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# JOURNAL OF MATHEMATICS EDUCATION AT TEACHERS COLLEGE

*A Century of Leadership in Mathematics and Its Teaching*

## Examining Practices and Resources from Mathematics Classrooms

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The *Journal of Mathematics Education at Teachers College (JMETC)* is a recreation of an earlier publication by the Program in Mathematics Education at Teachers College, Columbia University. As a peer-reviewed, semiannual journal, it is intended to provide dissemination opportunities for writers of practice-based or research contributions to the general field of mathematics education. Although many of the past issues of *JMETC* focused on a theme, the journal accepts articles related to any current topic in mathematics education, from which pertinent themes for future issues may be developed.

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AT TEACHERS COLLEGE

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# TABLE OF CONTENTS

# PREFACE

The Spring 2022 Issue of the *Journal of Mathematics Education at Teachers College* features three articles that discuss a broad range of topics, from engaging students in peer feedback and supporting positive mathematical identities, to the impact of reform efforts on a chapter in a calculus textbook. Additionally, we have two *Notes from the Field* articles that reflect on the practitioners' implementation of Algebra activities and teaching strategies. In this edition, educators will find research and descriptions of specific tasks designed to meaningfully engage students in mathematics learning, as well as some of their challenges.

Husband and Nikfarjam begin with a report on the results of a study they conducted on the impacts of peer feedback in an elementary mathematics classroom. To expand on the research of peer feedback, Husband and Nikfarjam investigated the possible benefits for the providers of peer feedback. By collecting and examining student-to-student feedback on a number of mathematical tasks and activities, they found that when students are directed to "comment on the mathematics," students who provide feedback have the opportunity to self-reflect, make connections, and engage in mathematical discourse.

After providing an overview of the current state of research in the field of mathematical identity, Barba invites readers to consider a variety of research-based practices that mathematics educators can employ to promote students' mathematical identities. Engaging students in such activities, Barba contends, may increase interest, as well as success in Science, Technology, Engineering, and Mathematics (STEM). Moreover, it may help students to see themselves as doers of mathematics and, therefore, contributing members of a mathematics community.

Lastly, Pogorelova investigated calculus textbook reform by evaluating the contents of a chapter in a reform-minded calculus textbook. By identifying and analyzing the type and characteristics of questions contained in the chapter, Pogorelova described how well the contents aligned to typical reform-based mathematics teaching and learning practices. The results showed that while a number of important reform-based strategies were employed, there was also a larger than expected amount of more traditional practices. Pogorelva also reflects on some of the challenges faced by reform textbook writers.

Each of these articles highlight important aspects of mathematics teaching and learning. The authors remind all educators to reflect on the strategies and practices we use to engage our students, as well as the physical materials we often rely on.

Ms. Alyssa MacMahon
Mr. Davidson Barr

*Guest Editors*

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# Peer Feedback in the Mathematics Classroom

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**ABSTRACT** This study explores peer feedback in a combined fifth and sixth-grade classroom. Drawing on Hattie and Timperley's (2007) model for feedback, we analyzed 334 peer feedback comments gathered during six mathematics lessons. Our analysis revealed evidence of peer feedback being beneficial to the students who provide it as well as those who receive it. Specifically, we share examples of how peer feedback can support opportunities for providers of feedback to 1) self-regulate by choosing mathematics strategies, 2) make connections between their own mathematical ideas and those of their peers, and 3) engage in ongoing back-and-forth conversations. Findings from our study point to recommendations for teachers to be more purposeful in their prompts to students about the types of feedback they might provide one another.

**KEYWORDS** *feedback, peer feedback, classroom practices, elementary mathematics*

## Introduction

Peer feedback is a specialized form of feedback that is "provided by equal status learners" (Gielen et al., 2010, p. 305). In the classroom, this translates into students providing each other with feedback, rather than the teacher. Peer feedback is an important feature of peer learning (Falchikov, 2001; Topping, 2005, 2009) as well as formative assessment processes (Black & Wiliam, 1998). Researchers suggest that peer feedback can be comparable to teacher feedback when the goals are clear and the criteria are set (Falchikov & Goldfinch, 2000; Hamer et al., 2015). As mathematics educators, we are interested in exploring peer feedback in an elementary mathematics classroom—what it looks like, sounds like, and feels like for teachers and students. Despite extensive literature on feedback in general, research on peer feedback is still in its youth (Kollar & Fisher, 2010). Of the limited research that has investigated the topic of peer feedback, studies have examined how peer feedback relates to teacher feedback (Falchikov & Goldfinch, 2000), effectiveness of peer feedback for learning (Gielen et al., 2010), and how peers perceive the feedback provided by their peers (Strijbos et al., 2010). Much of the literature about feedback has focused on potential benefits to the receiver. In contrast, this paper analyzes feedback for traces of there being mutually reciprocal benefits to the provider of feedback. In this pursuit, we will share examples of peer feedback that unfolded during mathematics lessons in a combined fifth and sixth-grade classroom. Our examples show how peer feedback can benefit the student who provides the feedback by 1) supporting their self-regulation in choosing mathematics strategies, 2) facilitating making connections between their own mathematical ideas and those of their peers, and 3) offering the opportunity to engage in ongoing back-and-forth conversations. Finally, we will discuss how teachers can be purposeful and more explicit when guiding students to provide peer feedback.

## Description of the Project

This study was part of a larger collaborative research project between researchers at a large urban university and an inservice teacher at an elementary school. Taking place over the course of four months, the larger study gathered data from six lessons, each lesson focusing on one mathematics task. The tasks ranged across a variety of topics, including geometric growing patterns, proportional reasoning, and tasks that involve using data to investigate mean, median, and mode. The study reported here uses data from all six lessons.

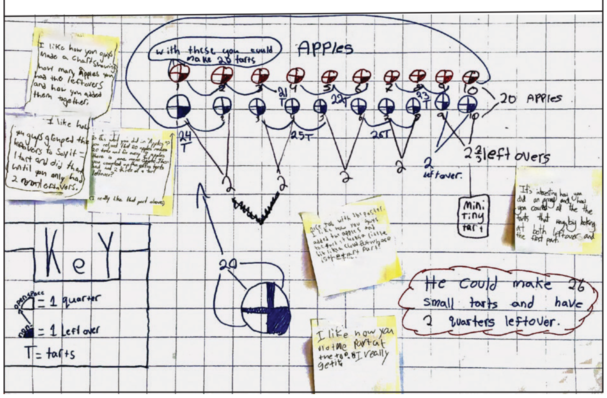
Within this study, we report on the feedback the students in a combined fifth and sixth grade mathematics classroom provided to one another. The teacher structured the observed lessons so that first, students worked in pairs on a mathematics task to co-create posters that displayed their thought process. Then, following the completion of the posters, students were directed to review one to two other posters and provide written feedback on sticky notes (see Figure 1).

Writing phrases such as "good work" or "I like your poster" was discouraged because the teacher wanted her students to focus feedback on the mathematics. This was evidenced by the teacher's prompt for her students to "comment on the mathematics."

**Figure 1**

*Example of a Poster Given a Mathematics Task*

**Task:** Dad makes small apple tarts using three-quarters of an apple for each small tart. He has 20 apples. How many small apple tarts can he make?




## A Model for Peer Feedback

To analyze our data, we first needed a model or framework that would help us categorize and describe different types of peer feedback provided by the students. A review of the literature revealed that there was not a well-developed and generally agreed-upon model for categorizing and describing peer feedback. In the absence of such a model, we decided to apply Hattie and Timperley's (2007) model of feedback. The use of this model seemed appropriate as their definition of feedback acknowledges peers as potential generators of feedback. Furthermore, other researchers (e.g., Harris et al., 2014) have also used Hattie and Timperley's (2007) model to study peer feedback. Hattie and Timperley's (2007) model for feedback consists of four non-hierarchical levels, in which each level describes the focus of the feedback (see Table 1).

Our coding process involved identifying statements and/or elements in each of the feedback comments that related to the descriptions for the four levels of feedback. This was an iterative process where the two researchers individually coded the data and then compared findings with the purpose of seeking agreement. There were instances where we did not agree on the coding of a particular feedback comment. To resolve these discrepancies we continually referred back to Hattie and Timperley's (2007) model and examined the criteria and description for each level and discussed the comments in relation to the mathematics on the poster.

**Table 1**

*Hattie and Timperley's (2007) Levels of Feedback*

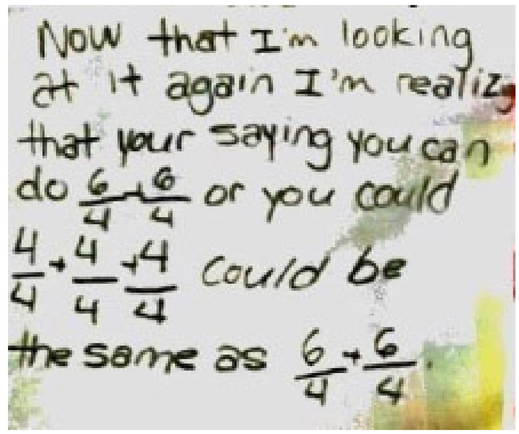
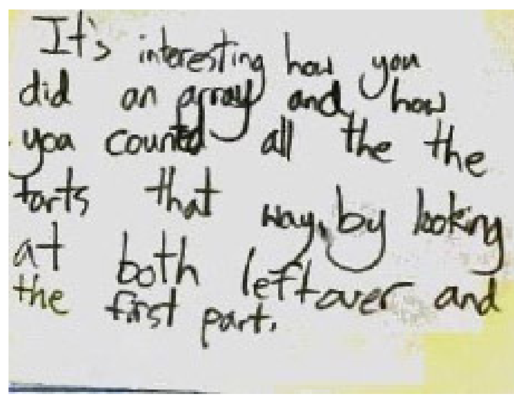
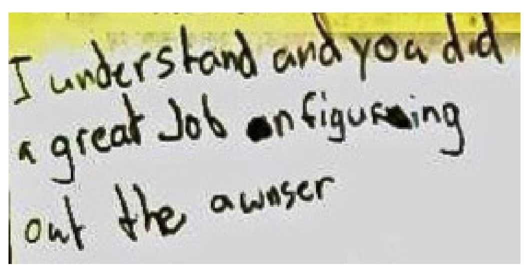
| Feedback Level | Description |
|---|---|
| Feedback on Task (FT) | Feedback about the learner's presentation, organization, and correctness of the task. |
| Feedback on Process (FP) | Feedback about the learner's thinking and strategies. |
| Feedback on Self (FS) | Feedback that is about the learner and not the task. |
| Feedback on Self-regulation (FR) | Feedback that engages the learner in self-regulation. |

## Analysis and Findings

At the end of each lesson, we took photos of all the posters that were created by the students. Each poster contained between four and six sticky notes that were used to provide peer feedback (see Figure 1). Based on the descriptions for each level of feedback, we then coded the students' peer feedback comments using the four levels of FT, FP, FR, FS. We found that out of 334 peer feedback comments, 282 (84%) were feedback on task (FT), 26 (8%) were feedback on process (FP), 26 (8%) were feedback on self (FS), and 0 (0%) were feedback on self-regulation (FR). Table 2 provides examples for each of the four levels of peer feedback.

