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AIMS AND SCOPE

The *Journal of Mathematics Education at Teachers College (JMETC)* is a recreation of an earlier publication by the Program in Mathematics Education at Teachers College, Columbia University. As a peer-reviewed, semiannual journal, it is intended to provide dissemination opportunities for writers of practice-based or research contributions to the general field of mathematics education. Although many of the past issues of *JMETC* focused on a theme, the journal accepts articles related to any current topic in mathematics education, from which pertinent themes for future issues may be developed.

JMETC readers are educators from pre-kindergarten through twelfth-grade teachers, principals, superintendents, professors of education, and other leaders in education. Articles appearing in the *JMETC* include research reports, commentaries on practice, historical analyses, and responses to issues and recommendations of professional interest.

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PREFACE

The Fall 2022 Edition of the *Journal of Mathematics Education at Teachers College* presents three research-based articles related to the development and communication of mathematical ideas. Following these articles, two short reports present methods to provide support to students in the mathematics classroom in two different ways: by encouraging long term cooperative learning and using empathy in teaching practices.

To begin this issue, Markopoulos et al. investigated students' thinking as they learned about representations of geometric solids, contributing to the literature about dynamic geometry environments. The conversations between the students and the teacher were captured while students engaged with two computer-based tasks. The researchers used this data to highlight how student thinking about geometric solids changed over the course of the task and posited that dynamic geometry environments can benefit developing students' understanding of three-dimensional figures.

Next, Ducharme et al. analyzed the language techniques preservice teachers used to facilitate mathematical discussion. Using data gathered from a simulated class activity, Ducharme et al. categorized the teachers' dialogue with students into six specific "discourse moves." The type and frequency of each "discourse move" were examined over the course of the exercise, with some notable differences appearing between teachers. Ducharme et al. concluded by using their observations to create a framework regarding discourse moves used during class discussions.

Finally, Mphalele et al. examined the challenges rural South African schools face when using indigenous languages for the teaching and learning of mathematics. The researchers collected data regarding the languages used for teaching and learning in these communities and conducted focus group interviews to gain insight into the difficulties of using indigenous languages in the mathematics classroom. The authors also provided recommendations for improving the use of indigenous languages in the teaching and learning of mathematics.

Mr. Davidson Barr
Mr. Baldwin Mei

Guest Editors

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Exploring Students' Geometrical Thinking Through Dynamic Transformations Using 3D Computer-Based Representations

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ABSTRACT This paper examines Grade 6 students' thinking about geometrical solids and their properties in the context of a computer-based task. The main focus is to explore how students interact with a number of tasks based on the dynamic transformation of computer-based representations of 3D solids in a geometry classroom with the aid of a Dynamic Geometric Environment. The analysis of two teaching experiments suggests that the dynamic transformations of geometrical solids encourage the students to investigate relationships between the solids and their properties and their 2D representations. Through the tasks, students realize the existence of non-trivial conventions in 2D representations and become aware that angles and edge dimensions may be distorted. The study provides insight into student learning around geometric solids and their properties, and consequently an opportunity to enhance the teaching of 3D geometry.

KEYWORDS *3D geometry, computer-based representations, geometric solids*

Introduction

Students' difficulties in conceptualizing three-dimensional (3D) solids (Ma et al., 2009; Pittalis & Christou, 2010, Sack, 2013) are often attributed to ineffective geometry teaching (Battista, 2007; Ho & Lowrie, 2014; Accascina & Rogora, 2006; Markopoulos et al., 2015a). Ineffective teaching of geometry can sometimes be characterized by procedural pedagogical approaches that merely focus on identifying the properties and definitions of 3D solids without relating the solids' properties to the formation of the solids themselves. Indeed, many students appear not to have the required spatial reasoning skills needed to interpret and understand images of 3D transformations (Suselo, Wünsche & Luxton-Reilly, 2022). In cases where the students experiment with three-dimensional models, learning activities are often limited to the recognition of the solids and to the identification of the critical properties that define the solid (Besson et al., 2010; Markopoulos et al., 2015b; Lowrie et al., 2019). In their study evaluating the types of reasoning that students develop in a dynamic geometry environment, Marrades & Gutierrez (2001) also highlighted the contribution of dynamic computer environments in the development of empirical justifications and their transformation into more analytic and abstract ways of thinking.

Geometrical concepts can be taught to students explicitly in the context of developing and modeling specific tasks in, for example, the field of technology education. However, the use of spatial geometry in programming is often neglected and the teaching of 3D geometry in high school geometry is often absent. (Mammana, Micale & Pennisi, 2012). Indeed, a common pedagogical approach is teaching through rote memorization, often limited to the recognition and naming of geometrical solids and their critical properties, while common teaching and learning activities include "static" exploration of 2D plane representations of solids presented in textbooks (Glasnovic, 2018; Gutierrez, 1996).

The Focus

In this paper, we examine a teaching experiment in which Grade 6 primary school students engage in geometrical thinking about solids and their properties. The study provides insight into student learning around geometric solids and their properties, and consequently, reveals an opportunity to enhance teacher understanding for the teaching of 3D geometry. We explore the role that dynamic transformations can play in creating a learning environment that encourages the development of students' ability to focus on the properties of geometrical solids and, importantly, build relationships between these properties and various solids. Specifically, the research question for the study is: what are some of the mathematical ideas about cubes and rectangular prisms elicited by students working with Dynamic Geometric Environments?

In defining the dynamic transformation of a geometric solid, we consider a process where the solid changes its form through the variation of some of its characteristics and the conservation of others. For example, when transforming a cube to a rectangular prism by increasing the length of one dimension, four faces of the initial cube change from a square to a rectangle, whilst two faces are conserved as squares. It is important to give students opportunities to explore the relationships between the properties of the geometrical solids, so that they are able to properly conceptualize the solids. We explore the implementation of tasks based on the dynamic transformation of computer-based representations of geometrical solids in a mathematics classroom and reflect on how students learn 3D geometry.

