematical literacy areas, and (c) the three mathematical processes. These eight measures were analyzed separately. "There is theoretically no minimum or maximum score in PISA; rather, the results are scaled to fit approximately normal distributions, with means around 500 score points and standard deviations around 100 score points" (OECD, 2019, p. 43). Consequently, all eight measures of mathematics achievement had a mean of 500 and a standard deviation *(SD)* of 100.

[1] PISA devotes extra attention to one of the three subject areas in each three-year cycle. The focus of PISA 2012 was on mathematics; therefore, additional space was allocated to tests and questionnaires for important issues in mathematics. Although PISA 2015 and 2018 contained a mathematics component (thus providing newer data), the student help-seeking scale concerning mathematics was absent in both cycles because mathematics was not the focal subject area.

Independent Variables

The independent variables came from student and school questionnaires (see Kastberg et al., 2014). The key independent variable was student help-seeking behavior measured on the student questionnaire (see OECD, 2014b). From a number of items, we selected three to construct a composite variable to measure student help-seeking behavior (see Table 1). These items were selected because they all involve an individual seeking help by interacting with another person, which fits well with the PISA definition of student help-seeking behavior (a propensity to depend on the knowledge and intellect of other people, including both peers and teachers, when attempting to solve problems) (OECD, 2014b). The response options for these items were recoded so that a higher value is indicative of more proactive seeking of help. We aggregated scores on these items to create the composite measure of student help-seeking behavior.[2]

Table 1

Description of Items for Construction of Composite Variables

Variable	Possible Responses
Student Help-Seeking Behavior	
Scenario 1: Suppose you have been sending text messages from your phone for several weeks, but today you can't send text messages. You want to you do? • I would ask a friend for help. Scenario 2: Suppose you arrive at a train station. There is a ticket machine that you have never used before. You want to buy a ticket. What would you do? • I would ask someone for help. • I would try to find a ticket office at the station to buy a ticket.	1 = I would definitely do this. 2 = I would probably do this. 3 = I would probably not do this. 4 = I would definitely not do this.
Teacher-Directed Instruction	
How often do these things happen in your mathematics lessons? • The teacher sets clear goals for our learning. • The teacher asks me or my classmates to present our thinking or reasoning at length. • The teacher asks questions to check whether we have understood what was taught. • At the beginning of a lesson, the teacher presents a summary of the previous lesson. • The teacher tells us what we have to learn.	1 = never or hardly ever 2 = some lessons 3 = most lessons 4 = every lesson
Student Orientation	
How often do these things happen in your mathematics lessons? • The teacher gives different work to classmates who have difficulties learning and/or to those who can advance faster. • The teacher assigns projects that require at least one week to complete. • The teacher has us work in small groups to come up with solutions to a problem or task. • The teacher asks us to help plan classroom activities or topics.	1 = never or hardly ever 2 = some lessons 3 = most lessons 4 = every lesson
Formative Assessment	
How often do these things happen in your mathematics lessons? • The teacher tells me about how well I am doing in my mathematics class. • The teacher gives me feedback on my strengths and weaknesses in mathematics. • The teacher tells us what is expected of us when we get a test, quiz, or assignment. • The teacher tells me what I need to do to become better in mathematics.	1 = never or hardly ever 2 = some lessons 3 = most lessons 4 = every lesson

Note. For each bulleted item, students were instructed to choose one of the possible responses.

[2] PISA 2012 originally constructed a composite variable measuring student help-seeking behavior with several items; however, we were concerned with the level of internal consistency (Cronbach's α = .54). Table 1 presents the three items we used to construct our own composite variable which resulted in a higher internal consistency (Cronbach's α = .58). Although this is still not optimal, we decided to use our composite for two reasons. First, it represented a small improvement in internal consistency over the PISA composite (also with a more efficient structure of only three items). Second, the three items are conceptually clear and consistent, all emphasizing interactions of students with real people.

Other independent variables functioned as control variables to account for their effects on the relationship between student help-seeking behaviors and student mathematics achievement. At the student level, the control variables included gender, socioeconomic status (SES) (indexed from parental education, parental occupation, and family possessions), family structure (single parent versus other structure), and home language (English versus other language). At the school level, the control variables included school enrollment size; proportion of girls; school location (city or large city versus village, small town, or town); school mean SES (aggregated from SES of students within a school); school type (public versus private); and proportion of mathematics teachers with a bachelor's or master's degree majoring in mathematics, statistics, physics, or engineering. These student- and school-level variables are exogenous in nature. We also controlled for three teacher instructional practices: teacher-directed instruction, student orientation, and formative assessment (see OECD, 2014b). PISA 2012 used information obtained from the student questionnaire to create these three composite variables (see Table 1). Each teacher instructional practice was aggregated within each school to generate measures at the school level.

Statistical Analyses

We used a two-level hierarchical linear model (HLM) to account for the multilevel structure of the U.S. data with students nested within schools. The HLM analysis was performed in three stages. The first stage was the null model, which included no independent variables at either level. This model was used to determine the means for the eight (dependent) measures of student mathematics achievement and the variance in these outcomes at both levels. At the second stage, the variable of student help-seeking behavior was added to the null model to determine the absolute effects of student help-seeking behaviors on the eight measures of student mathematics achievement. The variable of student help-seeking behavior was treated as random at the school level to examine the variance across schools in the effects of student help-seeking behaviors on student mathematics achievement.[3] The results obtained at this stage were used to address the second research question. At the third stage, student- and school-level variables were added to the model developed at the previous stage to determine the relative effects of student help-seeking behaviors on the eight measures of student mathematics achievement. The results obtained at this stage were used to address the first research question.

Results

With the descriptive statistics for student- and school-level variables presented in Table 2, we focus on the results from various HLM analyses. For the key independent variable of student help-seeking behavior, the (U.S. national) mean was 2.21 (on a scale of 1 to 4) with an *SD* of 0.73.

The null model estimated the mean and partitioned the variance for each mathematics achievement measure (see Table 3). In Table 3, the fixed effects indicate average mathematics achievement, and the random effects indicate the variation in mathematics achievement across

Table 2

Descriptive Statistics for Student-Level Variables and School-Level Variables

Variable	*M*	*SD*
Student-level variables		
Student help-seeking behavior (on a scale of 1 to 4)	2.21	0.73
Gender (1 = male, 0 = female)	0.50	0.50
Socioeconomic status (SES) (a standardized variable)	0.22	0.97
Family structure (1 = single parent, 0 = other structure)	0.21	0.41
Home language (1 = English, 0 = other language)	0.88	0.33
School-level variables		
School enrollment size (in hundreds)	13.37	8.70
Proportion of girls	0.49	0.07
Location (1 = city or large city; 0 = village, small town, or town)	0.38	0.49
School mean SES	0.21	0.54
Public vs private (1 = public, 0 = private)	0.91	0.29
Proportion of mathematics teachers with math-related degree	0.67	0.37
Teacher-directed instruction (a standardized variable)	0.30	0.52
Student orientation (a standardized variable)	0.26	0.47
Formative assessment (a standardized variable)	0.32	0.48

Note. A standardized variable has a mean of 0 and an *SD* of 1. For dichotomous variables, the mean indicates the percentage for the category that is coded as 1 (e.g., English is the home language for 88% of the students).

[3] In case the variance is statistically significant across schools, school-level variables can be used to explain this variance.

schools. While PISA mathematics achievement is scaled to have a mean of 500 and an *SD* of 100, for the eight measures of student mathematics achievement, the mean for the U.S. students ranged from 468.72 to 494.41. The partition of variance for the eight measures showed that between 76% and 79% of the variance was attributable to students, while 21% to 24% was attributable to schools. The variance at the school level was statistically significant ($p < .05$) for all eight measures, indicating that the U.S. schools were significantly different in terms of various measures of student mathematics achievement. For example, in the U.S., the mean for the overall mathematical literacy was 486.32. Meanwhile, 78% of the variance in the overall mathematical literacy was attributable to students, while 22% was attributable to schools (which was statistically significant).

Table 3

Means and Partition of Variance for Multiple Measures of Mathematics Achievement

Variable	Fixed Effects	
	Coefficient	*SE*
Intercept (mathematics achievement)		
Overall	486.32*	4.52
Change and relationship	492.03*	4.59
Space and shape	468.72*	5.01
Uncertainty and data	494.41*	4.46
Quantity	484.69*	5.07
Formulating	481.22*	5.13
Employing	485.45*	4.41
Interpreting	494.02*	4.60
Variable	**Random Effects**	
	Variance	***df***
Between-school variability		
Overall	1772.02*	137
Change and relationship	1874.71*	137
Space and shape	2077.15*	137
Uncertainty and data	1839.98*	137
Quantity	2201.20*	137
Formulating	2230.16*	137
Employing	1682.15*	137
Interpreting	1920.38*	137
Within-school variability		
Overall	6123.07	
Change and relationship	6816.64	
Space and shape	7247.03	
Uncertainty and data	5845.28	
Quantity	7420.53	
Formulating	7410.57	
Employing	6232.59	
Interpreting	6806.72	

$^*p < .05$

Absolute Effects of Student Help-Seeking Behaviors

Table 4 presents the results on the absolute effects of student help-seeking behaviors on the eight measures of student mathematics achievement. In this table, the fixed effects refer to the effects of student help-seeking behaviors on student mathematics achievement, and the random effects refer to the variation in the effects of student help-seeking behaviors on student mathematics achievement across schools. Student help-seeking behaviors showed statistically significant absolute effects on all eight measures of student mathematics achievement. Specifically, a 1-point increase in student help-seeking behavior was associated with an increase of between 7.24 and 10.18 points in student mathematics achievement. Because PISA mathematics achievement has a mean of 500 and an *SD* of 100, we can easily turn our effects into an effect size measure. The corresponding effect sizes ranged from 7.24% to 10.18% of an *SD*. For example, a 1-point increase in student help-seeking behavior was associated with an increase of 8.17 points in the overall mathematical literacy, with a corresponding effect size of 8.17% of an *SD*. According to Cohen (1988), 20% of an *SD* is a small effect, 50% of an *SD* is a medium-size effect, and 80% of an *SD* is a large effect. Therefore, in the U.S., student help-seeking behaviors demonstrated statistically significant positive, though relatively small, effects on all eight measures of student mathematics achievement.

On the other hand, the effects of student help-seeking behaviors on student mathematics achievement did not vary statistically significantly from school to school for any of the eight measures of student mathematics achievement; that is, the effects of student help-seeking behaviors on student mathematics achievement were quite similar for all U.S. schools. Because the effects of student help-seeking behaviors on student mathematics achievement did not vary from school to school, it was not necessary to model the effects of school-level variables.