Table 2

*Examples of Peer Feedback Categories*

| Feedback Level | Example |
|---|---|
| Feedback on Task (FT) |   |
| Feedback on Process (FP) |   |
| Feedback on Self (FS) |   |
| Feedback on Self-regulation (FR) | We found no examples of feedback that engages the receiver in self-regulation. |

Our initial analysis of the data revealed that some peer feedback comments had elements that could arguably place them in more than one of the four levels described in Hattie and Timperley's (2007) model. Being provided with more than one level of feedback is beneficial to learners because it allows more opportunities for taking up feedback. Our analysis of the comments also revealed that the benefits of the feedback were not exclusive to the students receiving them. In fact, some of these comments had the potential to benefit the person providing the feedback.

## Peer Feedback that Benefits the Provider

As a peer-to-peer form of communication, peer feedback has the potential to support both the providers and the receivers. Below, we share examples of comments that offered the feedback provider opportunities to 1) self-regulate, 2) make connections between mathematical ideas, and 3) elicit further conversations with peers.

### Self-Regulation: "I'll use your strategy next time!"

Self-regulation involves self-assessment, self-appraisal, and self-management (Hattie & Timperley, 2007). Self-regulation includes thoughts, feelings, and actions that support the attainment of personal goals (Zimmerman, 2000). For example, students engaged in self-regulation might consider accepting feedback provided to them or seek further information in order to better understand and apply the feedback. In this study, we did not find samples of peer feedback that would engage the student receiving the feedback in self-regulation. However, we noticed peer feedback comments that illustrated self-regulation on behalf of the student who provided the feedback, which we have coded as *Feedback on Self-regulation on behalf of the Provider* (FR-P).

**Example 1:** "You [have a] unique way of doing this poster! Me and my partner never did that kind of work! Maybe I'll use your strategy next time!"

The student begins by complimenting the uniqueness of the poster (FS), goes on to self-assess by sharing that they had never used that strategy (FR-P), and concludes by considering using that strategy next time. The comment, "I'll use your strategy next time" demonstrates how the student providing the feedback may be reconsidering their own problem solving strategies and, therefore, engaging in self-regulation. We think this is important because it exemplifies how peer feedback can serve the student who is providing it, not just the one receiving it.

### Connection-Making Feedback: "Me and my partner did …"

To grow mathematical understanding, the National Council of Teachers of Mathematics (NCTM) recommends that teachers provide opportunities for students to analyze and compare one another's mathematical approaches and arguments (2014). There is a personal quality to peer-to-peer feedback that helps students connect their ideas with those of their peers. While analyzing our data, we noticed that almost all of the peer feedback comments started with "I" statements such as "I like….", "I understand…", "I don't see how…," etc. In the following two examples of peer feedback comments, students made connections between their strategies and what they saw on the poster. We coded these comments as *Feedback on Connection-Making* (FCM).

**Example 2:** "I liked how you showed the bottom that one block + 1 block + 1 block = figure 2. I also like how you showed that on figure 2 there is 2 on the side and it keeps going, figure 3 has 3 on the side. Me and my partner did the same thing."

**Example 3:** "I like how you did the odd and even pattern and how 1 and 3 are odd and 6 and 10 are even. I also like how you did the old blocks and the new blocks. Me and my partner did something like the odd and even except we used the figure number as the odd and even instead of the full figure."

In both examples, the students describe what they see on the poster and compare it to their own solution strategy. In Example 2, the student providing the feedback appears to be making a connection (FCM) by noticing the same solution strategy: "me and my partner did the same thing." In Example 3, the student providing the feedback also appears to be making a connection (FCM) to their work by noticing a similar, yet slightly different, way of seeing the pattern by noting that they "used the figure number as the odd and even instead of the full figure."

The examples outlined here demonstrate how peer feedback facilitates the students providing peer feedback to make connections and, potentially, deepen their understanding of the mathematical concept.

### Elicit Further Conversation: "Talk to me after."

NCTM' (2014) *Principles to Action* recommends that teachers facilitate meaningful mathematical discourse. Such discourse supports students to a) clarify understandings, b) construct convincing arguments, c) develop language to communicate mathematical ideas, and d) learn to see things from different perspectives. In this study, of the 334 comments, 47 were either in the form of asking questions or eliciting a conversation between peers. We coded these comments as *Feedback as Conversation* (FC). The following examples show how peer feedback can support the beginnings of a mathematical conversation:

**Example 4:** "Me and my partner noticed the same thing that the 3 apples makes 4 tarts. Is this what you mean 26 ½ apples of ½ tarts (talk to me after)"

**Example 5:** "... I don't really understand the top left. Can you explain?"

In Example 4, the student shared a connection between their solution strategy and what their peers did (FCM). Then, they posed a question and invited their peers to talk afterward (FC). In Example 5, the student communicated their lack of understanding of a part of the strategy on the poster and asked for more explanation (FC). The comments, "talk to me after," and "can you explain" invite both students, the provider and the receiver of peer feedback, to have a future conversation, creating the conditions for further action. This is significant because it highlights how peer feedback can evolve from one-directional communication to an exchange of ideas, thus supporting student achievement (Lau et al., 2009).

## Discussion

Our use of Hattie and Timperley's (2007) model to analyze peer feedback comments revealed an uneven distribution of data among the four levels. Specifically, 84% of the comments belonged to the FT level. We think one possible reason for the large number of FTs could be attributed to the teacher's "focus on the mathematics" prompt at the launch of the feedback activity. Similarly, the teacher's decision to discourage students to use phrases such as "good work" or "I like your poster," could explain the comparably fewer number of comments that belong to the FS level. These results inspired us to consider how teachers can be more purposeful in instructing their students about the types of feedback they might provide one another. For instance, instead of simply telling students not to say, "I like your poster" or "good work," teachers can prompt students by asking questions such as the following:

- What can you learn from your peers' work? Are there ideas in the work that inspire you to do something different next time?
- How is the mathematics communicated on the poster similar to or different from your work? Specifically, how do your peers' strategies compare with the strategies you used?
- Is there something in the poster that you want to know more about or talk further about?

Prompting students with questions like these may elicit a range of feedback comments from students, thus promoting a more even distribution among the four levels. This is important because each level of feedback focuses on a different aspect for growth. Having a more even distribution of feedback levels offers learners more opportunities to use the feedback and improve their work.

## Conclusion

Hattie and Timperley's (2007) model is designed for studying and categorizing feedback provided by teachers. Consequently, when describing the levels of feedback, they only consider the benefits to the student who receives the feedback. However, as this study examines feedback provided by and for peers, it is only natural that both the provider and the receiver of feedback are impacted by the process. Our analysis of the examples indicate that peer feedback has the potential to benefit the students who provide feedback and those who receive it. Specifically, peer feedback may support the provider's ability to self-regulate, make connections, as well as present opportunities for mathematical discourse among peers.

We recognize that our study was limited by its scope and duration—six lessons in one classroom. However, thinking deeply about what students are saying and doing while providing peer feedback has expanded our perceptions of the potential for using peer feedback routinely in mathematics lessons. We hope this study inspires further inquiries into peer feedback and the ways that teachers can support students to engage in reciprocal feedback processes. Specifically, future studies might investigate how teachers and students can co-create criteria for peer feedback.

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# Mathematical Identity and the Role of the Educator

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**ABSTRACT** Mathematical identity is a socio-motivational construct known to be a predictor of mathematical achievement. Students who identify positively with mathematics are more likely to pursue advanced courses and Science, Technology, Engineering, and Mathematics (STEM)-related occupations. Although mathematical identity is shaped by a myriad of internal and external factors on both a small and large scale, educators play a significant role in the formation of their students' mathematical identities. This paper presents an overview of research and theory regarding those pedagogical practices for teaching mathematics that can foster the formation of positive mathematical identities.

**KEYWORDS** *mathematical identity, mathematical learning, pedagogical practices*

## Mathematical Identity and Mathematical Learning

Lauded by Gee (2000) as an analytic lens for research in education, mathematical identity is a burgeoning topic of study that has become increasingly prominent over the past two decades. Since its emergence on the scene during what has been coined by Lerman (2000) as the "social turn in mathematics education," an operational definition for mathematical identity has evolved to a "narrative rendering" of participative experiences that equates storytelling with identity-building (Sfard & Prusak, 2005). As a result, researchers have come to define mathematical identity as a socio-motivational construct that refers to the dispositions and deeply ingrained beliefs regarding one's ability to participate and perform effectively in mathematical contexts as a learner and user of mathematics (Bohrnstedt et al., 2020; Dingman et al., 2019; Martin, 2009).