Methodology

The research methodology was a classroom teaching experiment along the same lines as described by Cobb et al. (2001). The plan called for students to work in groups on a number of dynamic transformation tasks relating to geometrical solids and their properties, using a central digital projector as a visual reference for the class. In Grade 6, students had already been briefly introduced to the recognition of the properties of a cube and a rectangular prism, along with their construction by using the 2D nets of the solids. The data analyzed in this paper were collected from two Grade 6 classes with the students aged 10-11 years old. The teaching experiment involved the teacher acting as a researcher. The teacher-researcher implemented the tasks, observed students throughout the tasks, and intervened intermittently to explore and clarify students' understanding. Two teaching sessions were analyzed, which focused on the learning environment that was created and the possible cognitive changes concerning students' conceptions of a geometric solid.

3D Representational Models

The materials used in the research study were physical models, computer representations and written imaginary tasks. In this paper, we focus on the computer representations and the way they were used by the students during the teaching session. The computer representations were created using Cabri Geometre II software. A number of research studies have investigated the usefulness of this particular software program in promoting children's geometrical thinking (Laborde, 1993; Leung & Lopez-Real, 2002; Mariotti, 2000). It is acknowledged that computer-based materials allow students to move from physical to cognitive actions, and vice-versa. Research shows that 3D dynamic computer representations provide a learning environment with which students can interact and explore a variety of 3D geometrical objects quickly and easily (Leung, 2011; Højsted, 2019).

The computer-based representations of solids in this study consisted of the isometric and oblique projections of a cube and a rectangular prism. By convention, these types of projections are the most commonly used techniques of pictorial projections in textbooks. This is an important consideration in the teaching and learning of geometry because the use of 2D representations of geometrical solids often follow certain norms.

Figure 1 shows both an oblique projection and an isometric projection of the same cube. In both projections, certain properties of the original 3D cube are maintained while other properties are not. In the oblique projection on the left, the congruence of the edges and angles is only maintained for the front and back faces. On the other hand, in the isometric projection on the right, the equality of the edges is maintained but not the equality of the angles.

In the isometric projection in Figure 1, none of the angles of the cube are 90 degrees. The angles around one corner are 120 degrees. In the oblique projection, the angle between the front face and the base of the cube is 45 degrees. The width of the cube can be drawn in proportion 1:2 to the original, as a cabinet projection. This halving of the depth is an effort to maintain the aesthetic proportions of the cube, as any 1:1 maintenance of the dimensional sizes would result in a cavalier projection (Boundy, 1988).

The two representations of the cube (Figure 1) can be transformed into rectangular prisms by varying the dimensions of the solids, as shown in Figure 2. The changes in dimensions can be achieved by the use of the mouse by "dragging and dropping" the corners. Through these changes, a number of the properties of the solids change. The transformation of the solids in the computer environment does not have the same constraints as transformation of physical models, allowing for greater flexibility with the model. In particular, the length of each of the dimensions could vary from zero, when the solid becomes a plane figure, to any desirable length, though the dimensions of the computer screen constrain the variation so that it remains visible. This characteristic provides students with the opportunity to experiment with different forms of geometrical solids of any length.

Figure 1

An Oblique (left) and an Isometric (right) Representation of a Cube

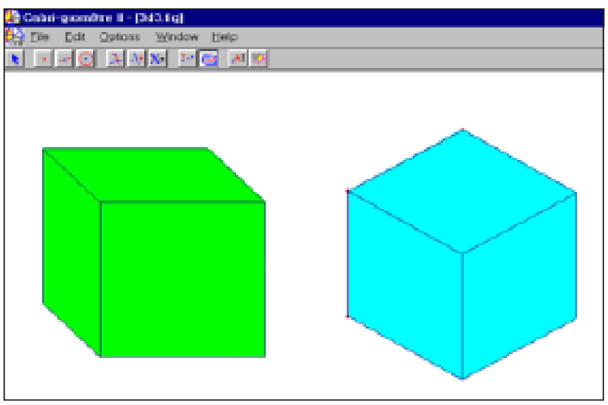
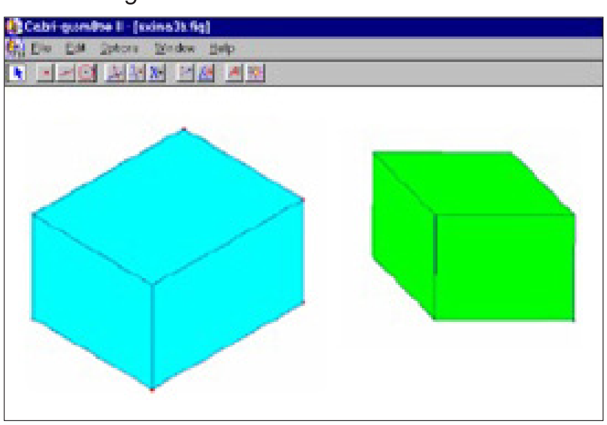


Figure 2

An Isometric (left) and an Oblique (right) Representation of a Rectangular Prism



In the design context, this ability to model and modify virtual shapes is what makes 3D computer representation versatile, as changes can easily be made to the object compared to the production of 3D representations with, for example, pen and paper. The software environment allows students to measure the length of the dimensions and the edges, as well as the size of the faces of the solids, and helps them to interact with the properties of the solids through dynamic transformation processes.

Analysis of the Grade 6 Teaching Experiment

We analyzed two teaching episodes, each from a different Grade 6 geometry classroom. These two teaching episodes were the last ones in the classroom teaching experiment and followed on from three teaching episodes in each of the classrooms where traditional geometric approaches with physical materials were used. The data consisted of field notes and transcribed video recordings. The analysis was a bottom-up process, where issues emerged and were tested during the teaching episodes. From the analysis, a number of issues emerged: the development of the tasks based on the computer representations, the type of interaction between the students and the teacher, and the development of students' thinking about the geometrical solid.