Table 4

Absolute Effects of Student Help-Seeking Behaviors on Multiple Measures of Mathematics Achievement

Variable	Fixed Effects	
	Coefficient	*SE*
Overall	8.17*	2.37
Change and relationship	7.24*	2.37
Space and shape	8.37*	2.80
Uncertainty and data	7.42*	2.42
Quantity	9.35*	2.56
Formulating	10.18*	2.70
Employing	7.93*	2.39
Interpreting	9.13*	2.40
Variable	**Random Effects**	
	Variance	***df***
Overall	27.46	135
Change and relationship	15.80	135
Space and shape	79.24	135
Uncertainty and data	64.72	135
Quantity	57.02	135
Formulating	38.72	135
Employing	24.57	135
Interpreting	38.29	135
$^*p < .05$		

Table 5

Relative Effects of Student Help-Seeking Behaviors on Multiple Measures of Mathematics Achievement

Variable	Fixed Effects	
	Coefficient	*SE*
Overall	6.61*	2.17
Change and relationship	5.99*	2.43
Space and shape	7.41*	2.73
Uncertainty and data	6.02*	2.36
Quantity	7.52*	2.49
Formulating	8.63*	2.90
Employing	6.59*	2.36
Interpreting	6.92*	2.32
$^*p < .05$		

Relative Effects of Student Help-Seeking Behaviors

Finally, student- and school-level variables were introduced as the control variables to estimate the relative effects of student help-seeking behaviors. Student help-seeking behaviors showed statistically significant relative effects on all eight measures of student mathematics achievement, even after controlling for student- and school-level variables (see Table 5).

In Table 5, the fixed effects refer to the effects of student help-seeking behaviors on student mathematics achievement; random effects were omitted from the table because the effects of student help-seeking behaviors on student mathematics achievement did not vary from school to school. Specifically, a 1-point increase in student help-seeking behavior was associated with an increase of between 5.99 and 8.63 points in student mathematics achievement; the corresponding effect sizes ranged from 5.99% to 8.63% of an *SD*. For example, a 1-point increase in student help-seeking behavior was associated with an increase of 6.61 points in the overall mathematical literacy, with a corresponding effect size of 6.61% of an *SD*. Therefore, in the U.S., even after control over student and school characteristics, student help-seeking behaviors still demonstrated statistically significant positive effects on all eight measures of student mathematics achievement.

HLM Performance

A statistical model often helps to explain the variance in a dependent variable. The proportion of variance explained by such a model is often used as a means of assessing the performance of the model. Across the eight measures of student mathematics achievement, our final models (see Table 6) accounted for between 8% and 11% of the total variance in student mathematics achievement at the student level and between 60% and 73% of the total variance in student mathematics achievement at the school level (see Table 6). Overall, across the eight measures, our final models accounted for between 20% and 25% of the total variance in student mathematics achievement. For example, for the overall mathematical literacy, our final model accounted for 11% of the total variance at the student level and 72% of the total variance at the school level. Overall, our final model accounted for 24% of the total variance in the overall mathematical literacy. According to the common standards in the social sciences (see Gaur & Gaur, 2006), these numbers indicate

Table 6

Proportion of Variance in Mathematics Achievement Explained by the Models

Variables	Student level	School level	Overall
Overall	.107	.715	.244
Change and relationship	.106	.661	.226
Space and shape	.084	.597	.198
Uncertainty and data	.088	.726	.241
Quantity	.100	.667	.230
Formulating	.086	.691	.226
Employing	.096	.684	.221
Interpreting	.108	.730	.245

a sound performance of our final models, providing confidence for our analytical claims.

Discussion

Revisiting Research Literature

Although the research literature concerning the relationship between student help-seeking behaviors and student mathematics achievement is limited, the small number of empirical studies all concluded that there is a positive association between the two (e.g., Ogan et al., 2015; Schenke et al., 2015). The populations for these studies involved students between the ages of 12 and 17. These studies employed nonrandom samples that were either small in size or selected from small geographical regions (or both). The statistical methods used in these studies included multiple regression analysis and ANOVA, with only one study employing HLM.

The present study provides stronger evidence than previous studies that student help-seeking behaviors are positively associated with student mathematics achievement. The term “stronger” reflects the fact that the present study employed a large, nationally representative, random sample to assess the issue. Further, we adopted multilevel techniques to account for the fact that students are nested within schools. Taken together, these represent a substantial improvement in research methodology. As a result, the present study makes unique contributions to the research literature with far more credible and precise generalizability.

Two major contributions of the present study can be made with confidence. First, the help-seeking behaviors of 15-year-old students (in the U.S.) have positive effects on their mathematics achievement. Although these effects are relatively small, they are robust (or stable) across all eight measures of mathematics achievement (even after controlling for student and school characteristics). Thus, for a wide range of domains in mathematics, an increase in student help-seeking behaviors contributes to an improvement in mathematics achievement. Not only does this conclusion provide further support for the relatively small number of previous empirical studies on this issue, but it also adds further evidence that the effects of student help-seeking behaviors on student mathematics achievement are rather comprehensive (or systematic).

Second, although students’ help-seeking behaviors have positive effects on their mathematics achievement, these effects do not vary statistically significantly from school to school (in the U.S.). In other words, the relationship between student help-seeking behaviors and student mathematics achievement is independent of which schools students attend. As a result, student help-seeking behaviors can have positive effects on student mathematics achievement in any school regardless of school characteristics. This conclusion has not been reported in previous studies and is thus very unique in the research literature.

Educational Implications

The robust finding that students’ help-seeking behaviors are positively associated with their mathematics achievement calls for efforts to educate teachers on how to encourage their students to be more proactive in seeking help in the learning of mathematics. The issue of student help-seeking behaviors may become an integral component of teacher professional development for the specific purpose of improving the help-seeking behaviors of students both inside and outside the mathematics classroom. Such professional development opportunities may emphasize a sound understanding of help-seeking behaviors (e.g., the nature, the unique relationship with mathematics as opposed to other school subjects, individual and cultural differences, related affective and cognitive conditions, and the role of technology) and effective techniques for creating an educational environment (both inside and outside the mathematics classroom) that invites and welcomes students to seek help.

The finding that the relationship between student help-seeking behaviors and student mathematics achievement is independent of which schools students attend is a piece of positive news.

Schools have a context (e.g., location, available resources, socioeconomic and racial-ethnic compositions of the student body, and education and experience levels of the teacher body) and a climate (e.g., administrative policies, instructional organization, and attitudes and expectations of students, parents, and teachers) (see Ma et al., 2008). It is challenging for educators to change school climate and nearly impossible for them to change school context. With that in mind, it is encouraging to find that, regardless of the school contextual and climatic characteristics, student help-seeking behaviors can have positive effects on student mathematics achievement. Therefore, improving students’ help-seeking behaviors can be a good strategy in any school for the purpose of improving students’ mathematics achievement.

Finally, although the present study supports the robust importance of student help-seeking behaviors to student mathematics achievement, the effect sizes are small. This finding implies that improving students’ help-seeking behaviors by itself in an isolated fashion may not generate dramatic improvement in their mathematics achievement. Instead, it should be more beneficial to combine efforts at improving students' help-seeking behaviors with other educational reforms aimed at improving their mathematics achievement. For example, Everyday Mathematics, a national reform-based curriculum that is widely used throughout the U.S. (http://everydaymath.uchicago.edu/), provides a good opportunity for mathematics educators to promote students' help-seeking behaviors. Specially designed student help-seeking activities can be implemented within the Everyday Mathematics curriculum framework as instructional strategies to address specific topics in mathematics, especially those that are traditionally considered to be difficult (e.g., fractions).

Limitations and Suggestions

The primary weakness of the present study is the relatively low internal consistency of the composite variable measuring student help-seeking behavior. Although the index we constructed did yield a better internal consistency than the index constructed by PISA, Cronbach's α is still not optimal. Ultimately, the decision was made to use our index because it is conceptually clear and consistent, emphasizing interactions of students with real people (as opposed to either books or machines). This emphasis is good in the sense that interacting with people is a common way to seek help; however, this emphasis seems to have focused on a very specific or unique aspect of student help-seeking behaviors. Further research may seek to improve the measurement, especially if researchers intend to design their own surveys.

By conducting secondary data analysis, we were limited to data collected by PISA. Ideally, more control variables at the student and school levels would be considered to improve the HLM analysis. For example, because information on race-ethnicity was not available in PISA, the racial-ethnic background of students and schools could not be controlled for, thus ignoring the importance of racial-ethnic differences in student mathematics achievement (Parks & Schmeichel, 2012). Further research that seeks to improve control over student and school characteristics may better reveal the robust nature of student help-seeking behaviors in relation to student mathematics achievement.

Although it is reasonable for us to use data from PISA 2012, ideally we would prefer to have more current information. Considering this, our results may provide more of a historical examination on the issue of student help-seeking behaviors. The next PISA cycle with a focus on mathematics will occur in 2021. Given that we have shown clearly that this is a worthy issue to explore in mathematics education, researchers may want to follow up with PISA 2021 for any updated measures on student help-seeking behaviors.

Many research extensions are possible with newer (or even 2012) PISA data. We offer two suggestions. One is to examine the interactions between student help-seeking behaviors and other student characteristics (e.g., gender, attitude toward mathematics). The interest is in whether, say, one gender shows stronger effects of student help-seeking behaviors on student mathematics achievement. The other suggestion is to link student help-seeking behaviors with other outcomes in mathematics (e.g., mathematics anxiety, motivation to learn mathematics). It is informative to examine whether student help-seeking behaviors can, say, reduce student mathematics anxiety.

Finally, modern forms of communication may have fundamentally changed the primary ways in which people seek help. Previously, help-seeking concerning mathematics relied almost exclusively on the availability of physical resources such as textbooks and teachers. Now, in addition to these (which are still quite important), help-seeking has a growing reliance on the availability of electronic resources such as online homework systems and instructional videos, adding further complexity to the conceptual structure of student help-seeking behaviors. Further research that taps into these new avenues of help-seeking can bring a more comprehensive understanding of the relationship between student help-seeking behaviors and student mathematics achievement.

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Metacognitive Skills of Students in a Mathematics Class with Supplemental Instruction and Online Homework

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ABSTRACT Improving students' performance in Calculus is a challenge for many colleges and universities. One way of improving students' performance as well as their metacognition and study skills is to provide opportunities for them to receive support outside of the lecture. A modified version of the Motivated Strategies for Learning Questionnaire (MSLQ) was used to reveal any significant differences in metacognition and study strategies between students in a class with supplemental instruction with peer tutors and a dynamic online homework software (WebAssign), and students in a traditional class without these additional supports. Surveys and interviews were utilized to provide anecdotal evidence on the influence of WebAssign and supplemental instruction sessions on study skills and metacognition and whether students preferred WebAssign to traditional homework. Overall, the study showed no significant difference between the two groups in seven out of eight sub-scales of metacognitive skills and study strategies. Students reported that the supplemental instruction sessions and the WebAssign software were beneficial to their success.

KEYWORDS *calculus, metacognition, online homework, retention, study skills, supplemental instruction*

Introduction

The need to improve the performance of U.S. students in science, technology, engineering, and mathematics (STEM) is widely recognized (McGivney-Burelle & Xue, 2013; Zerr, 2007). In response to a growing concern that the U.S. is facing a crisis in college attainment rates, many programs in the New York Metropolitan area are promoting college access, success, and completion. The Percy Ellis Sutton Search for Education, Elevation, and Knowledge (SEEK) Program is one such program offered by the City University of New York at their senior (four-year) colleges.

One benefit of SEEK programs is supplemental instruction hours attached to high-risk academic courses. High-risk courses are defined as traditionally difficult academic courses that have a 30% or higher rate of D or F final course grades and/or withdrawals (Blanc et al., 1983; Kenney, 1988; Martin & Arendale, 1992; Peacock, 2008). Calculus, which is a prerequisite for many STEM disciplines, is a high-risk course. McGiveney-Burelle & Xue (2013) observed that despite the numerous efforts in recent decades to modify the teaching and learning of first semester calculus, this course remains a "gatekeeper" for STEM majors.