Since identity-making is a "communicational practice" over which humans are active agents (Sfard & Prusak, 2005), social interactions are critical to the formation and perpetuation of mathematical identities. Every occasion for communication is an opportunity to (re)construct and (re)negotiate self-images through the discursive positioning of self and others (Davies & Harré, 2001; Waring, 2018). Although moments for mathematical identity formation are ubiquitous to daily life, of particular importance are those experiences that occur within the classroom setting (Anderson, 2007). In fact, teachers' roles in shaping their students' mathematical identities can be dramatic (Martin, 2009) because teachers communicate to their students what mathematics is, what learning mathematics entails, and who is considered a doer of mathematics. Indeed, extant research in sociocultural learning theory emphasizes the significant impact, both short term and long term, that teachers have in shaping identity, suggesting that learning and identity development are intrinsically intertwined (Martin, 2009). The National Council of Teachers of Mathematics (NCTM) believes mathematics educators can leverage students' identities to enhance mathematical learning (2014). As a result, in 2016 the NCTM renewed its focus on the ability and responsibility of teachers to foster positive student mathematical identity by reframing their principle of access and equity to "capture the additional and critical constructs of students' mathematical identities, students' sense of agency, and the teaching of mathematics for social justice" (Larson, 2018, para. 2).

Anderson (2007) describes learning mathematics as a "complex endeavor" consisting of three dimensions: (1) the development and application of skills, algorithms, and procedures; (2) the construction and acquisition of mathematics knowledge; and (3) the participation in social interactions that influence thoughts, actions, and membership within communities. Identity formation is a critical component of the third dimension wherein students "must participate within mathematical communities in such a way as to see themselves and be viewed by others as valuable members of those communities" (Anderson, 2007, p.8). Teachers are both architects and stewards of mathematical communities in the classroom, responsible for designing and supervising an environment that encourages students to view themselves and others as valued members and contributors.

Boaler (2002), likewise, views learning as a process composed of three components in which knowledge, classroom practice, and identity are interrelated in a disciplinary relationship. In her study with Advanced Placement Calculus students, Boaler (2002) found that classes with teacher-centered instruction positioned students as "received knowers," in which the role of the student was to passively receive mathematical knowledge. The teacher and the textbook were described as the mathematical authority, meaning that they were relied on as the holders of all mathematical knowledge (Boaler, 2002). Students classified as received knowers were more likely to dislike and disengage from mathematics, often seeking out academic opportunities in other disciplines that offered more interpretation and freedom to express their ideas (Boaler, 2002). Ultimately, the lack of classroom discourse left students with few opportunities to develop a positive mathematical identity. In contrast, students in discussion-centered classes formed a relationship with mathematics that gave them agency and authority over the learning process (Boaler, 2002). Rather than viewing mathematical learning as a mere reproduction of standard procedures, they understood their active role in the process and made plans to continue studying mathematics; notably, mathematical authority was conferred to the students whose opinions, ideas, and conjectures served as valuable contributions to the learning of mathematics (Boaler, 2002). When faced with challenging problems, those students performed a "dance of agency" in which they moved freely between established methods and their own creative process (Boaler, 2002). As a result, they not only mastered the content but, more importantly, appropriated the mathematical knowledge – a key component to the formation of positive mathematical identities. Fortunately, the positive results elicited by the discussion-centered pedagogical practices of the educators in Boaler's (2002) study can be replicated in any mathematics classroom through the adoption of similar practices.

## Pedagogical Practices That Promote Positive Mathematical Identities

"Students do not just learn mathematics in school classrooms, they learn to be" (Boaler & Greeno, 2000, p.188). Teaching mathematics is more than just the dissemination of content and the development of mathematical skills. It is also about empowering students to see themselves as participants and doers of mathematics, facilitating motivation and interest through opportunities of engagement and discourse in the classroom, and helping students to understand the value and relevance of mathematics in their own lives (Miller & Wang, 2019; Wang, 2012). Since the role of the teacher is critical to the formation of mathematical identities, educators should regularly engage in practices that foster positive associations with mathematics. Several of these pedagogical practices are described below.

### Fostering the Four Components of Identity

According to Anderson (2007), there are four components of identity associated with mathematical learning that a teacher can enforce through intentionally designed mathematical tasks: engagement, imagination, alignment, and nature. He argues that students engage in mathematics when given the opportunity to explore and develop their own problem solving strategies. Through this experience, they not only connect with their classroom community but are also recognized as members and contributors to the greater mathematical community. Anderson (2007) describes imagination as the way in which students envision those engaging mathematical activities as fitting into their daily lives, as well as their future educational pursuits and careers. Anderson (2007) believes that when students respond to the imagine component of identity, they align their preconceived notions to conform with institutional expectations and behave in ways that align with a specific type of person (i.e., a "mathematics person"). Finally, Anderson (2007) describes the nature component of identity as being directly related to fixed mindset beliefs and the existence of a mathematics gene. To aid in the development of positive mathematical identities for all students, Anderson (2007) argues that teachers should discount the nature component, which can often lead to unfounded explanations for student participation or success in the mathematical community. Several of Anderson's (2007) recommendations for educators to foster positive mathematical identities are described in Table 1.

Table 1

*Methods for Fostering Positive Mathematical Identities*

| Anderson's (2007) Recommendation | General Strategy | Specific Examples |
| --- | --- | --- |
| **Use meaningful tasks** | Incorporate open-ended questions or projects that allow students to develop strategies, make use of mathematical tools, and focus on explanations over the brevity of responses. | A swimming pool project in which students explore concepts of surface area and volume through the design of a swimming pool according to particular standards or constraints.<br><br>An exercise in concept attainment that asks students to look at examples of polygons and non-polygons to generalize a rule for what constitutes a polygon (Brahier, 2016). |
| **Establish classroom norms centered on discourse** | Incorporate classroom expectations that encourage collaboration wherein the teacher comprises the role of facilitator. | Questions can be elicited from students by providing them with "Student Question Cards" that model various types of questioning techniques, such as "Why do you do that?" and "How does this compare to …?" (Brahier, 2016).<br><br>Teachers can demonstrate ways for students to listen with intention by creating a poster that describes specific actions students can take to (1) pay close attention, (2) show that they are listening, (3) provide feedback, and (4) respond appropriately (Hattie et al., 2016).<br><br>Teachers can encourage student contributions through the utilization of checklists that prompt students to consider ways that they can contribute to the learning process, such as "Have you considered looking for another way to solve the task?" (Hattie et al., 2016). |
| **Provide occasions for students to reflect upon their mathematical journey** | Incorporate success criteria, journal prompts, or concept maps. | At the culmination of a lesson, teachers can ask students to respond to "I can…" statements, such as "I can explain why a zero exponent produces a value of one," to make their learning, and the progress they've made, more visible (Hattie et al., 2016).<br><br>At the end of an assessment, teachers can ask students to write in their journals in response to the prompt, "How do you think you performed on today's test? What was the easiest part? What was the hardest part? Would you make any adjustments to how you prepared for the test? Why?" (Brahier, 2016).<br><br>In a Discrete Mathematics course, students can create a concept map for proof-writing strategies based on the logical form of the conclusion. |

Table 1 (Continued)

*Methods for Fostering Positive Mathematical Identities*

| Anderson's (2007) Recommendation | General Strategy | Specific Examples |
|---|---|---|
| **Keep students abreast of the role that mathematics plays in their success as a student and future employee** | Consistently remind students that mathematics is integral to college matriculation and their careers. | Create a reference sheet for, or display on a bulletin board, the progression of mathematics courses in their school with a particular emphasis on those courses that are required for entrance into college.<br>Invite professionals from outside the school to discuss ways in which they use mathematics in their professional lives (Anderson, 2007).<br>Include applications of mathematics that are relevant to the students or their prospective careers, such as using graph theory for the scheduling of final exams or modeling a mock election using different voting systems. |
| **Make students aware of high expectations** | Enforce the belief that all students can learn mathematics. | Maintain expectations that all students will continue their study of mathematics in every year of high school and beyond (Anderson, 2007).<br>Consistently communicate growth mindset beliefs and practices that emphasize effort attributions, learning goals, and a mastery-oriented response to failure (Dweck, 2000). |

### Fostering Identity Through Effective Mathematics Teaching Practices

According to Kebreab et al. (2021), a positive mathematical identity can be fostered in the classroom in accordance with the NCTM' *Effective Mathematics Teaching Practices* (MTPs) (2014). Kebreab et al. (2021) based their concept of identity on Gee's (2000) four perspectives: natural, institutional, discursive, and affinity. Gee (2000), like Anderson (2007), describes the nature perspective of identity as a view developed from natural forces beyond one's control, such as genetics. The institution perspective of identity is how one derives their identity from authorities within institutions, such as how students derive their identity as mathematicians from their teachers. The discourse perspective is described as a social construct recognized through dialogue, such as through social interactions in the classroom that influence views regarding who is a learner and user of mathematics. Finally, the affinity perspective is derived from shared experiences with "affinity groups," such as the members of a mathematics class or group (Kebreab et al., 2021).

Kebreab et al. (2021) argue that natural identities can be developed when teachers support students in productive struggle (MTP 7). In particular, when educators afford opportunities for students to persevere in problem solving, it communicates confidence in students' natural ability to overcome challenges and reaffirms their agency over mathematical learning. Kebreab et al. (2021) suggest students' institutional identities can be developed through the implementation of tasks that promote reasoning and problem solving (MTP 2), as well as using and making connections between mathematical representations (MTP 3). For instance, educators can position students as capable knowers and doers of mathematics when using guiding questions such as "How would you describe this?" or "Where would be a good place to start?" Such questions should be used in conjunction with rich mathematical tasks, which include nonroutine problems that can be solved using a variety of methods (Brahier, 2016). This serves to shift the mathematical authority to the student which, therefore, encourages them to make their own connections and justify their thinking. Kebreab et al. (2021) posit that the discursive identity can be developed by posing purposeful questions (MTP 5) and building procedural fluency from conceptual understanding (MTP 6). For example, when educators question or prompt students they are sending the message that student thinking is valued which may encourage them to further engage with their peers and teachers. Finally, when educators establish clear mathematics goals to focus learning (MTP 1), Kebreab et al. (2021) postulate that students reestablish their roles and responsibilities as affinity members of the classroom mathematical community. Thus, the affinity perspective of identity can be fostered by the teacher via frequent redirection towards the learning goals and classroom expectations.