Episode 1

At the beginning of this episode, the teacher-researcher showed an oblique representation of a rectangular prism on the screen and asked students to describe it. The students measured the three dimensions of the solid by using the software and found that the length was 3.4cm, the height was 2.9cm and the width was 2.5cm. Therefore, it was clear that they recognized the solid as a rectangular prism in line with their past teaching experiences. Through follow-up class discussion, the teacher-researcher asked the students to predict which properties would need to be changed in order to transform the solid into a cube. The teacher-researcher used language such as "If you want to transform this solid to a cube what properties of the rectangular prism would you need to change?"

Initially, the students approached the task of transforming the rectangular prism into a cube rather intuitively and transformed the solid by simply decreasing the length of one of its dimensions. Some of the students recognized the new solid as a 'cube', while others thought there was a need to check if the new solid's dimensions were equal.

The teacher-researcher changed the context and asked the students to draw a cube at the board and identify whether the faces were congruent. Students drew an oblique representation of the cube, and tried to justify their opinions:

Teacher-Researcher: *Very well, why is this a cube?*

Student 1: *Isn't this a cube?*

Student 2: *What is it then*?

Student 1: *But we are trying to measure it…*

Student 3: *Despite the fact that it has all the sides equal, in order to be certain, we have to measure them.*

Teacher-Researcher: *Ok, why do you call this a cube? What did you do to get a cube? What did you do first?*

Student 2: *I made two equal faces.*

The teacher-researcher extended the activity aiming to encourage the students to focus their attention on the properties of the cube and to identify the implied drawing conventions:

Teacher-Researcher: *And what are these faces?*

Student 3: *Squares.*

Student 1: *Squares.*

Teacher-Researcher: *Thus, first I make the two faces which are square and then these lines to link the two faces. …*

Student 2: *These (lines) are the edges…*

Teacher-Researcher: *These are the edges. Now the faces now on the side … let's imagine them, as you see them, are they squares?*

Students: *No.*

Student 2: *No, it is a rectangle, no…*

Teacher-Researcher: *Why, why does this happen?*

Student 2: *Because they are oblique …*

Here, the students came to a conflict concerning the concept of the solid. In their previous experiences with the static and dynamic physical models, they had not considered the drawing conventions of the solids. The teacher-researcher introduced the isometric representations as a way to resolve the conflict. He asked the students to recognize an isometric representation of a rectangular prism and then asked them to predict what changes would be required to transform the rectangular prism into a cube. The students correctly identified that a cube must have equal dimensions and began assessing whether the rectangular prism's current edges were all equal. The transformation took place, and a cube with 3.8cm edges was created. The teacher-researcher intervened in the activity and asked the students to justify why the resulting solid was a cube.

The students based their justification on whether the edges were equal, but they did not pay attention to the fact that the angles were not equal. A progression of the students' reasoning was observed. Initially, the students considered the equality of the edges as the only crucial property for a solid to be considered a cube. Next, the students related the cube with the equality of its faces, and lastly, they intuitively attempted to relate the solid with the equality of its angles.

Initially, the students supported the view that the angles were equal. Then they looked for a way to justify their stance. Student 1, for example, proposed to consider the angles of a piece of A4 paper as a model for comparing the angles in the 3D representation of the solid. Student 1 held the corner of the paper up to the screen as a means of comparison. This comparison led the students to conclude that the angles of the solid on the computer were not equal. However, the students still understood that the solid was a cube, although they had discovered that the angles were not equal. Student 1's approach to the teacher-researcher's challenge to justify their opinion was rather intuitive:

Student 1: *We turned around the cube, something like this.* (Showing with her hands an imaginative movement.)

Teacher-Researcher: *Could you explain this to us?*

Student 1: *We turned the cube around to the front and the angles did not appear to be equal.* (She seemed to start to realize conventions.)

Teacher-Researcher: *How?* (He turns a physical model of a cube around in his hands.)

Student 1: *No, we have to turn it …. It is standing on an edge.*

Teacher-Researcher: *This way?* (He turns the physical model around to have an edge in a vertical position.)

Teacher-Researcher: *How does it appear? How do we see it? Like this?* (He kept the cube as an oblique projection where the two faces appear as squares.)

Student 1: *No, turn it, like this ….* (She turned the cube to appear as in the isometric projection.)

Teacher-Researcher: *How? Like this?*

Student 1: *Yes, something like that.*

Students: *Yes.*

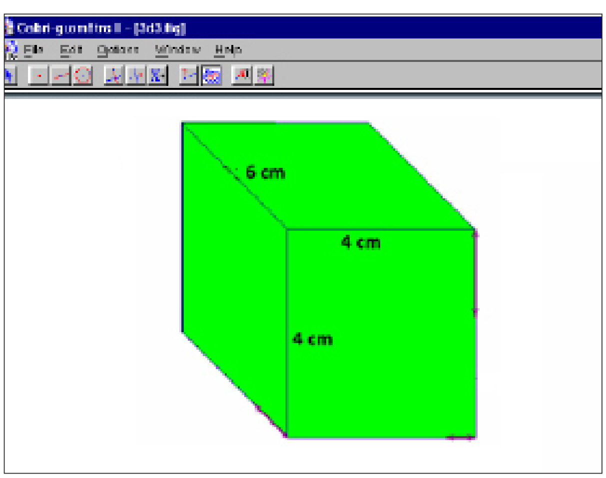
In this exchange, Student 1 was starting to understand the drawing conventions. Initially, she used gestures to assist in explaining her justifications. Later, the teacher-researcher gave her the opportunity to clarify her intuition through the instantaneous use of the computer representation and a physical model. Through the analysis of the previous interaction between the student and the teacher-researcher, it became apparent that the concept of representing 3D geometrical solids and the conventions used to design their plane representations require advanced intellectual processes from students.