Supplemental Instruction and Online Support

Martin and Arendale (1992) define Supplemental Instruction (SI) as an academic assistance program that may increase student academic performance, retention, and metacognitive skills in high-risk courses. Metacognition can be defined as skills that learners acquire which demonstrate an awareness of their own knowledge and their ability to understand, control, and manipulate their own cognitive processes (Pintrich et al., 1991). SI is a proactive model where the peer tutor, called the SI leader, would attend all classes, and then hold tutoring sessions outside of class time. Martin and Arendale (1992) felt that the tutor could more effectively assist the students in this manner. SI differs from a tutorial program in that it integrates study skills for the course with content.

Martin and Arendale (1994) also defined the features of an SI program that contribute to students' success as follows:

- SI is proactive rather than reactive; it begins from the first day of classes.
- SI is attached to specific courses.
- SI leaders attend all class sessions.
- SI is not a remedial program.
- SI sessions are designed to promote a high degree of student interaction and mutual support.
- SI provides a way for the course instructor to receive feedback from the students through the SI leader.

Such features separate an SI program from other academic assistance programs.

In order to improve students' performance as well as their metacognitive and study skills, and to provide opportunities for students to receive peer and instructor support outside of the traditional lecture, out-of-class time should be highly structured to best prepare students for in-class activities (McGivney-Burelle & Xue, 2013). WebAssign is a versatile, web-based homework service for educators who want to offer expanded learning opportunities to their students (Risley, 1999). WebAssign is bundled with the textbook it uses. WebAssign and SI sessions are examples of structured out-of-class support that gives students immediate feedback on their homework.

WebAssign has several tools for instructors to use for teaching and assessing their students. It offers exercises, problems, simulations, videos, and tutorials for instructors to choose from in creating an assignment. Instructors can also access a dynamic gradebook, which shows students' performance on questions and topics throughout the course. Additionally, they can reuse the assignments created for other sections of the same course. Instructors can reply to extension requests on assignments made by their students. If instructors choose to administer graded examinations online, the WebAssign "LockDown Browser" prevents students from doing anything on the computer other than the examination.

Students have access to many features on WebAssign as well. WebAssign instantly grades the question as soon as they submit it, which allows students to receive immediate feedback. In addition to this feedback, WebAssign provides the opportunity to redo the question or a similar one. Students have multiple attempts to answer each question without losing credit for those questions. WebAssign has videos of course concepts and solved problems that students can use as a resource for learning. Students can click on "Practice Another Version" to solve a similar question for no credit if they need additional practice. Another resource for students is "Master It" tutorial which shows the solution to a problem step-by-step. The "My Class" Insights feature allows students to see the questions and concepts they mastered and those that require additional practice. When students need help or guidance on an assignment or a specific question, they can contact their instructors by using the "Ask Your Teacher" feature. There are additional questions, quizzes, and tests available that students can do, but are not graded for extra practice. Study guides are also available for students to review current and past concepts.

Methodology

The current study investigated the effects of SI and online homework on the metacognitive and study skills of first semester calculus students. The study addressed two research questions: (1) How do the metacognitive and study skills of students in a calculus class with SI and online homework differ from those of students in a traditional class? and (2) What are students' experiences with SI and online homework?

Theoretical Framework

McGivney-Burelle and Xue (2013) observed that in a typical calculus lecture, emphasis is placed on the lower cognitive level as defined in Bloom's Taxonomy. The six major categories of Bloom's Taxonomy are: Knowledge, Comprehension, Application, Analysis, Synthesis, and Evaluation (Bloom & Krathwohl, 1956). Typically, when the students are solving homework questions on their own, they are expected to engage in the higher-level skills of *analysis, synthesis,* and *evaluation* without the support of their peers and instructor. SI sessions and WebAssign provide students the opportunity to receive immediate feedback on their homework.

In search of a theory of learning to improve metacognitive and study skills, Casazza & Silverman (1996) developed a theory consisting of four assumptions:

(a) Learning is an active process rather than a passive one,
(b) Individuals have to thinkabout a problem and reduce ambiguity before they can reach a solution,
(c) Motivational drive is intrinsic, and
(d) Before a learner can solve a problem, he/she needs to be able to look at the pieces of information that define the problem in different ways (p. 292).

The SI program was designed to increase students' metacognitive and study skills by fostering their growth as independent learners (Peacock, 2008).

Design

This study was conducted at a large, urban four-year college in the borough of Queens. It utilized Campbell and Stanley's (1963) Nonequivalent Control Group model since a true randomized design could not be achieved. The participants are students in two calculus classes: one where the students were given traditional homework assignments (control group), and the other where students were offered SI and online homework with WebAssign (experimental group). In this mixed-methods study, all participating students responded to items on a modified version of the Motivated Strategies for Learning Questionnaire (MSLQ) which is displayed in Appendix A. Due to the nature of the study, only the items in the learning strategies section of the MSLQ were included in the questionnaire that was used. To help gauge students' attitudes towards, and experience working with SI and online homework, a survey was created and administered. In addition, fourteen students from the experimental group were interviewed to determine what elements of the programed did and/or did not work for them.

The validation of true group equivalence in the control and experimental groups was not possible due to the lack of information regarding students' prior academic records. All students in the study were between the ages of 18-25 years. The control and experimental groups were similar with regard to demographic variables such as age, gender, class level, reason for taking the course, and the number of courses they were currently taking.

Three classroom observations were conducted throughout the semester. Both instructors began their classes by discussing homework questions which students voluntarily wrote and/or solved on the whiteboard. Following the explanation of new mathematical concepts or rules, students had the opportunity to solve some practice questions by themselves. In the experimental group, the supplemental instruction leader walked around the room helping students. At the end of the problem-solving session, both instructors explained and gave the answers instead of facilitating discovery by the students or asking students to explain their solutions. Both instructors referred to their prepared notes during the lecture. Based on the observations, it is safe to conclude that both instructors' presentation of course material and pedagogical strategy were similar. The only pedagogical difference between the two groups was the means by which homework was assigned.

Findings

Metacognitive and Study Skills

The items on the MSLQ related to Self-Regulation learning strategies measured how often students think about what they are reading, doing, or studying as they solve mathematics problems. This involves connecting new concepts to relevant aspects of prior knowledge, self-testing and questioning oneself to ensure understanding of the material, and continuously checking and correcting one's behavior as they proceed on a task. An independent samples *t*-test was conducted (see results in Table 1) to determine whether the students in the control and experimental groups differed significantly with regard to their Self-Regulation learning strategies. No statistically significant difference was found, which suggests that students in the SI/online homework sections and students in the traditional sections did not differ significantly in their awareness, knowledge, and control of cognition (Pintrich et al., 1991).

Similar results were found for Rehearsal, Organization, Critical Thinking, Time and Study Environment, Effort Regulation and Help Seeking. Basic rehearsal strategies involve rereading class notes and course readings and memorizing lots of key words and concepts (Pintrich et al., 1991). They are used to help students to retain concepts and recall them when needed. Organization strategies refer to students' ability to select the main ideas from class readings and organize what they need to learn in the course (Pintrich et al., 1991). These strategies require students to be actively engaged with the course material. Critical thinking refers to the degree to which students apply previous knowledge to new situations to solve problems, reach decisions, or make critical evaluations with respect to standards of excellence (Pintrich et al., 1991). Essentially, they enable students to look for evidence and/or alternative solutions and question theorems and other mathematical statements before accepting them as true. Time and study strategies involve scheduling, planning, and managing one's study

Table 1
Means for Metacognitive and Resource Management Strategies

Strategy	*N*	Mean	Std. Dev.	*t*	*p*
Self-Regulation				0.056	0.955
Control Group	56	3.62	0.51		
Experimental Group	56	3.61	0.61		
Time and Study Environment				-1.819	0.072
Control Group	56	3.74	0.49		
Experimental Group	56	3.91	0.49		
Effort Regulation				0.000	1.000
Control Group	56	3.16	0.51		
Experimental Group	56	3.16	0.51		
Help Seeking				0.482	0.631
Control Group	56	3.57	0.75		
Experimental Group	56	3.50	0.71		
Rehearsal				-0.287	0.775
Control Group	56	3.97	0.84		
Experimental Group	56	4.01	0.81		
Elaboration				-2.032	0.045*
Control Group	56	3.67	0.90		
Experimental Group	56	3.99	0.77		
Organization				-0.113	0.910
Control Group	56	3.68	0.77		
Experimental Group	56	3.70	0.89		
Critical Thinking				0.575	0.566
Control Group	56	3.34	1.03		
Experimental Group	56	3.23	1.02		

Note: The items are scored on a 5-point Likert scale with 1 meaning *strongly disagree* and 5 meaning *strongly agree.*
$^*p < .05$

time and the use of a place of study (Pintrich et al., 1991). They entail the effective use of study time in a setting where the student chose to do his work. Effort regulation refers to students' willingness to try hard on their schoolwork, even when the work is difficult (Pintrich et al., 1991). These strategies include students' ability to stay focus on their schoolwork in the presence of difficulties and/or distractions. Help seeking involves students learning to manage the support of others (Pintrich et al., 1991). This enables students to seek help from others such as peers, tutors, and instructors outside of the classroom.

Elaboration strategies help students store information into long-term memory by building internal connections between items to be learned (Pintrich et al., 1991). These strategies include paraphrasing or summarizing course concepts and connecting new information to prior knowledge. An independent samples *t*-test (displayed in Table 1) revealed there was a statistically significant difference between the elaboration learning strategies of students in the two groups. The Cohen's *d*, which is used to measure effect size, for the learning strategy of elaboration was 0.4. This indicates a small effect size. That is, elaboration strategies have a small effect on students in the experimental group.

Online Homework and SI

Approximately 80% (45 out of 56) of the students agree (either "strongly" or "somewhat") the online homework using WebAssign helped them to better understand the course material and improve their problem-solving skills. One student said "WebAssign is helpful. It helps me answer the question(s) and it helps me to understand the material better." Almost all the students appreciated the benefits of having tutorial videos to watch when they needed help in solving a problem or understanding a particular concept. Another student commented "My favorite features are the videos, because I can actually see someone doing it [hands-on]." Ten out of fifty-six students mentioned that the feature that allows students to submit multiple attempts for a problem is one of the best aspects of completing online homework using WebAssign. One student mentioned "I like best that WebAssign gives some, at least five, chances without being penalized to get the answer right." On the other hand, one student noted that one could earn a good grade without actually understanding the concepts or even how to solve the problem since they can continue using the multiple attempt feature until they are correct.

Additionally, 48 out of 56 students responded that WebAssign helped them to be better prepared for tests and 42 out of 56 responded that the software helped them score higher on tests. Other responses included students responding that their time spent on WebAssign was worthwhile (40 out of 56) and that homework using WebAssign was more beneficial to their success than traditional homework completion (35 out of 56). Ten of the fourteen students interviewed agreed that "WebAssign homework was more helpful than the textbook homework." One student explained:

> There are more helpful tools on WebAssign than there is with me doing the homework on my own where all I have is my notes and the textbook to rely on. On WebAssign I can switch to a different question, or I can watch a video on how it's done, or I can go to the exact spot in the textbook as to where I can find the problem. So, it is better.