### Fostering Positive Mathematical Identities Through Discussion-Centered Pedagogical Practices

According to Boaler and Selling (2017), “the idea that students develop different mathematical identities in mathematics classrooms that include beliefs about oneself, ideas about mathematics, and an eagerness to engage actively with mathematics draws from a situated perspective that attends to forms of engagement” (p. 83). Boaler and Selling (2017) delineate two forms of engagement: active and passive. Active engagement in the mathematics classroom occurs when students are engaged in problem solving, discourse, and the application of mathematical methods. In contrast, passive engagement takes place when students are positioned as “received knowers” in which their role in problem solving is to listen to the authority of the teacher and reproduce the teacher’s methods (Boaler & Selling, 2017).

Educators that employ discussion-centered pedagogical styles can consistently offer opportunities for active engagement that encourage students to have agency over validating mathematics methods, generating questions, and developing ideas. For instance, consider placing students in cooperative learning groups and presenting them with a challenging mathematical task situated in the real world. As the students work, encourage them to make conjectures, justify their reasoning, and critique the approaches and methods of their peers (Seeley, 2004). Additionally, when instructional strategies are employed that shift the mathematical authority to the students, educators can enhance the autonomy of their students by welcoming questions, acknowledging frustrations, and encouraging independent investigations (Skilling, 2014). This can be done through the establishment of classroom discussion norms that create a mathematical community which values inchoate ideas and making mistakes. Educators may also consider utilizing flipped or blended classroom approaches to afford more opportunities for inquiry-based, constructivist learning. When the classroom structure is reversed, students can take advantage of classroom time to build conceptual understanding and engage in activities that lead them to discover that mathematics is a process and not a “universal truth handed down by some disembodied, non-human force” (Becker, 1995, p.168). Ultimately, when educators adopt these practices they “share the process of mathematical problem solving with students … making mathematics more equitably accessible, and also encouraging larger numbers of students to explore mathematics as a career” (Boaler & Greeno, 2000, p.189). Unsurprisingly, positive mathematical identities flourish in these types of learning environments because students are given agency over the mathematical process, are encouraged to view themselves as mathematicians, and experience first-hand the value of mathematics in their lives (Boaler, 2002; Boaler & Greeno, 2000).

## Conclusion

The significance of the relationship between mathematical identity and mathematical learning is underscored by its increasing prominence in research. Mathematical identity is considered an indicator of academic performance, persistence, and success (Bohrnstedt et al., 2020; Cribbs et al., 2015; Marsh et al., 1988). Additionally, forming positive mathematical identities can empower students, especially those marginalized by race, gender, class, or ethnicity (Larson, 2016). Moreover, those students that positively identify with mathematics are more likely to pursue future mathematics courses as well as STEM-related occupations (Cribbs et al., 2015; Watt et al., 2017). According to the National Science Foundation (NSF), perpetual innovations in STEM fields are critical for maintaining a competitive edge in the increasingly knowledge-based, technological, and global economy in which we live (2007). The NSF (2007) contends that increasing the STEM literacy of citizens is crucial to ensuring their full and active participation in sustaining a high quality of life.

This paper presented an overview of research and theory regarding pedagogical practices for teaching mathematics that can foster the formation of positive mathematical identities. To begin this work it is important that we, as educators, first self-assess and reflect upon our own beliefs that are both intentionally and unintentionally communicated to students. As stewards of the classroom mathematical community, in what ways are we communicating ideations of who is a learner and doer of mathematics? Likewise, it is important that we analyze our classroom norms and expectations in consideration of the message they send to students regarding the nature of mathematical learning. Are we presenting mathematics in such a way so as to foster intuition, creativity, and connectedness (Burton, 1998, 1999)? The 70 research mathematicians in Burton's (1998, 1999) study identified these three notions as consistent with the discipline of mathematics itself. Further, they described feelings of euphoria in mathematical exploration and a distinct need for collaboration with others (Burton, 1998, 1999). As teachers evaluate their own pedagogical practices, they should be asking themselves: Are we eliciting positive feelings in the classroom? To present mathematics in any other way would be to disenfranchise students from the joy of identifying with the subject and, therefore, developing a positive mathematical identity.

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# How Much Reform? An Analysis of a Chapter in a Reform-Based Calculus Textbook

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**ABSTRACT** Though the curriculum reform movement that began in 1989 has had a significant impact on calculus pedagogy and content (Windham, 2008), the question remains to what extent reform-based calculus textbooks reflect these reforms. This paper explores this question by analyzing the mathematical content and pedagogical approach of the chapter on functions in Hughes-Hallett et al.'s (2018) reform-based college textbook *Calculus: Single and Multivariable (CS&M)*. To conduct this analysis, the paper first synthesizes Hwang et al.'s (2021) framework for analyzing mathematics textbooks along with Sood and Jitendra's (2007) framework for comparing traditional and reform-based mathematics textbooks. The paper then applies the synthesized frameworks to identify elements of reform from the big ideas, contextual features, mediated scaffolding, problem solving activities, and STEM integration found in the *CS&M* chapter. The results suggest that, although the chapter integrates many reform-based principles, there may be scope for further integration. Such efforts, however, may be constrained by influential stakeholders.

**KEYWORDS** *calculus, reform, textbook, curriculum, mathematics pedagogy*

## Introduction

Considering that mathematics textbooks exert significant influence on pedagogy as well as the topics teachers present in class (Chang et al., 2016; Johansson, 2005), they may also help teachers implement educational goals promoted within a curriculum (Hwang et al., 2021). In recent decades, debate has centered on whether to adopt traditional mathematics curriculum and textbooks or reformed curriculum and textbooks grounded in *Principles and Standards for Mathematics Education* (NCTM, 2000). Traditional curriculum and textbooks emphasize systematic explanation of algorithms, practice of problems to demonstrate concepts, teacher-centered instruction, memorization, and procedural knowledge (Schoenfeld, 2002; Sood & Jitendra, 2007; Waite, 2000). In contrast, reform-based curriculum and textbooks emphasize conceptual knowledge and critical thinking, engage students in real-world problem solving, focus on explanation, and encourage active learning (Sood & Jitendra, 2007; Waite, 2000). Reform-based approaches may also encourage Science, Technology, Engineering, and Mathematics (STEM) integration further to promote real-world problem solving (Bybee, 2013).

With regard to calculus, reform arose out of concern that students had a weak understanding of the subject and lacked interest in pursuing higher mathematics (Todd, 2012). Characteristics of traditional calculus include a highly rigorous and rigid curriculum, a pen-and-paper approach to problem solving, a heavy emphasis on theorems and proofs, memorization, and a format in which the teacher is the primary source of knowledge (Garner & Garner, 2001; Windham, 2008). Situated as the primary source of knowledge, the teacher explains concepts and provides examples during lectures, while students take notes, ask clarifying questions, and study from textbooks in a mostly individual experience (Garner & Garner, 2001; Windham, 2008). Defenders of reformed calculus contend that finding correct answers to procedural questions has little value if one cannot explain why algorithms work and therefore cannot develop conceptual understanding. Accordingly, reformed calculus courses often place less emphasis on doing proofs and more on understanding what proofs mean, as well as on applications of calculus (Garner & Garner, 2001; Windham, 2008). These courses often employ group work to help students construct their own meaning of concepts and performance-based assessments that push students to reason and justify their ideas (Garner & Garner, 2001).

When decisions about adoption of reform-based textbooks are made with the idea of integrating reform-based pedagogical principles in calculus courses, it is useful to know to what extent these textbooks actually reflect such principles. This paper begins exploring this issue by analyzing the mathematical content and pedagogical approach of Chapter 1, "Foundation for Calculus: Functions and Limits," in Hughes-Hallett et al.'s (2018) reform-based college textbook *Calculus: Single and Multivariable (CS&M)*. This textbook was chosen because the lead author is a primary proponent of reformed calculus (e.g., Hughes-Hallett, 2006), and their textbook would likely reflect reform-based principles. Additionally, this textbook, in its various editions, has been one of the most popular reform-based calculus textbooks (Bressoud, 2011; Windham, 2008). Chapter 1 was chosen because functions display the relationship between variables and are essential for learning algebra as well as calculus (Chang et al., 2016). The analysis of the mathematical content of the chapter focuses on big ideas, context, and STEM integration. The analysis of the pedagogical approach of the chapter focuses on mediated scaffolding and problem solving activities. The rationale for these choices will be provided later in this paper. Ultimately, this paper seeks to answer the question: To what extent does Chapter 1 of *CS&M* reflect the principles of a reform-based curriculum?

## Literature Review

Although many forces, including standards and assessments, shape the curricula that classes adopt, textbooks play a central role (Stein et al., 2007). This is partly because they are concrete objects rather than abstract ideals that teachers and students can use in the classroom (Ball & Cohen, 1996). Given the influence mathematics textbooks have on curriculum and teaching (Chang et al., 2016; Johansson, 2005), a reform-based calculus textbook may be able to promote reform-based teaching and learning practices. To demonstrate further the influence of textbooks on the choices both teachers and students make, the following sections outline the relationships between mathematics textbooks and teachers, as well as between mathematics textbooks and students.

### Relationships Between Mathematics Textbooks and Teachers

According to Ball and Cohen (1996), there is a close relationship between mathematics textbooks and teachers' practice. While teachers rely on their professional experiences and beliefs to interpret the content within a textbook (Liakos et al., 2021), they still look to textbooks as a "dialogic partner" (Dietiker et al., 2018, p. 522) that contains vital material to be used in the classroom. Liakos et al. (2021) conducted a qualitative study of the curriculum planning activities of a seasoned teacher who switched from teaching calculus with a traditional approach to teaching reform-based calculus. Observing the teacher using the textbook extensively in his planning and adding his own related materials, they concluded that the textbook significantly influenced his teaching. In addition, research suggests that in more recent years, despite the increased use of digital resources, textbooks continue to play a vital role in influencing how and what teachers teach (Glasnović Gracin & Jukić Matić, 2021; Polikoff, 2015).

Research suggests that, during periods of reform, mathematics textbooks are especially influential among teachers (Glasnović Gracin & Jukić Matić, 2021; Howson, 2013). This may be due to the fact that mathematics textbooks often serve as an authority not only on reformed mathematical content, but also on new curriculum (Glasnović Gracin & Jukić Matić, 2021; Valverde et al., 2002). Moreover, often new textbooks are published during times of reform as a cost-effective way to implement new curriculum (Polikoff, 2015), as teachers often seek help in the implementation of a new curriculum in textbooks and corresponding teacher guides (Howson, 2013). New textbooks under a reformed curriculum may also encourage teachers to undergo professional development so that they can use the textbooks effectively (Glasnović Gracin & Jukić Matić, 2021). Thus, while textbooks may not directly determine teachers' pedagogical approaches or strategies, they may influence the mathematical content taught in the course.