Episode 2

In a different classroom, the students were presented with a 3D computer representation of an isometric projection of a rectangular prism. Students were asked to predict the changes required to transform this isometric projection of the rectangular prism into an isometric representation of a cube. The students proposed changing the lengths of the edges, by altering the height and the width of the solid, so as to make it "look like a cube." Apart from varying the dimensions and the edges that corresponded to each dimension in the solid, students made connections between the solid and its angles. They did that by referring to the definition of a cube: namely, that a cube has all lengths equal and all face angles equal to 90 degrees. In their effort to confirm that the solid was indeed a cube, they related the length of its dimensions to the length of its edges and the shape and size of its faces. The dynamic representation on the computer provided the opportunity to measure the length of the edges, the dimensions and area of the faces, and even the volume of the solid (Figure 3). Thus, using this particular strategy, they accurately transformed the solid to a cube.

Figure 3

An Oblique Representation of a Rectangular Prism with measurement functions enabled



Teacher-Researcher: *Are you sure that the (resultant) solid is a cube?*

Student 1: *If you can make it a bit shorter* (pointing in one dimension) *it would look like a cube.*

Teacher-Researcher: *How much shorter?*

Student 1: *A little bit.*

Teacher-Researcher: *Is this enough?*

Student 1: *Yes.*

Teacher-Researcher: *Are you 100% sure.?*

Student 1: *Yep.*

Students: (looking sceptical) *it may be….*

Student 2: *We need to make sure that all the edges have the same length and all the faces are congruent.*

Student 3: *Yes, we need to make sure that all edges have the same length.*

Teacher-Researcher: *How many are the edges?*

(Students are counting the edges).

Student 2: *There are 12 edges. On the screen we can see 9 edges and 3 edges are hidden on the back corner.* [Approach 1]

Teacher-Researcher: *OK. Do I need to check every single one of them?*

Student 2: *Yes, all edges should have the same length in order to be a cube.*

Student 3: *No, we don't need to measure every single one. Only three are enough. One for the length, one for the width and one for the height.* [Approach 2]

At this point, the students developed two different approaches for justifying their assumption that the solid they produced through the transformation was indeed a cube (as noted above). Both approaches required the measurement of certain properties. The first approach was based on relating the cube to the length of its edges, and so they proposed the measurement of this property. The second approach considered the length of the three dimensions of a solid as a key property to justify that the resultant solid was indeed a cube. The main difference between the two approaches was that, in the first case, the cube as the end product of the task was seen as independent of the original solid, while in the second approach, the cube was considered as being derived from the rectangular prism. In particular, the first approach was successful only in the case where the students correctly interpreted the definition of a cube, as a three-dimensional figure, which has 6 square faces, 8 vertices and 12 edges. The students focused only on the equality of the 12 edges of the final solid (a cube) not considering the properties of the original solid (a rectangular prism). They simply tried to make all 12 edges equal in length. On the other hand, the second approach was more constructive, because the students made connections between the original solid and the final solid. In the second approach, students investigated the properties of both solids and identified key properties that link the two solids, such as the lengths of the dimensions and the equality of the face angles. They faced a cognitive conflict that was counter-intuitive. Consequently, they tried to apply the first approach, but they realized that it did not work. Then, using the first approach, they transformed the solid into a cube. The appropriateness of the two methods was negotiated in the classroom discussion, and students realized that the measurement of the lengths of the edges of a cube was adequate.

Finally, the cube was transformed into a rectangular prism by increasing the length of one of its dimensions. During the investigation, students made comparative relationships concerning the variation of the form and the size of the faces of the initial solid (cube) and the final solid (rectangular prism). Moreover, the students compared the length of the edges and the dimensions of the two solids, and ultimately correlated the form of the faces with the length of the dimensions of the solid. While the students initially approached the concept of the solid intuitively, without paying attention to the solid's properties, the use of the dynamic representations allowed them to develop their analytic reasoning. Students built relationships between the properties of the cube and between different solids (the cube and the rectangular prism).

Discussion

The analysis of how the dynamic environment developed in mathematics classrooms highlights the constraints that plane representations of solids pose to students and their thinking. In particular, students, through the study of dynamic transformations, may realize some of the drawing conventions that exist in creating plane representations of solids. In addition, the dynamic transformations of geometrical solids on the computer software encouraged the students to dynamically study geometrical solids and to investigate relationships between the solids and their properties. Moreover, the environment helped students to build hierarchical relationships between different solids, the cube and the rectangular prism. By implementing a dynamic geometry, teachers may be able to improve student outcomes in 2D and 3D geometry. This highlights the importance of developing teacher knowledge in the area of dynamic geometric visualization tools.

It should be noted here that 3D computer representation software may not be accessible to all students. However, schools are gradually becoming more technology focused in the mathematics education environment (Kaput, Hegedus & Lesh, 2007). The increasing availability of educational technology allows for more widespread use of such software.

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Pre-Service Teachers' Discourse Moves During Whole Class Mathematical Discussions: An Analysis and Proposed Framework

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ABSTRACT Learning to facilitate whole-class mathematical discussions is a complex task. Teachers must become adept at using discourse moves to elicit and extend student thinking and move discussions towards a mathematical point. Practice-based mathematics methods courses give pre-service teachers opportunities to rehearse these moves in a controlled setting. Providing opportunities for practice, often called rehearsals or approximations of practice, have been shown to be effective at developing pre-service teachers' abilities to use discourse moves, but we still do not have a clear understanding of the types of discourse moves pre-service teachers use during discussions or how these moves change over the course of a discussion. In this article, we analyze. whole-class discussions facilitated by pre-service teachers to determine the types of discourse moves used as the discussion progresses. Based on this data and relevant literature, we propose a framework for the structure of whole-class mathematical discussions.