The overwhelming sentiments of the students (51 out of 56) were that they liked that "WebAssign immediately grades their homework" and 52 liked that "WebAssign showed them the step-by-step solution of a similar problem when they asked for help." The students' interviews corroborated those findings. One student said "WebAssign shows you an example of how a problem is done and then you learn from that and fix your mistakes; the instant feedback is great." Another student, commented:

> I like best that WebAssign tells you right away if the question you did is correct or incorrect and it also has practice examples so if you're unable to do the homework question, you can try the practice example and then attempt the actual homework question.

However, approximately 30% (17 out of 56) of students felt that the questions on WebAssign do not match the questions solved in class.

More than 42 (out of 56) of the students strongly agreed or somewhat agreed that SI enabled them to better understand the course content, become better mathematics problem solvers, and perform better on tests, thus improving their grades. Problem solving strategies and the use of index cards were emphasized in SI sessions. One student claimed, "I feel like it [SI session] was very helpful, and it helped me learn the material better and reinforced what I learned the day before." The majority of students (47 out of 56) believed that SI sessions were helpful and 45 of them agreed that the SI leader was available to assist them from the first week of classes which confirms one of the distinguishing features of SI. All the students interviewed agreed that SI sessions were helpful. One student said, "I think SI sessions are very helpful; they help us to solve and understanding the questions, and the SI leader is very helpful." Similarly, another student commented, "SI provides us with an extra resource from day 1, so in case we need to approach someone besides the professor."

Thirty-nine (out of fifty-six) students believed the time spent in SI sessions were worthwhile while forty-three agreed that they learned appropriate study strategies in SI sessions. Activities in SI sessions focused on notetaking, text reading, problem solving, and study habits. One student noted, "I found better ways of memorizing formulas and knowing when and where to use the correct formula in SI sessions." Another student had the same opinion: "I found recopying my notes after class helped me in understanding the course. This is something I learned from the SI leader." Approximately 75% (42 out of 56) of students believed SI sessions motivated them to study. Five out of fourteen students believed attending the SI sessions motivated them to get their schoolwork done. One student noted, "Sometimes I felt like I was slacking then I would go to SI sessions and realized I have to review my class notes and complete the homework in order to have questions for the next session."

Discussion

The purpose of Research Question 1 was to determine if there was a significant difference in metacognitive and resource management strategies (Rehearsal, Elaboration, Organization, Critical Thinking, Metacognitive Self-Regulation, Time and Study Environment, Effort Regulation and Help Seeking) between students in a class with SI/online homework and students in a traditional class. The MSLQ is designed to determine the cognitive and metacognitive strategies and resource management strategies of students. Based on the responses to the modified MSLQ, there were no statistically significant differences between students in a class with SI/online homework and students in a traditional class in seven out of eight sub-scales of metacognitive and resource management learning strategies. Recall that these seven strategies were metacognitive self-regulation, time and study environment, effort regulation, help seeking, rehearsal, organization, and critical thinking. However, students enrolled in the course with SI/online homework showed higher levels of elaboration learning strategies compared to the students who were not exposed to SI/online homework. This could be due to the fact that students with SI/online homework had more opportunities to relate new concepts with prior concepts/knowledge and exposure to more examples and practice. Students in the treatment group are required to attend SI sessions so maybe that can be a reason why they viewed their effort to study calculus as low. In a similar study, Peacock (2008) found that students in courses with SI had higher levels of organization skills.

The purpose of Research Question 2 was to investigate students' attitudes toward, and experience working with SI and online homework using WebAssign. Based on the survey results, more than 75% (42 out of 56) of the students agreed that attending SI sessions and/or doing online homework using WebAssign have helped them to understand the course better, improved their problem-solving skills, and be better prepared for tests which boosted their performance in the class. One student said "WebAssign is helpful. It helps me answer the question(s) and it helps me to understand the material better." When asked about SI sessions, she noted "The best part is it [SI session] reviews what we learned in the past and prepared us to do well in tests." McGivney-Burelle and Xue (2013) noted out-of-class time should be highly structured to best prepare students for in-class activities. WebAssign and SI sessions are designed to guide and support students as they are studying the course content. Results from the survey and interviews showed that the overwhelming majority of the students liked the extra resources, WebAssign and SI, available to them outside of class. SI sessions also emphasized appropriate study skills and strategies for succeeding in the course. The findings reinforced the use of WebAssign to improve students' metacognitive and study skills.

Casazza and Silverman (1996) stated that an effective teaching/learning process increases awareness of one's own thought processes and encourages the learner to gradually assume the responsibility for learning. Students are taking responsibility for their own learning when they are solving problems on WebAssign and are using the resources (videos, tutorials, and Practice another question) available to them whenever they need them. Some students also reach out to their instructors and/or SI leaders when they realized they need additional guidance and support. These behaviors enable students to be actively engaged in the course material and be aware of their own thought processes and learning.

Implications and Limitations

A study to determine if there is a significant difference in the test scores between students in a class with SI/online homework and students in a traditional class can be conducted. Also, additional research about the effects of SI and online homework on the metacognitive and study skills of students is needed with larger class sizes. An investigation of the qualifications and training of SI leaders for their role is also warranted.

Even though every student in the study answered the questionnaire and survey, it is difficult to say if they read and thought about each question thoroughly before answering it. Moreover, it is not always clear if they answered the question truthfully or if they chose the answer that was sociably acceptable. Students grades were also not analyzed in this study. Students' academic records were not available, and the groups were not created by a randomization process. Students from four intact sections of a first semester calculus were utilized. The validation of true group equivalence in the control and experimental groups was not possible due to the lack of students' prior academic records. Also, there were two instructors in the study. Instructor quality (knowledge, degrees, etc.) was not measured and may have influenced the academic achievement between the groups. Furthermore, the lack of thorough training of SI leaders could also be impacting the way they conduct their sessions. This study was conducted for one semester in one course at a single college. The small sample size limits claims of generalizability to larger population.

Conclusion

Due to technological advancements, educators need to adapt accordingly to these changes in order for the United States to retain its standing in the world economy. Educators should continue to search for ways to increase students' metacognition and study skills. Incorporating WebAssign and SI sessions into the calculus class may help students to take responsibility for their own learning. WebAssign and SI sessions are additional resources and support that are available to students outside of the classroom. Both are designed to give students immediate feedback on their homework and/or course material. Although the results from the modified MSLQ showed a significant difference between the elaboration learning strategies of students in the two groups, the effect size is small. According to the survey and interviews, most students showed support for the SI program and the online homework management system, WebAssign. This may represent another step towards developing students' metacognition and study skills in college calculus courses.

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Appendix A

Modified MSLQ: STUDY HABITS AND LEARNING SKILLS – A Truncated List

PLEASE RESPOND TO THE QUESTIONNAIRE AS ACCURATELY AS POSSIBLE, REFLECTING YOUR OWN ATTITUDES AND BEHAVIORS IN THIS COURSE.

5) Choose the reason(s) for taking this course.
   - ☐ It is a required course for my major.
   - ☐ It is an elective course which fits my schedule.
   - ☐ It will improve my career prospects.
   - ☐ It was recommended by someone.
   - ☐ Content seems interesting.

6) How many courses are you taking this semester (including this course)?
   ☐ One ☐ Two ☐ Three ☐ Four ☐ Five or more

7) During class time I often miss important points because I'm thinking of other things.
   - ☐ Strongly disagree
   - ☐ Somewhat disagree
   - ☐ Neither agree nor disagree
   - ☐ Somewhat agree
   - ☐ Strongly agree

7) When I become confused about something I'm reading or doing for this class, I go back and try to figure it out.
   - ☐ Strongly disagree
   - ☐ Somewhat disagree
   - ☐ Neither agree nor disagree
   - ☐ Somewhat agree
   - ☐ Strongly agree

9) Before I study new course material thoroughly, I often skim it to see how it is organized.
   - ☐ Strongly disagree
   - ☐ Somewhat disagree
   - ☐ Neither agree nor disagree
   - ☐ Somewhat agree
   - ☐ Strongly agree

10) I try to change the way I study in order to fit the course requirements and instructor's teaching style.
   - ☐ Strongly disagree
   - ☐ Somewhat disagree
   - ☐ Neither agree nor disagree
   - ☐ Somewhat agree
   - ☐ Strongly agree

11) When studying for this class I try to determine which concepts I don't understand well.
   - ☐ Strongly disagree
   - ☐ Somewhat disagree
   - ☐ Neither agree nor disagree
   - ☐ Somewhat agree
   - ☐ Strongly agree

12) If I get confused taking notes in class, I make sure I sort it out afterwards.
- ☐ Strongly disagree
- ☐ Somewhat disagree
- ☐ Neither agree nor disagree
- ☐ Somewhat agree
- ☐ Strongly agree

13) I usually study in a place where I can concentrate on my course work.
- ☐ Strongly disagree
- ☐ Somewhat disagree
- ☐ Neither agree nor disagree
- ☐ Somewhat agree
- ☐ Strongly agree

14) I make good use of my study time for this course.
- ☐ Strongly disagree
- ☐ Somewhat disagree
- ☐ Neither agree nor disagree
- ☐ Somewhat agree
- ☐ Strongly agree

15) I find it hard to stick to a study schedule.
- ☐ Strongly disagree
- ☐ Somewhat disagree
- ☐ Neither agree nor disagree
- ☐ Somewhat agree
- ☐ Strongly agree

16) I have a regular place set aside for studying.
- ☐ Strongly disagree
- ☐ Somewhat disagree
- ☐ Neither agree nor disagree
- ☐ Somewhat agree
- ☐ Strongly agree

17) I make sure I keep up with the weekly assignments for this course.
- ☐ Strongly disagree
- ☐ Somewhat disagree
- ☐ Neither agree nor disagree
- ☐ Somewhat agree
- ☐ Strongly agree

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Streamlining Time Spent in Alternative Developmental Mathematics Pathways: Increasing Access to College-Level Mathematics Courses by Altering Placement Procedures

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ABSTRACT Developmental mathematics, which is designed to prepare students for college-level mathematics courses, can be a barrier to students' success. In the United States, the majority of students placed into developmental mathematics courses fail to complete the developmental sequence. Alternative mathematics pathways offer some benefits when integrated with "just-in-time support" or expedited instruction on specific prerequisite concepts needed solely for the current lesson. This study compares two statistics courses taught at a public community college: a complete course taught in one semester and a two-semester version with just-in-time developmental content integrated into the course. The study found that students placed into the one-semester statistics course accumulated significantly more credits after one and two years of college. These students also completed an associate's degree within two years at a significantly higher rate than students placed into the two-semester statistics course. The study also found students deemed non-proficient by the college's placement exam, who also had a strong high school average were significantly more likely to earn a grade of A- or higher or a grade of B- or higher compared to students originally deemed proficient.

KEYWORDS *community college, developmental education, placement, co-requisite remediation, remedial mathematics, higher education, statistics pathways*

Introduction

In the United States, despite efforts to reform K-12 education, between 40% and 60% of first-year college students are deemed to be unprepared for college-level course work and require remediation in mathematics, English or in both subjects (Jimenez et al., 2016). To make decisions about mathematics course placement, nationwide, in the United States, 100% of community colleges and 85% of four-year colleges rely on high-stakes tests (Fields & Parsad, 2012). Remediation refers to the courses that students enroll in in order to prepare them for college-level course work. Similarly, developmental education was designed to strengthen students' academic skills, preparing students to be successful in credit-bearing college courses. It refers to a wide array of support services provided to students who are unprepared for college-level course work. However, instead, students assigned to one, two, or even a series of three levels of remedial courses often fail to complete the course sequence (Bailey et al., 2010). Although students assessed to be non-proficient could have gaps in their background or weaker study skills, mathematics remediation has been shown to be a significant obstacle, preventing students from earning a college degree (Bailey, 2009; Bailey & Smith Jaggars, 2016; Chen & Henke, 2017; Hoyt, 1999; Logue et al., 2016). In this article, following the lead of Bailey et al. (2009) and to vary the language, developmental education and remediation will be used interchangeably with no negative or positive connation implied in the word choice.