### Relationships Between Mathematics Textbooks and Students

Research into traditional and reformed calculus is motivated not just by the relationship between calculus textbooks and teachers, but also by the relationship between calculus textbooks and students (Bressoud et al., 2016). Research reveals that the interactions among mathematics textbooks, teachers, and students are complex (Sevimli, 2016). Additional research suggests that most of the mathematical tasks students undertake and most of what they learn are influenced by the content of the textbook (Begle, 1973; Thompson et al., 2012). This may be because textbooks are written for students with tasks specifically addressed to them (Glasnović Gracin & Jukić Matić, 2021; Valverde et al., 2002). Unfortunately, scant research explores mathematics textbooks from the perspective of students. Nonetheless, it is evident that textbooks may influence students as significantly as they do teachers.

### Research on Reformed Calculus Textbooks

Although reform in calculus has garnered much attention from researchers, particularly in relation to pedagogy and curriculum (Bressoud et al., 2016; Dunnigan & Halcrow, 2020; Garner & Garner, 2001; Keynes & Olson, 2000), most of these researchers provide little analysis of textbooks. For example, Dunnigan and Halcrow (2020) describe a restructuring of the course Applied Calculus at their university, which focused on increasing students' conceptual understanding and eliminating large lectures, but failed to address the role of textbooks. Similarly, Keynes and Olson (2000) describe changes in the content and pedagogy of the calculus sequence at the University of Minnesota, devoting their attention primarily to such innovations as the development of group work among students and the use of new technologies, but saying little about the use of textbooks.

What little research explores the impact that reform-based calculus textbooks have on teaching and learning focuses on a few specific issues. For instance, Chang et al. (2016) investigate uses of coordinated multiple representations in calculus textbooks for pedagogical and scaffolding purposes. Özgeldi and Aydin (2021) explore the levels of competency demand used by three calculus textbooks, including traditional and reformed. Neither Chang et al. (2016) nor Özgeldi and Aydin (2021) address other issues, such as STEM integration in calculus textbooks or the use of understanding, estimating, exploring, resolving, and explaining in the solving of mathematical problems. To address this gap in the literature, this paper provides a more comprehensive evaluation of a chapter in a reform-based calculus textbook that focuses not only on content, but also on pedagogical characteristics reflecting the principles of the reformed calculus movement.

Specifically, this paper seeks to answer the questions:

1. To what extent do the big ideas in the *CS&M* chapter on functions reflect the principles of a reform-based mathematics curriculum?
2. To what extent does the context in the *CS&M* chapter on functions reflect the principles of a reform-based mathematics curriculum?
3. To what extent do the examples and problems in the *CS&M* chapter on functions integrate the STEM disciplines?
4. To what extent do the examples and problems in the *CS&M* chapter on functions use multiple representations?
5. To what extent do the examples and problems in the *CS&M* chapter on functions require different problem solving activities, such as understanding, estimating, exploring, resolving, and explaining?

## Methodology

### Materials

*CS&M* is a product of the reform movement that has its roots in the 2000 publication of the NCTM's *Principles and Standards for Mathematics Education* (Windham, 2008). The data source of this study consists of materials from the chapter on functions, including definitions of concepts, illustrations of these concepts, practice exercises, and end-of-chapter exercises.

### Analytical Framework

Two frameworks are synthesized to evaluate the content and pedagogical approach of the *CS&M* chapter on functions, henceforth referred to as "Chapter 1." These two frameworks are chosen because they allow for an analysis of the presence or absence of a wide array of features that are central to the 1989 reform movement. This synthesis of the two frameworks allows not only an analysis of the content of Chapter 1 for evidence of reform-based principles, but also expounds on how that chapter integrates reform-based pedagogical approaches. The first framework, developed by Hwang et al. (2021), is a commonly used framework to analyze mathematics textbooks by distinguishing two dimensions. The first is a horizontal dimension, which includes topic sequence and frequency. The second is a vertical dimension, which includes contextual features, cognitive demands, and problem solving activities. The second framework, developed by Sood and Jitendra (2007), is used to compare number sense between traditional and reform-based mathematics textbooks. It is rooted in the principles of effective instruction for students at risk in mathematics and includes criteria such as the teaching of big ideas, conspicuous instruction, mediated scaffolding, and judicious review (Sood & Jitendra, 2007).

The present study focuses on big ideas (Sood & Jitendra, 2007) and context (Hwang et al., 2021) to evaluate the mathematical content of Chapter 1. Unlike Sood and Jitendra (2007), who, when analyzing teachers' manuals and other instructional materials, assume a teacher's perspective, the present study analyzes the student edition and thus assumes a student's perspective. STEM integration, a feature of interest in the 1989 mathematics reform movement (Maass et al., 2019; Williams, 2011), is additionally included to analyze the content of Chapter 1 for its use of examples and problems relating mathematics with science, technology, or engineering. The use of mediated scaffolding (Sood & Jitendra, 2007) and problem solving activities (Hwang et al., 2021) are used to analyze the pedagogical approach within Chapter 1. The following is a summary of the analytical framework the present study adopts.

1. Mathematical content
   A. Big ideas: Collections of related concepts that help students acquire a broad set of skills and knowledge (Sood & Jitendra, 2007)
   B. Context: How a textbook illustrates math problems (Hwang et al., 2021)
   C. Integration with STEM: The use of examples and problems relating mathematics with science, technology, or engineering
2. Pedagogical approaches
   A. Mediated scaffolding: Support provided to students through teachers (e.g., instructional feedback), materials (e.g., visual prompts and representations), or tasks (e.g., the systematic introduction of more difficult materials) (Sood & Jitendra, 2007)
   B. Problem solving activities: The use of understanding, estimating, exploring, resolving, and explaining in the solving of mathematical problems (Hwang et al., 2021)

#### *Mathematical Content: Big Ideas, Context, and STEM Integration*

Big ideas are what the authors of a textbook consider important (Sood & Jitendra, 2007). They are discernible through chapter headings, the amount of space in the chapter devoted to them, and the number of problems and examples that illustrate them (Sood & Jitendra, 2007). Figure 1 provides an example of a big idea that illustrates the use of different representations to understand, interpret, and analyze functions.

**Figure 1**

*Example of a Big Idea in Chapter 1*

**Big Idea: Understanding, interpreting, and analyzing functions using different representations**

### 1.1 FUNCTIONS AND CHANGE

In mathematics, a *function* is used to represent the dependence of one quantity upon another.

Let's look at an example. In 2015, Boston, Massachusetts, had the highest annual snowfall, 110.6 inches, since recording started in 1872. Table 1.1 shows one 14-day period in which the city broke another record with a total of 64.4 inches.

**Table 1.1**

*Daily snowfall in inches for Boston, January 27 to February 9, 2015*

| **Day** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Snowfall** | 22.1 | 0.2 | 0 | 0.7 | 1.3 | 0 | 16.2 | 0 | 0 | 0.8 | 0 | 0.9 | 7.4 | 14.8 |

You may not have thought of something so unpredictable as daily snowfall as being a function, but it is a function of day, because each day gives rise to one snowfall total. There is no formula for the daily snowfall (otherwise we would not need a weather bureau), but nevertheless the daily snowfall in Boston does satisfy the definition of a function: Each day, $t$, has a unique snowfall, $S$, associated with it.

We define a function as follows:

> A **function** is a rule that takes certain numbers as inputs and assigns to each a definite output number. The set of all input numbers is called the **domain** of the function and the set of resulting output numbers is called the **range** of the function.

**Example 1**

The function $C = f(T)$ gives chirp rate as a function of temperature. We restrict this function to temperatures for which the predicted chirp rate is positive, and up to the highest temperature ever recorded at a weather station, 134°F. What is the domain of this function $f$?

**Solution**

If we consider the equation

$$C = 4T - 160$$

simply as a mathematical relationship between two variables $C$ and $T$, any $T$ value is possible. However, if we think of it as a relationship between cricket chirps and temperature, then $C$ cannot be less than 0. Since $C = 0$ leads to $0 = 4T - 160$, and so $T = 40$°F, we see that $T$ cannot be less than 40°F. (See Figure 1.2.) In addition, we are told that the function is not defined for temperatures above 134°F. Thus, for the function $C = f(T)$ we have

$$\begin{aligned} \text{Domain} &= \text{All } T \text{ values between } 40°\text{F and } 134°\text{F} \\ &= \text{All } T \text{ values with } 40 \le T \le 134 \\ &= [40,134] \end{aligned}$$

**Example 2**

Find the range of the function $f$, given the domain from Example 1. In other words, find all possible values of the chirp rate, $C$, in the equation $C = f(T)$.

**Note.** Adapted from *Calculus: Single and multivariable,* by D. Hughes-Hallett et al., 2018, pp. 2-3. Copyright 2018 by John Wiley & Sons, Inc.

Context refers to how a textbook illustrates math problems (Hwang et al., 2021). *CS&M* explicitly states that it uses the "Rule of Four," illustrating concepts geometrically (G), numerically (N), algebraically (A), and verbally (N) (Hughes-Hallett et al., 2018). See Table 1 for examples.

Figure 2 provides an example of a STEM integration problem that connects the use of functions with the discipline of physics.