KEYWORDS *discourse moves, whole-class discussion, pedagogies of practice, teacher education*

Introduction

Facilitating mathematical discourse among students is a key component of reform mathematics teaching (NCTM, 2014). The type and quality of classroom discourse can be associated with differences in student affect and self-regulation (Turner et al., 2003), as well as differences in learning (O'Connor et al., 2015). Developing effective discourse within a learning community takes time and requires attention to posing purposeful questions, developing students' abilities to explain their mathematical ideas, balancing classroom power dynamics, and sharing responsibility for learning among the teacher and students (Hufferd-Ackles et al., 2004). Teachers play an active role in facilitating productive mathematical discussions. They employ discourse moves, which Krussel et al. (2004) define as, "the deliberate actions taken by a teacher to mediate, participate, or influence discourse in mathematics education" (p. 307). These moves elicit student thinking, orient students to each other's ideas, and move the discussion towards a mathematical point. For instance, a teacher might ask the class to explain whether or not they agree with a student's reasoning and provide support for or against the student's argument, or they might restate a student's idea to ensure it is understood.

While there are many resources to support teachers in leading mathematical discussions (e.g., Lamberg, 2019; Smith & Stein, 2011), it is challenging for pre-service teachers (PSTs) to develop the skills necessary to lead rich, productive discussions. When facilitating mathematical discourse, PSTs struggle with anticipating student thinking (Yilmaz & Yetkin-Ozdemir, 2021), knowing when and what information to give students (Selling & Baldinger, 2016), and steering discussions to a mathematical point (Baldinger et al., 2016).

Practice-based teacher education models have the potential to address the difficulties many PSTs have integrating the instructional moves they are learning about into their teaching (Grossman et al., 2009;

Lampert, 2010). In a practice-based curriculum, PSTs rehearse and enact core teaching practices as part of their teacher education courses and fieldwork (Grossman et al., 2009). Approximations of practice play an important role in supporting PST's learning to facilitate meaningful discourse in practice-based mathematics methods courses (Baldinger et al., 2021; Ghousseini, 2009). These activities help PSTs deepen their understanding of discourse moves, recognize the importance of anticipating student thinking, realize the need to steer discussions to a mathematical point, and improve their overall ability to facilitate student-centered dialogue (Baldinger et al., 2016; David, 2020; Freeburn, 2015; Spangle & Hallman-Thrasher, 2014).

However, we do not yet have a clear picture of how PSTs in a practice-based course facilitate whole class discussions. What discourse moves do PSTs use, and how do the moves change over the course of the discussion? In this paper, we examine the types of discourse moves pre-service teachers implement while facilitating student-centered, whole class mathematical discussions, and we draw from our data and from relevant literature to propose a framework for structuring whole-class mathematical discussions.

Background

Discourse Moves

While teachers can enter discussions with a mathematical point in mind and a general plan for how the discussion might play out, specific discourse moves cannot be scripted in advance. In the midst of the classroom discussion, teachers must interpret the situation and draw on these routines used for specific purposes, such as eliciting student contributions, responding to student thinking, and ensuring the discussion stays on track. A number of researchers have identified various moves used by teachers to facilitate discourse (e.g., Chapin et al., 2003; Ellis et al., 2018; Franke et al., 2015; Herbel-Eisnmann et al., 2016; Steele, & Cirillo, 2013; Staples & Colonis, 2007). Ghousseini (2008) describes five types of discourse routines: *revoicing*, orienting students to each other's thinking, pressing students for explanations, *connecting* students' ideas, and modeling or pointing to specific aspects of discourse. O'Connor et al. (2015) found evidence that students learn more in classrooms where teachers use moves that encourage students to share their thinking and engage with their peers' ideas.

Through approximations of practice, PSTs can increase their ability to use discourse moves effectively (Baldinger et al. 2021; David, 2020; Spangle & Hallman-Thrasher, 2014). For instance, Freeburn (2015) found that PSTs in a practice-based mathematics methods course developed the ability to pose advancing questions that prompted mathematical connections. Also, David (2020) found that PSTs increased their skill in facilitating mathematical discussions over the course of a practice-based methods course, becoming more adept at using talk moves intentionally to support and extend student thinking.

Structuring Mathematical Discussions

As teachers use discourse moves to promote math talk, they need support in knowing how to structure whole class discussions (Walker, 2014). They must navigate the tension between valuing student ideas as the basis for mathematical discussions while ensuring the discussions are mathematically productive (Sherin, 2002). Multiple frameworks exist to support teachers in structuring discussions that lead to a mathematical point. Smith and Stein (2011) have outlined five practices for orchestrating productive mathematical discussions. In these practices, teachers anticipate student thinking prior to the discussion, actively monitor students while they work, select students to present their ideas, sequence the presentations in strategic order, and draw connections among ideas. In another framework, Lambert (2019) breaks down mathematical discussions into three "levels of sense making." The first level, making thinking explicit, is where students communicate their answers. The goal of this level is for students to understand each other's thinking. The second level, analyze each other's solutions, engages the whole class in critically evaluating the ideas presented, and finally, the third level, develop new mathematical insights, is where the teacher scaffolds students' thinking toward a "big idea."

While these frameworks are helpful in breaking down discussions into manageable parts, we know little about how PSTs discussions are structured in practice. Some studies have examined aspects of PST planning for whole class discussions. Tyminski et al. (2014) examined how PSTs using Smith and Stein's (2011) five practices framework planned discussions. They found that PSTs were able to identify mathematical goals for their discussions and plan the discussion in ways that could support those goals. Meikle (2016) looked at the rationale behind PSTs' selecting and sequencing strategies.

In terms of examining how PSTs structure their discussions in practice, Conner, Singletary, Smith, Wagner, and Francisco (2014) analyzed the ways two student teachers supported collective argumentation in their student teaching placements. They identified three types of support the student teachers used to promote argumentation: direct contributions to the argument, posing questions, and using other supportive actions. Still, we do not have a clear picture of how the pieces of a discussion practically fit together from start to finish.

Therefore, the purpose of this study is to identify the discourse moves PSTs use over the course of a whole class discussion and then build a new framework for discussions informed by this data and previous discussion models.