Although placement into remedial courses may negatively impact students, the impact on women, minorities and low-income students may be more pronounced.

American College Testing (ACT) reports benchmark scores that assess algebra college readiness (ACT, 2017). Students who score above the benchmark on the ACT algebra exam have a 75% chance of passing a college credit-bearing algebra course with a grade of C or higher (ACT, 2017). When assessing students' mathematics college readiness, a higher percentage of women, Blacks and Hispanics fell below the benchmark (ACT, 2017). As a result, female, Black and Hispanic students are overrepresented in remedial mathematics courses (Hagedorn et al., 1999) relative to White males. Low-income students are also more likely to be placed into a remedial course (Jimenez et al., 2016). The 2018 Student Success Score for the state of California tracked students' ability to exit mathematics remediation within a six-year period in community colleges. The percent of Black, Hispanic and White students who have not successfully exited mathematics remediation within six years was 78%, 64% and 59%, respectively (Student Success Initiative, 2018). This gap in remedial course completion contributes to a racial gap in degree completion. An unpublished study found that more than half of the racial community college degree completion gap was a result of remedial mathematics placement decisions (Edley, 2017). As a result, the single remedial track leading to college algebra has been called a civil rights issue (Edley, 2017). Developmental education, though a well-intentioned intervention designed to boost students' skills, has not only failed to meet this goal but, in fact, may be contributing to widening rather than closing the race, gender and socioeconomic status education attainment gaps. For readers who are working to make mathematics education more accessible and the outcomes of placement more equitable, this research can serve as a primer by demonstrating how colleges can streamline alternative mathematics pathways to help close persistent race and gender based degree completion gaps.

For community college students, mathematics may be an impediment to graduating. Based on a review of the general education requirements posted on colleges' websites, many schools stipulate that all students must successfully complete at least one college-level credit-bearing mathematics course to graduate (The City University of New York [CUNY], n.d.; The State University of New York [SUNY], n.d.). To study persistence, researchers sampled 250,000 students across 57 community colleges and found that less than 50% of students completed the entire remedial course sequence to which they were assigned (Bailey et al., 2010). Therefore, reform in developmental education is needed. Although researchers have promoted and described the benefits of alternative mathematics pathways, (Blumenthal, 2016; Steen, 2004), these paths often span multiple semesters (Carnegie Foundation for the Advancement of Teaching, n.d.) and, as a result, delay the accumulation of college credits. This article sheds light on the process of placing students who will not major in a science, technology, engineering and mathematics (STEM) field in alternative mathematics pathways that meet for more days and hours per week, or are spread out over multiple semesters. It expands the discussion of developmental alternative credit-bearing mathematics pathways for students who will not major in a STEM field. Additionally, it describes how colleges can streamline alternative mathematics pathways, which may be better aligned with non-STEM students' interests and majors. Students majoring in a STEM field need to complete a traditional sequence of mathematics courses consisting of algebra, pre-calculus and calculus, which is outside the scope of this paper. The paper investigates how non-STEM students can effectively and efficiently bypass long sequences of mathematics courses designed to fulfill a college's general quantitative requirement for graduation when given the opportunity to enroll in a streamlined, relevant mathematics pathway.

One potential remedy for the persistent low completion rate is to offer non-proficient students who will not major in a STEM field the opportunity to enroll in a college-level credit-bearing mathematics course such as quantitative reasoning, statistics or financial literacy with just-in-time academic support for developmental prerequisite skills. Recognizing the time and money students spend attempting to complete long sequences of developmental courses that have a negative stigma, just-in-time support gives students the opportunity to enroll directly in a college-level credit-bearing course. Timely academic support is provided to assist students in learning the subset of prerequisite skills just prior to material that builds upon them. For example, if a student assessed not to be proficient in mathematics wanted to take an elementary statistics course, many colleges would require that the student first complete a sequence of one or more remedial pre-algebra courses before being allowed to enroll in elementary statistics. By contrast, colleges that offer just-in-time developmental support allow non-proficient students to enroll in statistics without completing any developmental prerequisite courses. Just-in-time support provides timely instruction on prerequisite skills that will arise in the lesson being taught. A student assessed not to be proficient in mathematics who enrolled in a statistics course would not relearn how to factor binomials, since learning statistics does not depend on knowing how to factor. However, a developmental student who enrolled directly in statistics would receive a refresher on how to plot points on a coordinate system and interpret the slope of a line just prior to learning how to create scatterplots and how to interpret the line of best fit. Just-in-time support removes the stigma associated with remediation and offers students the opportunity to accumulate credits.

Developmental Education: Cost Analysis

There is a psychological cost of referral to developmental education. Underprivileged students may not be well-prepared for college-level course work and, as a result, may be more likely to be assessed as needing remedial courses to catch up academically to their peers. However, placement into remedial courses stigmatizes students whose skills are lagging (Arendale, 2010; Juszkiewicz, 2016). This can reinforce the stratification of privilege and access. In addition to the psychological cost of developmental education, there is also a financial cost borne by both students and taxpayers. Developmental education costs students approximately $1.3 billion dollars per year (Jimenez et al., 2016). Taxpayers contribute approximately $7 billion per year to cover the cost of developmental education through loans and grants to students (Carter, 2017) with little return on the investment, given the low rate of persistence. To fully understand how severe the barriers to advancing are, it is important to describe why proficient students are sometimes incorrectly assessed to be unprepared for college-level course work.

Mathematics Placement Procedures

Typically, upon entry to college, before registering for classes, students take a high-stakes mathematics placement test to determine which course they should take. The four tests most often used to make decisions are the Scholastic Aptitude Test (SAT), American College Test (ACT), ACCUPLACER and Compass (Fields & Parsad, 2012). Because community colleges are generally open-access (College Board, n.d.) and hence less-selective, students are, on average, less prepared for college. Although some community college programs such as nursing or computer technology could have special admission requirements which include SAT or ACT scores, generally the open-enrollment policy for many other programs would mean that any high school graduate could enroll (College Board, n. d.). Therefore, many community college students either may not have taken the SAT or ACT exams or may not have scored highly enough to be exempt from having to take their college's placement exam. High school students must register in advance to take the SAT exam (College Board, n.d.) and, as the test is not a surprise, students can elect to prepare. Additionally, information about practice questions is available on the site used to register (College Board, n.d.). By contrast, many students are unaware that they will be required to take a college placement test. Additionally, students may not realize the consequence of performing poorly. Students from five community colleges in California, most of whom had not been out of high school for more than two years, participated in a focus group and reported that they received little information about college placement (Venezia et al., 2010). Based on a review of college placement policies posted on schools' websites, students may receive little or no help to prepare for the placement test, nor is the opportunity to be retested guaranteed. These factors can negatively impact the score earned.

Despite the heavy reliance on placement tests, such tests are poor predictors of students' performance in college courses as measured by grade point average (GPA) (Belfield & Crosta, 2012). Although placement test scores are associated with college GPA, the association is weak (Belfield & Crosta, 2012). Two possible explanations for the weak association are that the predictive validity of the test is limited (Jaggars & Hodara, 2011) and that the test does not measure non-academic traits necessary for success in college. Non-cognitive characteristics such having a growth mindset (Sole, 2019) and openness to seeking help may be better aligned with and predictive of, one's ability to pass a college mathematics course. A single score on an unanticipated mathematics placement test may not capture traits that are likely to predict success in college.

Placement tests tend to misassign proficient students to developmental courses (Scott-Clayton & Belfield, 2015). By comparison, multiple measures provide a more accurate means of assessing students (Barnett & Reddy, 2017; Ganga & Mazzariello, 2019) and could reduce the percent of students assigned to remediation by approximately between 8 to 12 percentage points (Scott-Clayton, 2012). Another more accurate way of placing students would be to use either their high school transcript or GPA. High school GPA assesses traits that could predict success in college such as drive (Ganga & Mazzariello, 2019). If students were given the opportunity to immediately be placed into credit-bearing college courses, they could save money and more quickly accumulate credits (Ganga & Mazzariello, 2019). This is significant because a stronger rate of momentum in the first year of college has been shown to be associated with a higher rate of graduation (Attewell et al., 2012). A recent study done statewide in California used decision trees to examine high school and college transcripts to shed light on what measures would most consistently predict success in college mathematics courses (Bahr et al., 2019). The research showed that cumulative high school grade point average was the most helpful measure used to predict how students would perform in college mathematics courses (Bahr et al., 2019).

Mathematics Alternative Guided Pathways

Some students are accurately placed but assigned to a traditional development course sequence preparing them for college algebra, which may be ill-aligned with their major and career aspirations. Non-stem students who need to take a mathematics course to satisfy a core general education requirement may find courses in quantitative reasoning, statistics or financial literacy more closely aligned with their interests. Often, these courses can be taken without first completing long sequences of remedial pre-algebra courses. Alternative mathematics course sequences may appeal more to non-STEM students. As new mathematics pathways emerged, the American Mathematical Association of Two-Year Colleges (AMATYC) made clear that for college-level mathematics courses that are not leading to calculus, students can be prepared for entry into credit-bearing courses without first mastering the material in developmental algebra courses (AMATYC, 2014). Two popular alternative mathematics pathways that emerged are quantitative reasoning and statistics. Both of these mathematics pathways can help students complete their college's quantitative graduation requirements in a timely manner, since neither relies on having mastered all of the material in a series of non-credit pre-algebra courses.

The benefits of quantitative reasoning have been observed (Steen, 2001; Steen & Madison, 2011). Steen (2004) supports including quantitative reasoning in the college mathematics curriculum, arguing that without the opportunity to study essential but simple mathematics ideas, students will fail to meet key tenets that are part of a college education. Researchers have also described the value of graduating statistically literate students (Carver et al., 2016; Sole, 2015; Weinberg & Abramowitz, 2020). Additionally, educators have stressed the importance of being able to understand the results of surveys (Carver et al., 2016; Sole, 2015) and use and interpret data (Carver et al., 2016; Sole & Weinberg, 2017; Weinberg & Abramowitz, 2020).

Recent research has shown that students randomly assigned to a college statistics course with a co-requisite workshop were more likely to pass the course than those assigned to a remedial elementary algebra course (Logue et al., 2016). Moreover, students who took statistics rather than elementary algebra graduated from two-year colleges at a higher rate (Logue et al., 2019). The benefit of offering non-STEM students the opportunity to complete alternative mathematics pathways is clear.

However, there are drawbacks. Alternative mathematics pathways can span two or more semesters and may be scheduled to meet for a large number of hours relative to the number of credits earned. The Carnegie Foundation for the Advancement of Teaching created courses in quantitative reasoning and statistics with just-in-time developmental content integrated into the courses. However, despite the benefit of accumulating college credits at the start of one's college career (Attewell et al., 2012; Attewell & Monaghan, 2016), these pathways can span a full academic year. Research is needed to investigate both the feasibility and benefit of streamlining alternative mathematics pathways. By examining the impact initial course placement has on credit accumulation and graduation rate, as well as comparing the performance of student assessed not to be proficient in a streamlined course, this article hopes to fill this gap in the literature.