**Table 1**

*Examples of Context in Chapter 1*

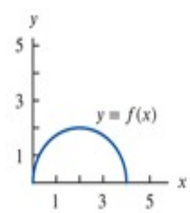
<table>
<tr><th>Context</th><th>Example</th></tr>
<tr><td>Algebraic (A)</td><td>Find the domain and range in Exercises 24–25.<br>24. $y = x^2 + 2$<br><br>Note. Adapted from Calculus: Single and multivariable, by D. Hughes-Hallett et al., 2018, p. 8. Copyright 2018 by John Wiley & Sons, Inc.</td></tr>
<tr><td>Verbal (V)</td><td>Problems 39–42 ask you to plot graphs based on the following story: "As I drove down the highway this morning, at first traffic was fast and uncongested, then it crept nearly bumper-to-bumper until we passed an accident, after which traffic flow went back to normal until I exited."<br><br>39. Driving speed against time on the highway<br><br>Note. Adapted from Calculus: Single and multivariable, by D. Hughes-Hallett et al., 2018, p. 9. Copyright 2018 by John Wiley & Sons, Inc.</td></tr>
<tr><td>Geometric (G)</td><td>For Exercises 20–23, give the approximate domain and range of each function. Assume the entire graph is shown.<br><br><br><br><br>Note. Adapted from Calculus: Single and multivariable, by D. Hughes-Hallett et al., 2018, p. 8. Copyright 2018 by John Wiley & Sons, Inc.</td></tr>
<tr><td>Numeric (N)</td><td>16. Find a linear function that generates the values in Table 1.3.<br><br>Table 1.3<br><table><tr><td>x</td><td>5.2</td><td>5.3</td><td>5.4</td><td>5.5</td><td>5.6</td></tr><tr><td>y</td><td>27.8</td><td>29.2</td><td>30.6</td><td>32.0</td><td>33.4</td></tr></table><br>Note. Adapted from Calculus: Single and multivariable, by D. Hughes-Hallett et al., 2018, p. 8. Copyright 2018 by John Wiley & Sons, Inc.</td></tr>
</table>

**Figure 2**

*Example of STEM Integration in Chapter 1*

75. When Galileo was formulating the laws of motion, he considered the motion of a body starting from rest and falling under gravity. He originally thought that the velocity of such a falling body was proportional to the distance it had fallen.

What do the experimental data in Table 1.7 tell you about Galileo's hypothesis? What alternative hypothesis is suggested by the two sets of data in Table 1.7 and Table 1.8?

**Table 1.7**

| **Distance (ft)** | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| **Velocity (ft/sec)** | 0 | 8 | 11.3 | 13.9 | 16 |

**Table 1.8**

| **Time (sec)** | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| **Velocity (ft/sec)** | 0 | 32 | 64 | 96 | 128 |

**Note.** Adapted from *Calculus: Single and multivariable,* by D. Hughes-Hallett et al., 2018, p. 14. Copyright 2018 by John Wiley & Sons, Inc.

#### *Pedagogical Approach: Mediated Scaffolding and Problem Solving Activities*

Mediated scaffolding provides students with support through teachers, materials, or tasks (Sood & Jitendra, 2007). Chapter 1 contains materials, such as multiple representations and it contains tasks, such as the systematic introduction of more difficult problems. Providing multiple representations of mathematical concepts counts as a form of scaffolding (Ngin, 2018), as it can support students' understanding of new or difficult concepts (Sood & Jitendra, 2007), particularly when students are directed to reason how different representations relate to each other (Chang et al., 2016). See Figure 3 for an example of a problem that uses mediated scaffolding by employing both algebraic and geometric representations.

Hwang et al. (2021) adapted Pólya's (1945) model for problem solving, maintaining that problem solving activities include understanding, estimating, exploring, resolving, and explaining. Understanding involves making sense of a problem, estimating involves approximating an answer or problem solving strategy, exploring involves investigating an answer, resolving involves finding an answer, and explaining involves providing the rationale behind an answer or problem solving strategy (Hwang et al., 2021). The present study adopts Hwang et al.'s (2021) classification to reveal the types of problem solving activities used in Chapter 1. See Table 2 for examples.

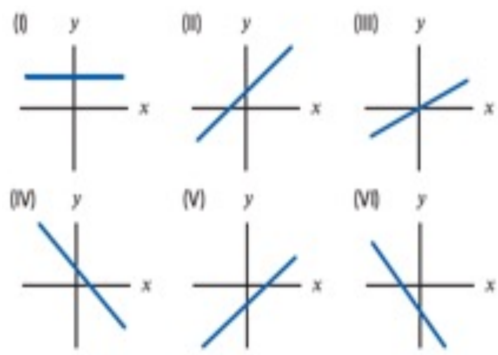
**Figure 3**

*Example of Mediated Scaffolding Using Multiple Representations in Chapter 1*

Match the graphs in Figure 1.9 with the following equations. (Note that $x$ and $y$ scales may be unequal.)

a. $y = x - 5$
b. $-3x + 4 = y$
c. $5 = y$
d. $y = -4x - 5$
e. $y = x + 6$
f. $y = x/2$

**Figure 1.9**




**Note.** Adapted from *Calculus: Single and multivariable,* by D. Hughes-Hallett et al., 2018, p. 14. Copyright 2018 by John Wiley & Sons, Inc.

Table 2

*Examples of Problem Solving Activities in Chapter 1*

| Problem Solving Activity | Example |
|---|---|
| Understanding | None in Chapter 1 |
| Estimating | 44. A certain region has a population of 10,000,000 and an annual growth rate of 2%. Estimate the doubling time by guessing and checking.<br><br>**Note.** Adapted from *Calculus: Single and multivariable,* by D. Hughes-Hallett et al., 2018, p. 23. Copyright 2018 by John Wiley & Sons, Inc. |
| Exploring | Investigate $\lim_{x\to\infty} \frac{1}{x}$ and $\lim_{x\to-\infty} \frac{1}{x}$<br><br>**Note.** Adapted from *Calculus: Single and multivariable,* by D. Hughes-Hallett et al., 2018, p. 73. Copyright 2018 by John Wiley & Sons, Inc. |
| Resolving | The functions in Exercises 5–8 represent exponential growth or decay. What is the initial quantity? What is the growth rate? State if the growth rate is continuous.<br><br>5. $P = 5(1.07)^t$<br><br>**Note.** Adapted from *Calculus: Single and multivariable,* by D. Hughes-Hallett et al., 2018, p. 20. Copyright 2018 by John Wiley & Sons, Inc. |
| Explaining | In Problems 83–88, is the statement true or false? Give an explanation for your answer.<br><br>83. For any two points in the plane, there is a linear function whose graph passes through them.<br><br>**Note.** Adapted from *Calculus: Single and multivariable,* by D. Hughes-Hallett et al., 2018, p. 14. Copyright 2018 by John Wiley & Sons, Inc. |

### Procedure

Using the above analytical framework, the author and a colleague independently examined Chapter 1 with the following aims:

1. to identify the big ideas;
2. to count the number of examples and problems illustrated algebraically, the number illustrated verbally, the number illustrated geometrically, and the number illustrated numerically;
3. to count the number of examples and problems that relate functions to either science, technology, or engineering;
4. to count the number of examples and problems that use multiple representations, noting which multiple representations are used;
5. to count the number of examples and problems that focus on understanding, the number that focus on estimating, the number that focus on exploring, the number that focus on resolving, and the number that focus on explaining.

After completing their independent examinations, the author and colleague compared their preliminary results, checking for any instances of disagreement. Two such instances were found. The author and colleague then consulted with experts, and, after discussion, reached unanimous agreement.

## Results

### Mathematical Content: Big Ideas

In Chapter 1, the big ideas include: (1) understanding, interpreting, and analyzing functions using different representations, (2) building functions, (3) constructing and comparing linear, logarithmic, trigonometric, power, polynomial, and rational functions, and (4) understanding limits and continuity using different representations (Hughes-Hallett et al., 2018). Chapter 1 includes no proofs, and it often introduces concepts with applications, suggesting that applications are the reason for, as well as the result of, doing calculus (Garner & Garner, 2001).

### Mathematical Content: Context

As Table 3 indicates, the problems in Chapter 1 are predominantly illustrated algebraically (A), followed by verbally (V), geometrically (G), and numerically (N).

**Table 3**
*Results for Context*

| Representation Type | Percent of the 854 Total Examples/Problems |
|---|---|
| Algebraically (A) | 71.2% |
| Verbally (V) | 24.2% |
| Geometrically (G) | 17.1% |
| Numerically (N) | 4.6% |

These results are noteworthy considering that traditional calculus textbooks use mostly algebraic representations (Todd, 2012), while reform-based textbooks, with their emphasis on multiple representations, would be expected to have a more even distribution of representations. Todd (2012) similarly found that the Hughes-Hallett (2009) text they analyzed, included few problems with numerical representations and, therefore, an uneven distribution of representations. The heavy emphasis on algebraic representations in Chapter 1 may result from the fact that asking and answering questions by mathematical means is among the chief purposes of mathematical activity (Niss & Højgaard, 2019). According to Niss and Højgaard (2019), such questions are about mathematical thinking, mathematical problem solving, mathematical modeling, or mathematical reasoning. Moreover, mathematical activity, by its nature, involves the ability to handle algebraic representations, which are connected with "mathematical language and tools" (Özgeldi & Aydin, 2021, p. 186). Thus, the widespread use of algebraic representations in Chapter 1 may be the authors' attempt to develop students' competency in mathematical language and tools.

### Mathematical Content: STEM Integration

Although Chapter 1 contains some problems and examples relating to science, technology, and engineering, these represent only a small percentage of all the problems and examples (see Table 4). These results run contrary to reform-based principles, which emphasize STEM integration.

**Table 4**
*Results for STEM Integration*

| Discipline | Percent of the 854 Total Examples/Problems |
|---|---|
| Science | 6.0% |
| Technology | 1.0% |
| Engineering | 0.2% |

### Pedagogical Approach: Mediated Scaffolding

Some problems in Chapter 1 contain more than one of the four types of representations: geometric (G), numeric (N), algebraic (A), and verbal (V). Considering that the use of multiple representations can be a form of mediated scaffolding (Ngin, 2018), problems were evaluated for the type and number of distinct representations used. Therefore, if a problem used both a geometric and algebraic representation, it was coded as GA. As Table 5 shows, of problems containing multiple representations,

**Table 5**
*Results for Mediated Scaffolding*

| Representation Types | Percent of the 854 Total Examples/ Problems |
|---|---|
| Geometric and Algebraic (GA) | 8.0% |
| Algebraic and Verbal (AV) | 2.1% |
| Geometric and Verbal (GV) | 1.4% |
| Numeric and Verbal (NV) | 1.4% |
| Numeric and Algebraic (NA) | 0.4% |
| Geometric and Numeric (GN) | 0.2% |
| Geometric, Algebraic, and Verbal (GAV) | 0.2% |
| Geometric, Numeric, and Algebraic (GNA) | 0.1% |

the most frequent contain geometric and algebraic (GA) representations, followed by algebraic and verbal (AV) representations.