Methods

The study was conducted in a semester-long mathematics methods course required for secondary education PSTs. The ten PSTs in the class were all pursuing an initial teaching license. The class met once a week for three hours for 15 weeks. No compensation was provided to the participants in the study. The second author was the course instructor. The first author was a student in the course; however, the first author did not join the research team in any way until after the course was complete.

Methods Course Instruction

One of the primary goals of this practice-based course was for the PSTs to develop their ability to plan and facilitate mathematical discussions. The students had been introduced to discourse moves and structuring student-centered discussions in a previous course, but the purpose of this course was to help students bring these skills together to facilitate high quality discussions. The instructor used the teacher learning cycle (McDonald et al., 2013) to facilitate this learning. This cycle has four phases (introduce, prepare, enact, and analyze) that help PSTs learn to enact core instructional practices. To provide context for this study, we describe all four phases of instruction. The data for our research was drawn from the mathematical discussions that PSTs facilitated during the *enact* phase.

In the *introduce* phase, the instructor modeled three different examples of mathematical discussions. The class analyzed these discussions. They also read about a range of discourse moves and about Smith and Stein's (2011) five practices. The instructor shared ideas for structuring whole class discussions drawn from Smith and Stein (2011) and Lamberg (2019), and the class engaged in an approximation of practice in which they were given student work and had to plan how they would facilitate a discussion centered on this work to highlight a given mathematical concept. Groups of students shared their plans, the class discussed them, and the teacher educator provided targeted feedback.

In the *preparation* phase of the learning cycle, the PSTs worked with a partner to design a problem-based lesson to teach to their classmates. Each lesson was centered on a single mathematical task and was composed of a launch, explore, and summarize portion. The PSTs did not teach the entire lesson at once. Instead, it was divided into sections to allow the PSTs to focus on each part individually. First, the pairs launched their lessons in class. During the launch the PSTs made sure their peers understood the problem the lesson was centered on. Following the launch, each PST completed the problems from each of their peers' lessons on their own for homework. The next class, each teaching pair reviewed all of their classmates' work and planned their whole class discussion, selecting and sequencing the strategies they wanted to highlight (Smith & Stein, 2011). The teacher educator provided feedback during this process.

In the *enact* phase, during the following class, each pair of PSTs led a mathematical discussion in which they selected and sequenced three different student (i.e. PSTs assuming the role of student) presentations of their work. Each discussion was video recorded and lasted about 25 minutes. Finally, in the *analyze* phase, the PSTs watched, analyzed, and reflected on the recordings of their discussions.

Data Collection and Analysis

The data for this study consisted of the five discussions from the enact phase. The video recordings of these discussions were transcribed for the analysis.

In order to determine the discourse moves PSTs use to facilitate mathematics discussions, we conducted a qualitative content analysis of the transcripts of the discussions. A qualitative content analysis uses a qualitative approach to assign codes to data, followed by a quantitative analysis of the frequencies of those codes (Mayring, 2015). We began the analysis by closely reading the transcripts multiple times to become familiar with the content to be analyzed. Then, we used a combination of deductive and inductive codes. We then created an initial list of 16 codes that described the discourse moves the PST utilized. Next, we combined these codes into groups of similar actions. This resulted in six categories of discourse moves described below.

Next, we found the frequency of each discourse move for each discussion. In order to determine if the types of discourse moves varied over the course of the discussion, we divided each discussion into three parts (Part 1, Part 2, and Part 3). The beginning of each part was marked by a new student presenter sharing their work with the class. For example, Part 1 of a discussion consisted of the first student's presentation of their solution method as well as all the discussion that followed up until the second student presenter began to share.

We then calculated the frequency of each discourse move for each part of each discussion. Then we considered how these frequencies aligned with components of discussions found in existing literature. We used these alignments to develop a framework for showing the ways discourse moves support whole class discussions. Last, we examined how closely our framework aligned with the five discussions and made final adjustments.

Categories of Discourse Moves in the PSTs' Discussions

Clarifying. This discourse move refers to questions posed by the PST to clarify student statements, as well as questions posed by the PST to prompt students to seek clarification. For example, to clarify a student's idea, one PST posed the question, "Like this or do you want me to add the ones in the middle?" By presenting this question to the student, the PST is asking the student to clarify which numbers to add in a problem to ensure the student's idea is properly represented in the discussion.

Revoicing. We categorized a discourse move as *revoicing* if the PST repeated or rephrased a student's idea or if they asked another student to repeat or rephrase the idea. Revoicing can be used to give the class more processing time and ensure all students understand an idea (Chapin et al., 2003). For instance, a PST asked the class, "Could I have someone..um..kind of restate what she said?" Another time the PST revoiced a student saying, "So she's asking, 'How do I use this to find the midpoint?'"

Probing Questions. We categorized moves as *probing questions* if the PST posed a question to assess or advance student thinking. Assessing questions are used by teachers to determine what students understand, while advancing questions aim to extend student thinking (Freeburn & Arbaugh, 2017; Smith et al., 2008). Both types of questions probe student thinking. For example, one PST asked, "Why did you divide by one fourth?" Another advanced student thinking by asking, "Can we [find the distance] diagonally without a measuring tool?"

Comparing and Evaluating. PSTs used *comparing and evaluating* moves when they asked students for, or provided, a comparison or an evaluation of student work. For instance, one PST prompted a turn and talk saying, "So let's take a minute and have everybody, in groups of two, just kind of discuss the similarities and differences between [this] method and [that] method." The same PST later posed the question, "So to kind of go off of that, which one would you guys say is more efficient?"

Initiating. The PST engaged in *initiating* by introducing a new mathematical idea into the discussion. For example, one PST described the idea of using a visual diagram to help with a fraction division problem saying, "We can use a visual diagram to split it up into fourths." While some teachers attempting to facilitate inquiry-based learning try to refrain from telling students anything, Lobato et al. (2005) argue that teachers can strategically insert new ideas into discussions to stimulate student thinking.