Method

Settings

Data from a small urban public community college located in the northeast was used to assess the value of streamlining a statistics pathway with just-in-time remediation by changing placement practices. The college serves a diverse, largely low-income population, demographically comprised of approximately 5% Asian, 28% Black, 58% Hispanic, 8% White and less than 1% in each of the other race/ethnicity categories. Sixty-seven percent of the students qualified to receive a Pell Grant. The majority of students are female (47% male; 53% female). These demographics are significant because research has shown that stereotype threat—the risk of conforming to negative stereotypes of one's racial, gender, ethnic, or cultural identity (Steele, 2011)—negatively impacts the test scores of females (Picho et al., 2013), Black students (Steele & Aronson, 1995), Hispanic students (Gonzales, Blanton, & Williams, 2002) and students with low socioeconomic status backgrounds (Croizet & Claire, 1998). This could increase the number of these students deemed not to be proficient placed into the two-semester statistics sequence.

The college chosen for this study is appropriate because it offers one-semester and two-semester statistics courses that all students are required to complete to satisfy the college's quantitative requirement for graduation. Students who are deemed to be proficient in mathematics enroll in the one-semester streamlined statistics course in the fall semester of their first year of college. Students who are deemed not to be proficient in mathematics take the two-semester statistics course. Proficiency is determined by an index that combines high school grade point average with SAT and Regents exam scores (City University of New York, n.d.). Proficiency can be demonstrated in other ways (See City University of New York, n.d.).

The content covered in the one-semester streamlined statistics course and the two-semester statistics course is identical. The two-semester statistics course covers half of the required material in the fall semester and the remaining required material in the spring semester. Just-in-time support for the developmental content is integrated into the two-semester statistics course on the small subset of pre-algebra topics needed to understand the course. For example, a brief refresher of the equation of a line and the interpretation of the slope and y-intercept would be taught before studying linear regression.

All students at this community college enroll full-time. The first year of community college is focused on completing several specific general education requirements and taking a few elective courses. Therefore, in the first year, all students' course schedules are quite similar. However, by taking the one-semester streamlined statistics course, students can accumulate credits more quickly. Additionally, if students complete only the first half of the two-semester statistics course, the credits could be difficult to transfer. Given the benefits of taking the one-semester statistics course, it is important to explore alternative means of evaluating proficiency to potentially increase the number of students who would be exempt from having to learn the same content in the two-semester statistics course.

Participants and Design

Before the start of the fall 2018 semester, all first-year students whose high school average was equal to or above 83% ($n = 26$) were contacted by email and informed that because of their strong high school average, although the college's placement procedures had deemed them non-proficient, faculty believed they were proficient based on the fact that placement tests are poor predictors of success in college. Therefore, it seemed to faculty that overall performance in high school might be better aligned with and predictive of students' academic performance in college. This study was undertaken to determine if a change to the college's placement procedures could help capable students accumulate college credits at an accelerated rate. Although conversions between percent grade, letter grade and grade point average on a four point scale are not standard, an average of 83% is approximately equivalent to a B- or a B letter grade, which converts approximately to a GPA of 2.7 to 3.0. According to Belfield (2014), a simpler, more accurate rule to assess placement into developmental education would be to rely only on high school GPA. Students with a GPA below a specific threshold of 2.7 or 3.0 could be assigned to remedial courses (Belfield, 2014). Because a GPA of 2.7 to 3.0 is approximately equivalent to a grade range from 80% to 86%, a cutoff for placing students in a streamlined statistics course of 83% was used, which was chosen to be in the middle of the range.

Students were informed that although they were assigned to take a two-semester statistics sequence, faculty believed that they had the skill set needed to learn the material at a more accelerated pace. Students were given the option to remain in the two-semester statistics course and were informed that they would automatically be placed in the one-semester statistics course if a response indicating a preference otherwise was not received. Because the college has a week-long required summer introductory program, students would be on campus and could speak with either a faculty member or an advisor if they had any questions. Of the 26 students who were contacted, 22 were switched into the one-semester statistics course and four indicated that they wished to remain in the two-semester statistics course. Each of the 22 students enrolled in the streamlined statistics course that fit their schedule. None of the instructors teaching the streamlined statistics course knew the proficiency status of the students in this experiment.

Foundations of the Current Study

Prior to offering the one-semester course to students with a strong high school average but placing in the "not proficient" category, previous years data was assessed to provide insight into the potential impact of using students' high school average to override the college's decision about placement. To help determine how students deemed not to be proficient with a strong high school average might perform in the one-semester statistics course, the connection between high school average, course grade in the first half of the 2016 two-semester statistics course, and the number of college credits accumulated was assessed. A score roughly equivalent to a student's cumulative high school average was used which considers outliers due to absences (E. Hertz, personal communication, November 20, 2019). The Pearson product-moment correlation coefficient indicated that there was a weak positive correlation between the high school average and the GPA students received in the first half of the two-semester statistics course [$r(225) = .21$, $p = .001$]. In general, the results suggest that students who do well academically in high school, tend to have higher GPAs in a college statistics course. This suggests that high school averages could be used to place students into a one-semester statistics course. To investigate, statistics course grades of "not proficient" students who had a high school average below 80 (an average below B-) and equal to or above 80 (an average above B-) were compared. The grade breakdown of equal to or above B- was selected based on placement recommendations Belfield (2014) suggested. Statistics course grades were divided into those equal to or above a C and those below a C, since students are placed on academic probation for scoring below a C in a course. Table 1 shows the distribution of grades students earned in two-semester statistics course.

The chi-squared test of independence showed there to be a statistically significant relationship between high school average and two-semester statistics course grade [$\chi^2(1, 309) = 4.19$, $p = .041$]. As indicated in Table 1, a higher percentage of students who had a high school average equal to or above 80 (53%) earned a grade equal to or above a C in the first half of the two-semester statistics course than students who had a high school average below 80 (38%). Given the strength of difference in the two-semester statistics course grades of students who had a high school average of 80 or higher, it seemed that some threshold for high school averages in the range of 80 to 86 could be used to more accurately place students deemed not to be proficient into the one-semester streamlined statistics course.

Next, examining how placement impacts the ability to accumulate college credits provides further support for placing more students in the one-semester statistics course. Table 2 gives the mean and standard deviation for the number of total credits accumulated after one year and after two years of college for students who took the streamlined one-semester statistics course and the two-semester statistics course. Table 2 presents the results of the two independent samples t-tests assessing whether students were accumulating credits at a rate fast enough to be on track to graduate in two years.

After completing one year of college, the results of the independent-samples t-tests showed that students who took the streamlined one-semester statistics course earned significantly more college credits 23.32 (9.24) than did students who took the two-semester statistics course 16.89 (9.88), [$t(422) = 5.99$, $p < .001$]. After two years of college, the results of the independent-samples t-test showed that students who took the streamlined statistics course earned significantly more college credits 48.92 (17.88) than did students who took the two-semester statistics course 43.98 (16.88), [$t(309) = 2.33$, $p = .010$]. One might argue that students deemed to be proficient in mathematics may inherently be more motivated and hence more likely to earn credits at an accelerated rate. However, it is equally possible that placement into longer pathways, and the potential stigma that comes with this placement, disadvantages students who have the same drive and motivation. It is quite possible that the under-placement is preventing students from earning credits at the same rate. Because students would need to earn 30 credits per year to graduate from a community

Table 1

Frequencies and Chi-Square Results for High School Average and Two-semester Statistics Course Grade (N = 309)

	High School Average Below 80		High School Average Equal to or Above 80		
	n	%	*n*	%	$\chi^2(1)$
Fall Semester Course Grade					
Equal to or Above C	96	38%	29	53%	
Below C or No Credit	158	62%	26	47%	4.19*

Note. First-year students receive grades of NC (no credit), W (withdrew), or WU (withdrew unofficially) in place of receiving grades of F. These grades were counted as receiving a grade below a C.

*$p < .05$.

Table 2

Means Standard Deviations and t-test Results for the Total Number of Credits Earned by Years of College and Statistics Course

	One-semester Statistics		Two-semester Statistics			
College Credits	*M*	*SD*	*M*	*SD*	*t*	*p*
Year						
Year 1	23.32	9.24	16.89	9.88	5.99***	<.001
Year 2	48.92	17.88	43.98	16.88	2.33*	.010

* $p < .05$. ***$p < .001$.

college in two years, which is highly desirable, the total number of college credits earned was assessed.

On average, students placed in the two-semester statistics course did not accumulate credits at a pace that would earn them an associate's degree in two years. After completing two years of college, the mean number of credits earned by students who took the two-semester statistics course in their freshman year, 43.98 (16.88), was significantly less than 60—the number of credits required to graduate [$t(215) = -13.95$, $p < .001$]. After two years of college, the number of credits earned by students who took the streamlined statistics course in their freshman year, 48.92 (17.88), was also significantly less than 60, the number of credits required to graduate [$t(94) = -6.04$, $p < .001$]. One possible explanation as to why students in the one-semester statistics course also fell short of accumulating 60 credits is that students took less than 15 credits per term since only 12 credits are needed to remain eligible for federal financial aid. Given that the average number of credits accumulated in the streamlined pathway after two years was 48.92, it is conceivable that the largely low-income student population that the college serves—many of whom must, by necessity, balance work and school—sought to take the minimum number of college credits to maintain their status as full-time students. It is also quite possible that students did not realize that by taking fewer credits it would be impossible to graduate in two years.

Table 3

Frequencies and Chi-Square Results for Statistics Course and Two-Year Graduation (N = 430)

	One-Semester Statistics		Two-semester Statistics		$\chi^2(1)$
	n	%	*n*	%	
Graduated Within Two Years					
Yes	48	43%	76	25%	
No	63	57%	243	76%	15.13***

***$p < .001$.

Goals and Hypothesis

Full-time students should be encouraged to take 15 credits per term to complete an associate degree in two years (Complete College America, 2013). To further demonstrate the benefit of accelerated mathematics pathways, the connection between the statistics course students take their first semester of college and the two-year graduation rate was assessed. Students who took the streamlined statistics course, as well as those who took the two-semester statistics course, completed less than 30 credits and less than 60 credits, respectively, on average, after one and two years of college. Given the difference in the number of credits students were earning and size of the standard deviation, enrolling in the two-semester statistics course could lessen the chance of completing a 60-credit associate's degree in two years. Table 3 shows the number and percent of students who completed an associate's degree in two years, broken down by the statistics course taken.

The chi-squared test of independence showed there to be a statistically significant relationship between placement into an accelerated statistics pathway and completing an associate's degree in two years' time [$\chi^2(1, 430) = 15.13$, $p < .001$]. A significantly higher percentage of students who took the streamlined one-semester statistics course (43%) compared to students who took two-semester statistics course (25%) were able to complete an associate's degree in two years.

The purpose of this research was to examine the impact of offering students with strong high school average who had been assessed not to be proficient in mathematics the opportunity to take the one-semester statistics course. Because placement tests are designed to predict success in college courses, the assumption was that students deemed to be proficient, by assessing a combination of high-stakes tests and high school grades, would earn higher grades in the streamlined one-semester statistics course than students deemed not to be proficient.