One possible explanation for these results is that Chapter 1 may be attempting to target areas that students find more difficult when studying calculus. For example, research suggests that students often have difficulty with coordinating multiple representations, particularly those that include graphical representations (Chang et al., 2016). Perhaps the prevalence of GA representations is intended to help students overcome this difficulty. The prevalence of AV representations may indicate the authors' attempt to help students review previously learned concepts and introduce more difficult or unfamiliar concepts (Chang et al., 2016). The prevalence of both GA and AV representations may also indicate coordination of the process and object perspectives of functions. A process perspective focuses on a function's procedural characteristics, with each $x$ value linked to a $y$ value, whereas an object perspective views functions as entities (Chang et al., 2016; Moschkovich et al., 1993). Therefore, students may tend to think of algebraic representations from a process perspective, but verbal or geometric representations from the object perspective (Chang et al., 2016; Moschkovich et al., 1993).

### Pedagogical Approach: Problem Solving Activities

Results reveal that most problems in Chapter 1 focus on resolving, followed by explaining, estimating, and exploring, and no problems focus on understanding (see Table 6).

Table 6

*Results for Problem Solving Activities*

| Representation Type | Percent of the 854 Total Examples/Problems |
|---|---|
| Resolving | 82.1% |
| Explaining | 15.3% |
| Estimating | 2.2% |
| Exploring | 0.4% |
| Understanding | 0.0% |

These results imply a structure in the lessons that is geared toward problem solving activities, which is consistent with the reformed curriculum approach (Sood & Jitendra, 2007). Moreover, the frequency of explaining in Chapter 1 is aligned with a reform-based pedagogical approach to teaching. Explaining is often associated with group activities and thus represents a more student-centered approach to teaching calculus (Sood & Jitendra, 2007). The absence of problems on understanding stands contrary to the goal of reformed-based mathematics textbooks to help students understand concepts (Sood & Jitendra; Waite, 2000). The unequal distribution of problem solving activities also stands contrary to the goal of reform-based mathematics textbooks to provide students with the opportunity to engage in multiple problem solving activities (Sood & Jitendra, 2007; Waite, 2000).

## Discussion

The above analysis reveals that significant steps seem to have been taken to adapt Chapter 1 to the principles of reformed calculus. This is evident, for example, from Chapter 1's emphasis on problem solving skills and multiple representations to help students visualize concepts. At the same time, however, Chapter 1 exhibits some similarities with traditional calculus. These similarities include a heavy emphasis on symbolic and algebraic representations, and less emphasis on STEM integration. The similarities between the traditional and reformed calculus approaches in Chapter 1 may stem from the fact that the subject of functions requires students to acquire competency in mathematical language that serves as a foundation for future topics (Niss & Højgaard, 2019). On the other hand, a variety of representations to illustrate function concepts may appeal to students who prefer more numerical or geometric representations. As Bressoud et al. (2016) acknowledge, calculus reform has led to "the recognition in almost all textbooks and most universities of the importance of graphical and numerical in addition to algebraic representations of derivatives and integrals" (pp. 17–18).

The publishers of *CS&M* may have had reasons not to incorporate more reform-based principles in Chapter 1.

For one thing, textbooks are commercial and political enterprises with various stakeholders, including government officials, influencing the selection of their content (Polikoff, 2018; Shapiro, 2012). For example, to reach a broader audience and larger market, textbooks must deliver the curriculum content that adoption states specify (Batista Oliveira, 1995). Thus, state regulations may influence design criteria, topics, objectives, and other important components of calculus textbooks (Batista Oliveira, 1995). Beyond the adoption criteria of states, publishers must also appeal to teachers (Batista Oliveira, 1995). Some of those teachers may be pedagogically conservative, and prefer textbooks with a more traditional approach (Batista Oliveira, 1995). The persistence of traditional calculus methods in a reform-based textbook may be an indication of the influence of multiple stakeholders who prefer traditional methods of calculus instruction.

Three limitations of the present study are worth noting. First, the present study analyzes just one chapter of one reform-based textbook. One might question how representative this chapter and this textbook are. Thus, an investigation of other chapters and other reform-based textbooks is recommended. Second, while the analytical framework the present study adopts is useful for analyzing functions, it may require expansion when analyzing derivatives, integrals, or other complex concepts in single variable calculus. Third, the present study focuses only on a reform-based calculus textbook and does not compare and contrast it with a traditional calculus textbook. Although Todd (2012) makes such a comparison between one reform-based and one traditional calculus textbook, a comparison and contrast of a larger sample of reform-based and traditional single variable calculus textbooks might shed further light on how much reform reform-based textbooks actually incorporate.

## Acknowledgements

The author is grateful to Professor Nicholas Wasserman for the inspiration for this paper from a class project in his Mathematics Curriculum Development course and for his continuous support for the creative exploration of the many facets of the field of mathematics education.

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# NOTES FROM THE FIELD

## REFLECTIONS ON THE ALGEBRA I CLASSROOM

The Spring 2022 issue features two *Notes from the Field* centered around teaching Algebra. Both authors report on their experiences implementing activities designed to engage students. First, Remijan begins with a description of a hands-on activity involving push toys, technology, and systems of linear equations from her eighth and ninth grade Algebra classrooms. Next, Longhitano reflects on a model for a high school Algebra Lab that includes focused sessions which help to develop essential mathematics skills, engage students in authentic mathematical tasks, and foster their social-emotional competencies.

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NOTES FROM THE FIELD

# Playing with Push Toys and Technology: Solving a System of Linear Equations

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## Mathematical Action Technologies

When students are given the opportunity to utilize technology and engage in hands-on activities within a mathematics class, they can experience mathematics in action. Mathematical action technologies "offer students opportunities to interact…in ways that are not possible (alone) with paper and pencil" (McCullough et. al, 2021, p. 739). One such mathematical action technology, for instance, involves a calculator-based ranger (CBR) which collects and displays motion data in real-time. As CBR activities impact students' abilities to interpret and model "physical phenomena" which enhances graphical understanding (Kwon, 2010), I have incorporated various CBR activities that involve students walking in front of a motion detector to create or replicate a particular given graph (Remijan, 2019). After engaging students with CBR activities within my classroom, as well as outdoor activities involving crash reconstruction and the Illinois State Police (Remijan, 2017), I developed an additional activity to model a "crash" within the classroom using push toys and a CBR.

As such, the following is an example of a technology-based activity involving push toys which I have personally conducted with eighth grade students in Honors Algebra and ninth grade students in Algebra 1A, a first-year course of a two-year Algebra I sequence. I have also performed this activity with students as a mathematics teacher in O'Fallon, Illinois and as a Curriculum Specialist with the Illinois Mathematics and Science Academy conducting statewide outreach. The lesson activity presented seeks to reinforce students' understanding of linear equations, graphing linear equations, and solving a system of equations by graphing.

**Figure 1**
*Push Toys*

To begin, students were told that two push toys, Turtle and Mickey (Figure 1), would be modeling a crash where they would travel in front of a motion detector before crashing into each other.

Additionally, I explained and showed students that the CBR was connected to a graphing calculator and would collect and graph distance and time data automatically. Lastly, students were informed that Turtle can travel 5 feet in 3 seconds while Mickey can travel 7 feet in 3 seconds.

Students were then given the following scenario:

### Scenario 1

Turtle is placed 2 feet from the motion detector and travels away from the motion detector.

a) Write an equation to model the path of Turtle in terms of distance versus time.

**Expected Answer:** $y = \frac{5}{3}x + 2$ where the slope is positive since Turtle is moving away from the motion detector and increasing its distance from the motion detector.

b) Graph the line representing the path that Turtle takes in terms of distance versus time.

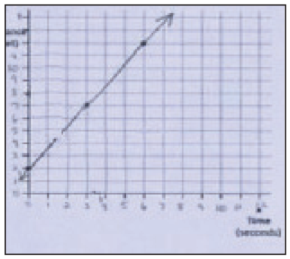
**Expected Answer:** A linear graph found in Figure 2.

c) Model the situation involving Turtle using a CBR and a graphing calculator.

**Expected Result:** A video of my former 9th grade Algebra 1A students modeling this situation can be found at https://youtu.be/sTMtWfvhlbc.

**Figure 2**

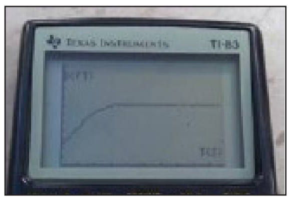
*Graph of the Line Representing Turtle's Movement*



After conducting the hands-on experiment involving Turtle, and reviewing the result acquired from the graphing calculator (see Figure 3), I asked my students "Why is the calculator showing something different than what you graphed?"

**Figure 3**

*CBR Model of Turtle's Path on Graphing Calculator*



With various students sharing their thoughts, students eventually came to a consensus that the graph on the calculator shows that Turtle eventually stopped while the original, hand-drawn graph suggests that Turtle never stopped. Students were guided to recognize that the oblique line on the calculator respresents Turtle moving away from the motion dector at the constant speed of 5 feet per 3 seconds, and that the horizontal line represents Turtle stopping. Additionally, the students determined that the slope of Turtle's path represented Turtle's speed. Thus, the slope of the horizontal line formed by Turtle's lack of movement is consistent with Turtler's speed being that of zero. This activity reinforces the concepts of speed comparing change in distance to change in time, as well as what it means for a linear function to be increasing or constant. Furthermore, the activity may help students build connections between objects in motion in relation to time, as well as the graphical representation and analysis of such a situation.

Next, students were given a second scenario:

### Scenario 2

Mickey is placed 8 feet from the motion detector and travels towards the motion detector.

a) Write an equation to model the path of Mickey in terms of distance versus time.

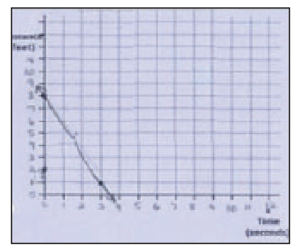
**Expected Answer:** $y = \frac{-7}{3}x + 8$ where the slope is negative since Mickey is moving towards the motion detector and decreasing its distance from the motion detector.

b) Graph the line representing the path that Mickey takes in terms of distance versus time.