Connecting. A discourse move was coded as *connecting* if the PST prompted students to make a connection among two or more mathematical ideas or if they explicitly stated a connection. For instance, in one discussion a PST asked, "So how does that connect to the $3(4^n)$ that you have below?"

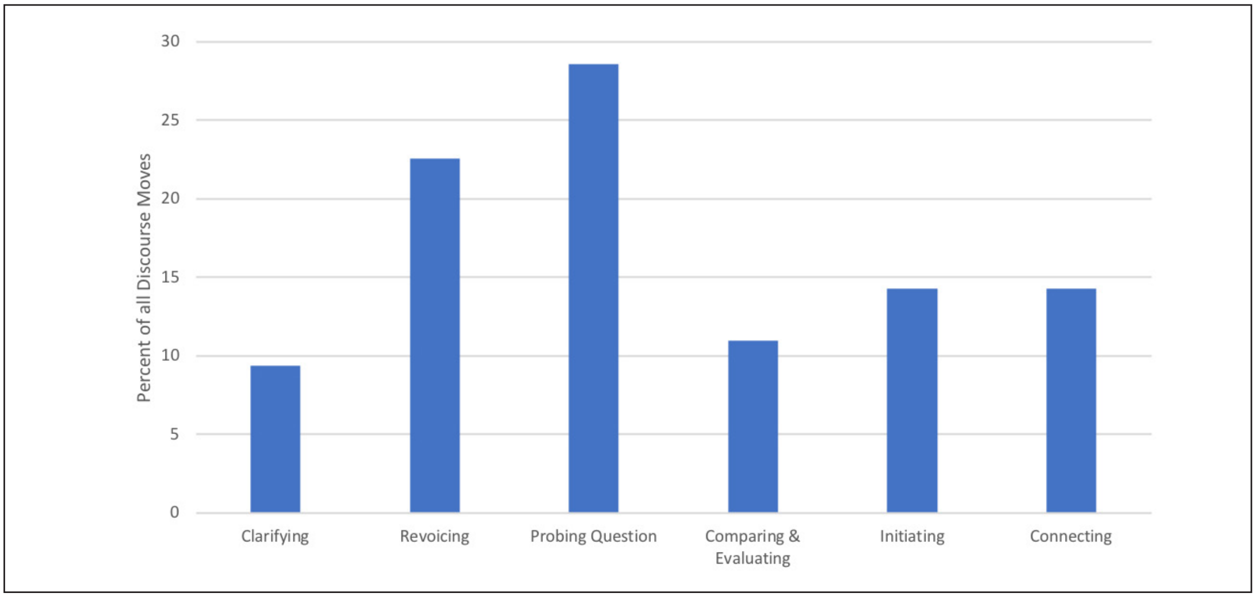
Results

Discourse Moves Used Overall

On average, each teaching pair of PSTs used 36.4 discourse moves over the course of their discussion (SD = 12.6). They used some types of moves more frequently than others. Figure 1 shows the percentage of discourse moves by category across all discussions. The most common discourse moves were *probing questions* (28.6%) and *revoicing* (22.5%), and the least frequent moves were *clarifying* (9.3%) and *comparing and evaluating* (11.0%).

Figure 1

Percentage of discourse moves by category



How Discourse Moves Changed Over Time

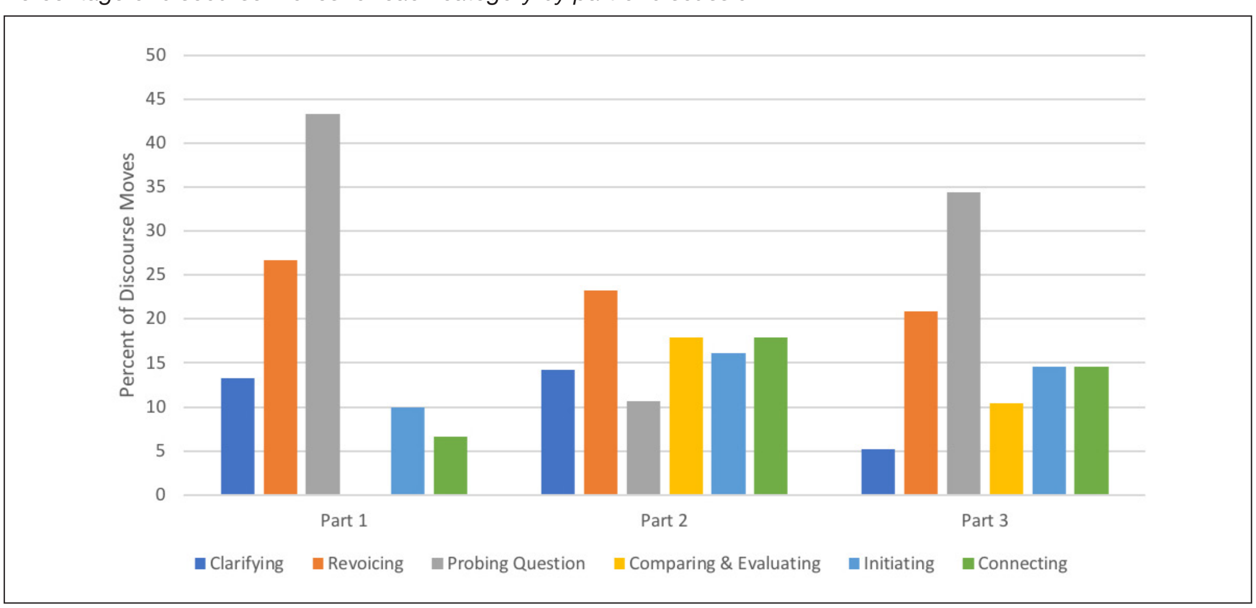
The way PSTs used discourse moves varied over the course of the discussion. In terms of number of moves, the PSTs used more discourse moves as the discussion progressed. They used an average of 6.0 discourse moves during Part 1 (SD = 2.1), 11.2 moves during Part 2 (SD = 7.6), and 19.2 moves during Part 3 (SD = 7.9). In other words, on average just over 50% of the discourse moves took place in Part 3. This demonstrates how the discussion builds as it progresses and how the PSTs did the most "work" in terms of discourse moves during Part 3.