Results

At the end of the semester, the grades of students deemed non-proficient and proficient were compared. Table 4 shows the number and percent of students receiving each letter grade in streamlined statistics course by proficiency status.

Students in the not proficient category were assessed not to be ready for the streamlined one-semester statistics course when using the college's proficiency index, which relies on a combination of high-stakes test scores and grade point average. To compare the groups and to assess the ability to transfer credits, three different grade ranges were used. The ranges used were: A- and higher, B- and higher and C- and higher. Three separate cutoffs were used to determine: (1) whether students would be less likely to excel, (2) if students would earn a grade that was high enough for admission into select programs, (3)

Table 4

Frequencies in Streamlined Statistics by Mathematics by Proficiency Status

Grade	Not Proficient *n*	Not Proficient %	Proficient *N*	Proficient %
A, A-	13	59%	96	38%
B+, B, B-	5	23%	52	20%
C+, C, C-	2	9%	42	16%
D	0	0%	17	7%
NC, W	2	9%	39	16%
WU	0	0%	9	4%
Total	22	100%	255	100%

Note. Grades of NC and W result when no credit is awarded. These grades do not factor into a student's GPA. Grades of F are not given in the first year. Grades of WU (withdrew unofficially) are given 0 quality points.

if the grade was high enough for the course to transfer and (4) if the grade was high enough for the course to transfer as a mathematics course rather than as an elective. Different colleges set different minimum grade requirements for courses to transfer (College affordability guide, n. d.). The grade ranges vary widely (College affordability guide, n.d.). If any difference were to be found, on the college's placement policy, the assumption was that proficient students would outperform non-proficient students, which clearly was not always the case.

The chi-squared test of independence showed there to be a statistically significant relationship between earning a grade of A- or higher in the one-semester statistics course and proficiency status [$\chi^2(1, 277) = 3.90$, $p = .048$]. In a one-semester statistics course, students assessed not to be proficient with a high school average of 83% or higher were significantly more likely to earn a grade of A- or higher compared to students assessed to be proficient. The results of Fisher's Exact Test showed there to be a statistically significant relationship between proficiency status and passing the one-semester statistics with a grade of B- or higher, $p = .040$. Students in the not proficient category were significantly more likely to earn a grade of B- or higher compared to students assessed to be proficient. Furthermore, the results of Fisher's Exact Test showed there to be no relationship between proficiency status and passing statistics with a grade of C- or higher, $p = .118$. The study was unable to conclude that students in the not proficient category were more likely to pass a one-semester statistics course with a grade of C- or higher compared with students in the proficient category.

Discussion

Because at least 50% of the differences in degree completion by race arise from the assessment and placement of students into non-credit developmental courses, placement into mathematics remediation has been called a civil rights issue (Edley, 2017). The pace of reforming assessment practices and increasing the number of mathematics pathways offered to non-STEM students to fulfill colleges' quantitative requirements has been slow. Therefore, although the American Mathematical Association of Two-year Colleges recognizes that pre-algebra and intermediate algebra need not be the prerequisite for alternative mathematics pathways (AMATYC, 2014), non-STEM students continue to be placed in long traditional remedial pathways. Faculty may believe this track provides a better foundation for all students; yet, research has shown that students are more likely to pass statistics than elementary algebra (Logue et al., 2016). Students might have been more motivated and, as a result, worked harder to complete a credit-bearing statistics course compared with a non-credit elementary algebra course. It is important to be mindful of the fact that students who complete an alternative mathematics course, depending on the major they select, may need to complete a sequence of courses leading to algebra.

This study builds upon research that has shown that the high school GPA could be used to more accurately assess and place students in college courses (Scott-Clayton & Belfield, 2015). Given the lack of alignment of placement exams, which typically focus on pre-algebra skills and statistics, this study relied on high school averages to place students into an accelerated statistics pathway instead. Although some colleges offer pathways in statistics that span multiple semesters, findings from this study (Table 4) highlight the value of placing students classified as "developmental" but also having a strong high school average directly into a one-semester statistics course. The students in this study with strong averages and originally classified as not proficient, were more likely to earn a grade of A- or higher and B- or higher in a one-semester college statistics course than those students classified as proficient. Hence, these students seemed to be appropriately placed. The results suggest that some students placed using a combination of high-stakes tests and grades were being under-placed. Two benefits of placing non-proficient students with sufficiently high averages into accelerated courses are that it removes any stigma that could be potentially associated with a slower paced course sequence and it accelerates the accumulation of credits (Table 2).

The results of this study also show that non-proficient students placed in a longer mathematics pathway do not accumulate as many college credits (Table 2) as their peers after two years. As this study has shown, the percent of students placed in the one-semester statistics course who graduated from a community college in two years (Table 3) was significantly higher than the percent of students placed in the two-semester statistics course. The data suggests students initially deemed non-proficient who had performed well academically in high school could be successful in an accelerated alternative mathematics pathway. Given the results of this study, it is unclear why some educators assert that the *best* option is to schedule courses for more hours spread out over more days for multiple semesters. Just as the "remedial" label negatively stigmatizes students, it is possible that perceptions about what "remedial" students can accomplish impacts decisions.

Limitations

Streamlined alternative statistics pathways seem more feasible now, having assessed these findings. However, there were two limitations relating to the study's design and execution. The most significant limitation was that the sample size was relatively small. Based on guidelines when using high school GPA to place students in college courses, (Belfield, 2014), only 26 students initially deemed non-proficient had an average that was high enough to be offered the opportunity to enroll in the more accelerated statistics course. Given the positive results of this study, further studies are needed with larger sample sizes to confirm the findings and to collect data over more than just one semester to measure the impact. Additionally, it is possible that students with documented learning disabilities may prefer to remain in the two-semester course and might perform better in the two-semester statistics course. Given a larger sample size, researchers may want to explore if students with documented learning disabilities would benefit from being placed in streamlined pathways. Another limitation of this study was that the pathway studied is unique and not offered at all colleges. The positive results from altering placement procedures for a statistics pathway may not apply to other mathematics pathways. Research that examines if these results could be extended to other alternative mathematics pathways or more traditional pathways would seem to be warranted, because it is beneficial for students to accumulate credits at a more accelerated rate.

Conclusion

This study demonstrated that altering the means of assessing and placing students and offering non-STEM students the opportunity to complete streamlined alternative mathematics pathways helps students excel and accumulate credits in an expeditious manner. Adding days and hours to courses can disadvantage students by making it more challenging to find part-time employment or by failing to fully develop the independence needed as students progress to more advanced courses. It is critical that colleges find ways to remove barriers preventing non-STEM students from satisfying general quantitative requirements for graduation, while simultaneously encouraging those students with an interest in majoring in STEM, to remain in these fields.

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Inverted Tasks and Bracketed Tasks in Mathematical Problem Posing

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ABSTRACT We present in this paper a pair of approaches to support mathematics educators and learners in formulating original tasks. In particular, we facilitate the posing of rich mathematical problems by using two novel methods that were created by a mathematics department at a K-12 school in the United States, and further developed alongside our students as well as a wider professional learning team of master teachers. We situate our work within the broader literature on mathematical problem posing and describe our strategies by including examples of their use in generating problems and by providing examples of authentic student-assigned tasks that were created with our approaches.

KEYWORDS *problem posing, assessment, evaluation, task design, inverted task, bracketed task*

Introduction

The literature on mathematical problem posing can be traced back at least to Polya's (1945) "How to Solve It," for which many of the heuristics around problem solving involve the asking of questions: *What is a related problem? What is a simpler problem? How can the problem be generalized?* The observation that problem solving involves posing, or reformulating, problems dates back at least to another work of the same year as remarked by Kilpatrick in Schoenfeld (1987, p. 125):

> Wherever the problem comes from, the problem solver is always obliged to reformulate it. In fact, as Duncker (1945) pointed out, one can think of problem solving itself as consisting of successive reformulations of an initial problem.

Other classical works on problem posing include Silver's (1994) article that connects problem posing with creativity, and Brown and Walter's (1983) treatise, "The Art of Problem Posing," in which an explicit model is described for modifying a given scenario (e.g., the graph of $y = x^2 + 6x + 9$) to create original problems: attribute listing, which involves writing out the various traits and characteristics of the initial setup (e.g., this quadratic function has one real root; the y-intercept is at $y = 9$); what-if-not-ing, which involves the new scenarios produced by asking about the changes effected if an attribute were assumed not to hold (e.g., What if the quadratic function had two real roots? What if the y-intercept were not 9?); and cycling, which involves combining multiple variations on a scenario to create a novel problem (e.g., What is a quadratic function that has two distinct real roots and a y-intercept at $y = 8$?).

We build on this earlier work with a pair of approaches to problem posing: the first approach, which we refer to as *inverted problem posing,* involves a new perspective on the relation between inputs, outputs, and methods/algorithms in mathematics courses; the second approach, which we refer to as *bracketed problem posing*, involves an approach to encouraging problem posing among teachers and learners of mathematics, and which we have incorporated into our own assessments. In doing so, we aim to answer the call of Kilpatrick that "problem formulating should be viewed not only as a goal of instruction, but also as a means of instruction. The experience of discovering and creating one's own mathematics problems ought to be part of every student's education" (1987, p. 123).

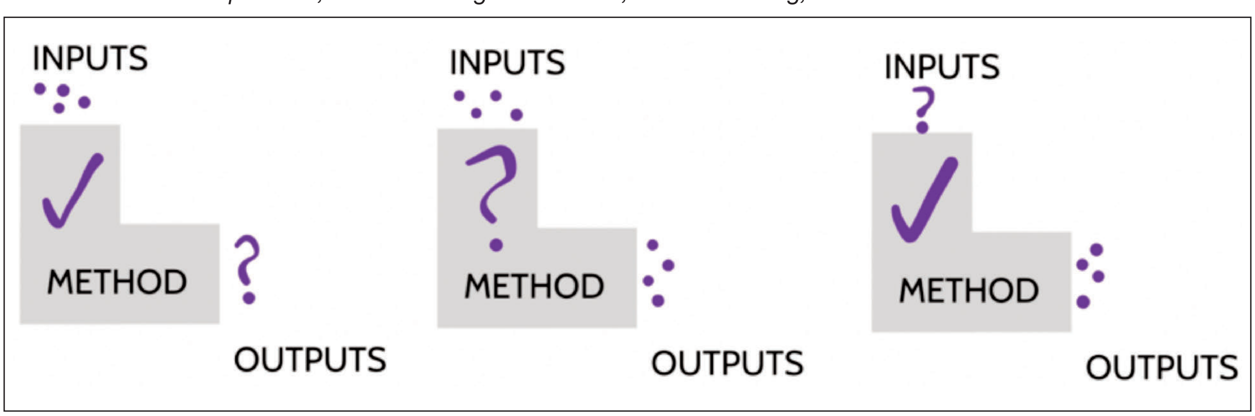
Inverted Problem Posing

We begin (see Figure 1) by characterizing three different approaches to mathematical problems in terms of their input (what is given), method of solution (technique or algorithm), and output (result of running the input through the method). For example, the input could be "4 and 6," the algorithm could be one to find the least common multiple (LCM), and the output would, therefore, be LCM(4,6) = 12. In traditional approaches to teaching mathematics, instructors might teach a method first (e.g., related to the prime factors) and then have students practice by giving them lots of inputs (here, lots of number pairs); so, the inputs are given, the method is known, and various outputs are found through a sequence of exercises. In a problem solving approach, instructors may set up an exploration in which the inputs are given or can be chosen, and students can find specific outputs. The challenge for students becomes the creation of algorithms or methods, scaffolded as necessary, to generalize their solution technique; so, individual inputs and outputs are available, but the method is to be found through investigation. For example, students may compute the LCM of many pairs of numbers and look for patterns to develop an algorithm that yields the LCM in general. We use *inverted problem posing* to refer to a different model: once students have understood a method, we provide sample outputs and ask for the possible inputs. For example, rather than providing natural numbers a and b for which LCM(a,b) = 12, we ask: Given that LCM(a,b) = 12, what could be the values of a and b? One consequence of this approach to problem posing is that the inversion may produce multiple inputs; in our example here, we could have, e.g., $a = 4$ and $b = 6$, or $a = 2$ and $b = 12$. Further employing Brown and Walter's (1983) scheme (what if there were not multiple answers?) can allow one to formulate problems with unique answers: one workaround is simply to ask for the set of all possible solutions; another approach is to impose additional constraints, e.g., for what a and b satisfying LCM(a,b) = 12 do we minimize $a + b$?