**Expected Answer:** A linear graph found in Figure 4.

**Figure 4**

*Graph of the Line Representing Mickey's Movement*



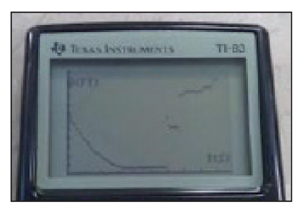
c) Model the situation involving Mickey using the CBR and a graphing calculator.

**Expected Result:** A video of my former 9th grade Algebra 1A students modeling this situation can be found at https://youtu.be/ZHNUxfq-7RQ.

After conducting the hands-on experiment involving Mickey, and reviewing the result acquired from the graphing calculator (see Figure 5), I asked my students again "Why is the calculator showing something different than what you graphed?"

**Figure 5**

*CBR Model of Mickey's Path on Graphing Calculator*



After a brief time of reflection, students explained that Mickey stopped right before the motion detector, making his speed zero, and showing a constant function. Similarly to Scenario 1, by guiding students to make these connections between the experiment and the algebraic and graphical representations, this activity may help to reinforce the concepts of speed, a decreasing function, and a constant function.

Lastly, students were presented with a third scenario:

### Scenario 3

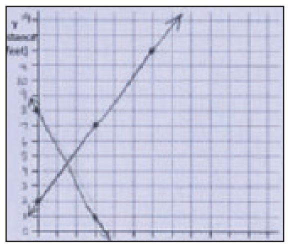
If both Turtle and Mickey leave at the same time, when and where will they "crash?"

a) Graph the two lines representing the paths of both Turtle and Mickey on the same coordinate plane.

**Expected Answer:** Two intersecting linear graphs as shown in Figure 6.

**Figure 6**

*Graph of the Two Lines Representing Turtle and Mickey's Movements*



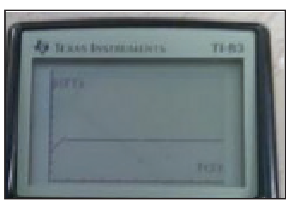
b) Model the situation involving Mickey and Turtle using the CBR and a graphing calculator.

**Expected Result:** A video of my former 9th grade Algebra 1A students modeling this situation with the CBR can be seen here at https://youtu.be/QMFeBmIgm2g.

After analyzing the graph, students determined that Turtle and Mickey will "crash" 1.5 seconds after they both leave and at a point which is 4.5 feet away from the motion detector. Additionally, after conducting the hands-on experiment, students analyzed the data captured from the motion detector (see Figure 7).

**Figure 7**

*Intersection of Turtle and Mickey's Paths*



With the calculator's graph again displaying a different result than the hand-drawn graph, I followed up with the question "Why is the decreasing function not visible?" In response, some students were able to use critical thinking skills to determine that the motion detector was only able to see Turtle because the motion detector could not "pick up" Mickey due to Turtle blocking the "laser beam." Lastly, students determined that the point shown on the calculator where the increasing and constant functions meet is the precise point in which the crash occurred. This point indicates the time and distance from the motion detector at which Turtle and Mickey crash into each other. Throughout this process of critical thinking, analysis, and discussion, students were able to acquire conceptual understanding and greater meaning behind the functions forming the graphs as well as the solution to the system of linear equations.

## Conclusion

With the help of CBR technology and push toys, students were able to model a situation involving the intersection of two objects in motion and collect real-time data. With such an activity, students were encouraged to connect graphing and solving a system of linear equations to a real-world situation. Hands-on activities, like this one, can make mathematical concepts come to life for students. Consequently, these activities may be able to promote students' ability to make connections between real-world actions and mathematical concepts that, potentially, build their conceptual understanding of the underlying mathematics. Moreover, as graphing and solving systems of linear equations appear as early as middle school mathematics curriculum all the way to post-secondary mathematics, mathematics teachers at all levels are encouraged to consider integrating activities that utilize CBR's and toys. Naturally, all teachers are recommended to use their professional discretion on the amount of guidance that is provided to help students gain conceptual understanding of the topic during the activity.

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NOTES FROM THE FIELD

# Evaluating the SSATSEL Algebra I Lab Model: Objectives and Challenges

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*Teachers College, Columbia University*

At the socio-economically diverse suburban high school where I teach outside of New York City, there had been a significant overrepresentation of students of color placed in the remedial level of a Common Core Algebra I course. This led to an unintended "in-school segregation" (Oakes, 1995). For the 2020-2021 school year, courses were restructured so that all Algebra I students were enrolled in a detracked, 41-minute class, while those needing additional support were also placed in a daily 41-minute Algebra Lab class taught by their Algebra I teacher. The new structure served to address the unintended in-school segregation taking place, as well as to promote more equitable access to mathematics education.

It should be noted that Algebra Lab students are evenly distributed throughout all sections of Algebra I. Each teacher is in charge of two sections of Algebra I, and one section of Algebra Lab that consists of students from their Algebra I courses. This structure is critical as it allows Algebra Lab students to receive the additional support needed from the same teacher who has detailed knowledge of their progress in Algebra I. The Algebra Lab course, which I am currently teaching, consists of skills support (SS), authentic tasks (ATs), and social-emotional learning (SEL), hence I refer to this as the "SSATSEL" model (Longhitano, 2021).

When the SSATSEL model was first implemented at the start of the 2020-2021 school year, a hybrid learning model was in use due to the COVID-19 pandemic. Many students were fully remote for the duration of the school year and students were exempted from the Common Core Algebra I Regents exam in New York. Hence, I cannot fully assess the efficacy of the SSATSEL model based on the 2020-2021 school year. However, I will discuss its structure and my experiences in this model from the perspective of a researcher and practitioner.

A weekly schedule, shown in Table 1, is embedded in the SSATSEL model.

**Table 1**
*Weekly Schedule for SSATSEL*

| Day of the Week | Targeted Activity |
|---|---|
| Monday | Vocabulary Development |
| Tuesday | Basic Skills Support (SS) |
| Wednesday | Mathematics-Related Games |
| Thursday | Regents Preparation |
| Friday | Authentic Task (AT) |

The SS component includes, but is not limited to, mini lessons on middle school mathematics content (such as combining signed numbers, operations with fractions, etc.), organization and study skills, and pre-teaching of topics in the Algebra I course. In our Algebra Labs, teachers often employ the use of Delta Math, an online platform that contains a bank of problems teachers can use to create and monitor assessments. In Delta Math, the teacher can select a variety of mathematics topics and skills for students to practice and, using a built-in algorithm, require that students are prompted with new problems until they answer a specified number correctly. When students submit an answer the full solution is revealed so they can learn from their mistakes and immediately try a new problem. In my Algebra Lab classroom, while students work on Delta Math, I circulate the room, answer questions, provide additional feedback, and monitor their progress. I believe this has helped my students tackle skill gaps and obtain greater fluency with the course content.

ATs require that students apply their knowledge and skills to a real-world problem with a real audience. For example, my Algebra Lab students applied their knowledge of writing and graphing linear equations and inequalities to develop a mathematical model for fundraising. The model prompted students to determine the number of donations they would need to reach a fundraising goal for a GoFundMe® page they had designed. The project required students to conduct research to determine the average donation amount, the cost of transaction fees, and how much money they would need to raise to support their cause. The task showed students the relevance of what they were learning and how they can leverage their mathematical knowledge to help others. ATs, like this one, are meant to spark students' interest in mathematics and inspire students to persist in their mathematics education.

Students participate in daily social-emotional learning (SEL) activities to promote their emotional health and growth. SEL has been shown to promote student achievement (Herrenkohl, 2020), which was the impetus of its inclusion in this model. Activities include analyzing a "quote of the week" on Mondays, giving themselves a "shout-out" on Tuesdays, participating in a guided "mindfulness meditation" on Wednesdays, choosing a word to describe their mood in a "mood meter" on Thursdays, and sharing their weekend plans on Fridays. These activities have served to build community in my Algebra Lab class and have helped me connect and build rapport with my students.

In my recent research (Longhitano, 2021), I focused on facilitating and studying teacher collaboration in the curriculum development process to create AT projects for the Algebra Lab course that align with the scope and sequence of Algebra I. The findings suggested that an overemphasis on the implementation of ATs in Algebra Lab may have negatively impacted student achievement and participation in the Algebra I class (Longhitano, 2021). This may have been due to the fact that the ATs did not directly attend to students' need to develop procedural fluency and conceptual understanding. While the teachers who participated in the study felt that the ATs were a valuable pillar of the course, they agreed that SS and SEL activities seemed to be most vital to the success of their students and, thus, should take priority. It was also noted that the COVID-19 pandemic made it difficult to reach students attending classes remotely. Teachers in the study commented that they believed the SSATSEL model would have been much more effective if all students were physically present. In addition, due to the number of fully remote students, recruitment of students to participate in my study was difficult, which limited my ability to investigate students' perceptions of the SSATSEL model.

Currently, in the 2021-2022 school year, I have observed an enormous gap in my Algebra I students' mathematical skills. I believe that this is primarily due to a loss of learning when we were in remote and hybrid classrooms during the COVID-19 pandemic. The efficacy of the SSATSEL model has been difficult to discern because it is confounded by the disruption of students' mathematics education during their formative middle school years. Recently, my Algebra I colleagues and I decided to replace vocabulary development on Mondays with more intensive SS, pre-teaching, and additional use of Delta Math and other platforms to address the skills gaps. The students still participate in vocabulary development activities in the Algebra I class, thus they are able to develop their mathematical vocabulary. I believe a few years of in-person learning will need to take place before I can again assess the efficacy of the SSATSEL model and its three components.

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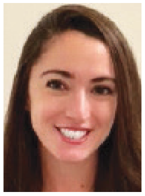

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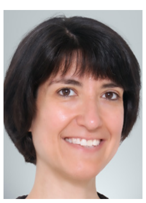

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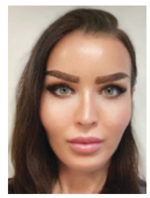

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