One of the advantages of inverted problem posing is that instructors can deploy this strategy whenever a problem has been solved, or an attribute has been noticed. For example, we used the Brown and Walter (1983) scheme to phrase our formulation of an earlier problem: What is a quadratic function that has two distinct real roots and a y-intercept at $y = 8$? This required a combination of attribute listing, what-if-not-ing, and cycling. The same question could, instead, be viewed as an inverted problem from the routine question, "What are the two roots and y-intercept of the quadratic function $y = x^2 + 6x + 8$?" Moreover, we can notice and incorporate additional features of this parabolic curve to pose richer problems. For example, the x-intercepts in this graph are at $x = -2$ and $x = -4$; so, they have a distance of 2 between them as measured along the x-axis. Given this additional observation, we may now consider the inverted problem that asks, "What is a quadratic function that has x-intercepts that are 2 apart, and a y-intercept at $y = 8$?" We can further enrich the problem by asking for all the quadratic functions that satisfy these criteria. Calling one root p, we have roots p and $2 + p$. Thus, the quadratic function has the form $y = a(x - p)(x - (2 + p))$, which has constant term $ap(2 + p) = 8$, i.e., $a = 8/(p(2 + p))$. An animated graph of this function, for which the parameter p varies between –5 and 5, can be found on Desmos [https://www.desmos.com/calculator/tr8yrpqalh]. In this way, we have taken the common context of quadratic functions and observations around standard features of their graphs, x-intercepts and the y-intercept, and inverted this information so as to pose a non-routine problem with a low-floor (e.g., give one example) and high-ceiling (e.g., parameterize all such functions).

Figure 1

Models of Problem Exploration, from Left to Right: Traditional, Problem Solving, Inverted



Bracketed Problem Posing

The second method of problem posing described here also builds outwards from standard problems. The general idea is captured by the following directions, which have been used across courses ranging from middle school algebra to high school calculus at the author's institution, as well as by additional master teacher fellows who attended a professional development course ["Problem Posing in Algebra Assessments"] co-facilitated by the author through the organization *Math for America*.

> **Directions:** For each of the following problems, please change only the portion in brackets to: (1) create a similar problem; (2) solve your similar problem; and (3) explain briefly how your problem is similar to the original.

The goal with these directions is to scaffold student problem posing by providing a framework within which they can create their own problems and encourage students to attend to structural rather than superficial features of mathematical tasks. The problem we developed in our previous question could be bracketed for students as follows:

> Show that the two x-intercepts of the graph of the quadratic function $y = [1]x^2 + [6]x + 8$ have a distance of 2 between them as measured along the x-axis.

The placement of the brackets is nontrivial and requires teacher expertise in thinking through what the resulting solution set might look like, how difficult the resulting problem is, and the potential for student misunderstanding of what is being asked. In the example above, directly answering the question is only a matter of finding the real roots (e.g., by factoring) and then verifying that they have a difference of 2. Much more challenging is for students to think through how the degree 2 and 1 coefficients can be chosen to yield another quadratic with real roots that differ by 2. In some cases, this may lead students into an exploration that looks more like our high-ceiling parameterization of the previous section; for others, finding a solution might involve playing with the constraints (e.g., observing $y = x^2 - 1$ has real roots that are 2 apart and multiplying through by -8 to get the desired y-intercept but not change the x-intercepts, i.e., using the quadratic $y = -8x^2 + 8$ as a solution). The considerations of where to incorporate bracketed tasks into a curriculum, how to grade or otherwise use summative assessments (if at all) of student work, and ways of formulating these tasks as a function of teacher goals all fall outside the scope of this brief paper. Instead, we close with a bracketed Algebra 2 examination that was given to students; sample copies of student work are available by request, as are corresponding materials from the professional development course referenced previously.

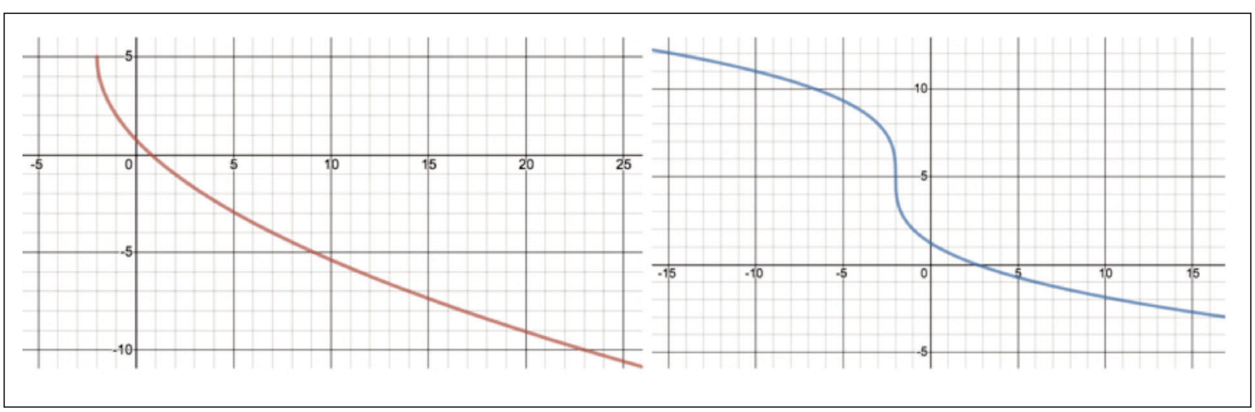
> **PROBLEM 1:** A polynomial with degree 4 has imaginary roots [$2i$ and $3i$]. Give two different possibilities for the polynomial; ensure that your examples do not both have the same end behavior.

> **PROBLEM 2:** Carry out the following polynomial division, by a quadratic expression, by hand in any manner that you wish, and verify that the quotient has a remainder of zero:
> [$(x^6 + 3x^5 - 3x^3 + 6x^2 + 9x + 2) \div (x^2 + 3x + 2)$]

> **PROBLEM 3**: Explain carefully which of the following two questions you would prefer on an in-class test, but you do **not** need to answer either of them.
>
> QUESTION A: Find all the rational roots of
> [$f(x) = x^5 - x^4 + 2x^3 - 5x^2 + 1$]
>
> QUESTION B: Find all the rational roots of
> [$g(x) = 2x^3 - 2x + 12$]

Figure 2

Two Bracketed Graphs of Functions Expressible as $y = a(x - h)^{1/n} + k$



PROBLEM 4: Write down radical functions corresponding to the two graphs in Figure 2 and explain carefully how you arrived at your answers. Consider the entire graphs bracketed!

Conclusion

In this paper, we named and briefly discussed two approaches to problem posing: inverted tasks and bracketed tasks. The former involves a reframing of how tasks are created in mathematics classrooms that is situated outside the binary of traditional instruction (tell methods, complete exercises) and certain problem solving alternatives (explore problems, discover methods). The latter involves a scaffolded approach to support student problem posing that encourages learners to focus on structural, rather than superficial, problem features. Our discussion of each is incomplete, and we look forward to developing and refining both—in theory, in practice, and in writing—as our thinking further evolves.

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Author's Note

This article was written for Topic Study Group 17, "Problem posing and solving in mathematics education," which was to be chaired by Teachers College alum Edward A. Silver at ICME-14 in Shanghai, China, during the Summer of 2020. However, the quadrennial International Congress on Mathematical Education has been postponed by at least a year due to the ongoing COVID-19 pandemic. As a bit of background for this 10th anniversary issue of the Journal of Mathematics Education at Teachers College: I arrived as a new doctoral student at Teachers College one decade ago, after spending the better part of the previous two years living in Nanjing, China, which I first called home during a 2008-09 Fulbright Fellowship to research Chinese mathematics teacher education. I am grateful to have had the opportunity, supported initially by the US Department of State, to live in China and form deep friendships during my time abroad. I am grateful to Teachers College for supporting me as an instructor for the graduate course "Teaching Mathematics in Diverse Cultures" while bringing a cohort of doctoral students to Shanghai during a study tour in Summer 2013. I am grateful to my present institution, The Hewitt School, for supporting me in language education studies at Nanjing Normal University in Summer 2017. It is within this context that I must also articulate my deep concern about the xenophobic/Sinophobic and racist rhetoric from United States politicians who were ostensibly elected to lead. My sincere hope is that when we look back in another decade we will see great progress and meaningful structural change, and that the present modes of targeting and othering individuals based on their identities—including, but not limited to, Asian Americans who have faced a recent uptick in hate crimes—will diminish. Yet, hope is sustained by shifting from thought to action, and few actors can be as powerful—and empowering—as educators.

Benjamin Dickman, April 15, 2020

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Benjamin Dickman majored in Mathematics at Amherst College and earned his Ph.D. in Mathematics Education from Columbia University. He studied as a Fulbright Fellow in Mathematics at Nanjing Normal University in Nanjing, China, and worked as a Postdoctoral Scholar in Math Education at Boston University. Benjamin is currently a math teacher and teacher coach at The Hewitt School in New York, NY. His interests include mathematical creativity, problem posing, and rehumanizing mathematics.

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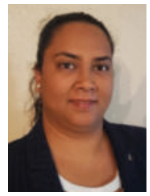

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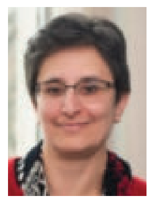

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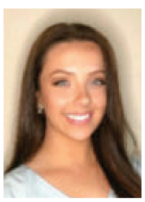

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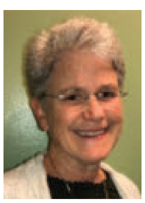

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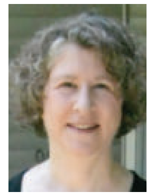

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Jonathan Thomas is an associate professor of mathematics education and chair of the department of STEM Education at the University of Kentucky. Dr. Thomas' research is focused on responsive mathematics teaching practices and children's mathematical cognition.

ACKNOWLEDGEMENT OF REVIEWERS

The Editorial Board would like to acknowledge the following reviewers for their effort and support in reviewing articles for this issue of the *Journal of Mathematics Education at Teachers College.* Without the help of these professionals, it would be impossible to maintain the high standards expected of our peer-reviewed journal.

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