In order to compare the types of discourse moves PSTs used in each part of the discussion, we computed the percentage of each category of discourse moves for each part of the discussion (see Figure 2). For instance, the first column on the left shows that *clarifying* moves made up 13.3% of all discourse moves in Part 1.

There are three results from Figure 2 that we would

Figure 2

Percentage of discourse moves for each category by part of discussion



like to highlight. First, when looking at the results for *comparing and evaluating*, we see that this move did not emerge until Part 2 where it accounted for 17.9% of the discourse moves used. Compare and evaluate was also used in Part 3, making up 10.4% of the discourse moves. Second, all of the other discourse moves, including *connecting* moves, were used throughout the three parts of the discussion. Finally, looking closely at *initiating* and *connecting* moves, we see that they make up similar proportions of the discussion as each other in each of the three parts of the discussion. For example, *initiating* moves make up 16.1% of the moves in Part 2, while *connecting* moves account for 17.9%.

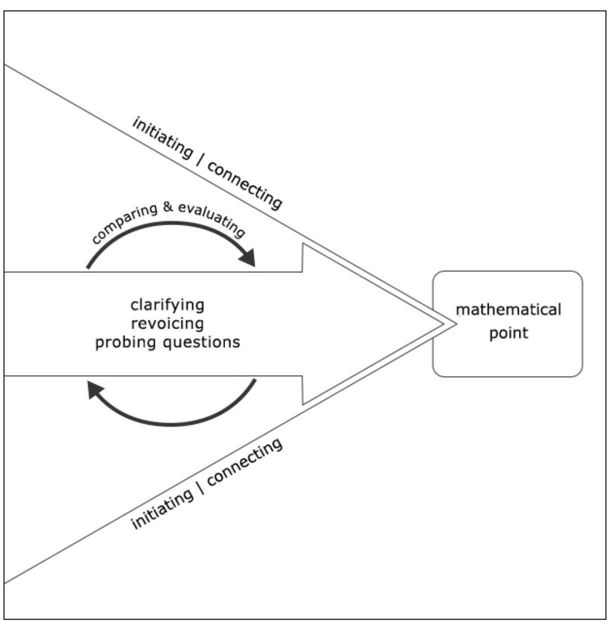
Discussion Framework

Examining how PSTs' use of discourse moves aligned with components of mathematical discussions identified in the literature has led us to propose a framework characterizing the way discourse moves support whole-class, inquiry-oriented discussions (see Figure 3).

Before the discussion begins, the teacher strategically selects and sequences student presentations in an order that will move the conversation toward the mathematical point of the lesson. In our study, each teaching pair selected three different students to share their strategies. However, this number may vary depending on

Figure 3

Framework for a Whole-Class, Inquiry-Oriented Discussion



the discussion. The PSTs elicited and advanced student thinking throughout each part of the discussion, using three discourse moves: *clarifying*, *revoicing*, and *probing questions*. These moves helped make student thinking explicit by ensuring that the teacher and the rest of the class understood what the student was saying, and they advanced student thinking by pressing them to think more deeply about their ideas. Taken together, these discourse moves are an example of how teachers can engage students in responsive listening (Empson & Jacobs, 2008) to support and extend student thinking. We represent them in our framework in Figure 3 by placing them inside an arrow to show how *clarifying*, *revoicing*, and *probing questions* serve to move the conversation forward.

Beginning in Part 2 of the discussion, we see PSTs prompt the class to compare and evaluate different ideas. This is not a separate, distinct phase of the discussion. Instead, *comparing and evaluating* moves are interwoven throughout Parts 2 and 3 as the teacher circles back to pull together strategies previously discussed with those currently being discussed. In Figure 3, this is represented by the circular arrows. These moves orient students toward others' thinking and prompts them to think analytically.

Finally, throughout the discussion, the teacher uses *initiating* and *connecting* to steer the discussion toward the intended mathematical point. Lobato et al. (2005) hypothesize that *initiating* moves can help students focus on important mathematical ideas. When used in the context of eliciting student thinking, *initiating* can play an important role in stimulating new ideas and helping students make sense of others. The PSTs in our study interwove *initiating* moves through all three parts of the discussion to help focus the discussion and move toward the mathematical point. Similarly, the PSTs used *connecting* moves in all three parts of the discussion. *Connecting* moves help students recognize relationships between mathematical ideas, often prompting the development of conceptual understanding (Gil, Zamudio-Orozco, & King, 2019). By intentionally and strategically *initiating* and *connecting*, teachers can "filter" (Sherin, 2002) through student ideas, sifting out those to focus on and use to build towards the mathematical point. This is represented in Figure 3 by two lines converging on the mathematical point showing how *initiating* and *connecting* serve to focus the conversation toward a learning goal.

Discussion

In summary, we found that PSTs used six different types of discourse moves to facilitate discussions: *clarifying*, *revoicing*, *probing questions*, *comparing and evaluating*, *initiating*, and *connecting*. The frequency of these moves varied over the course of the discussion, and the PSTs used more discourse moves as the discussion progressed.

We used this data to develop a framework for structuring discussions in practice. This framework has similarities to previous discussion models. First, our framework aligns with and builds on the work of Smith and Stein (2011). In preparation for a whole-class discussion, teachers engage in the first four of Smith and Stein's five practices—anticipating, monitoring, selecting, and sequencing. The framework proposed in this work focuses on Smith and Stein's fifth practice, *connecting*. By examining the primary discourse moves used during this phase and developing the framework to show how they work together, our work offers additional details on how teachers can structure the final *connecting* phase of the five practices.

Additionally, we see similarities between our framework and that of Lamberg (2019). The moves *clarifying*, *revoicing*, and *probing questions* align with the "making student thinking explicit" phase, *comparing and evaluating* has similarities to "analyze each other's solutions," and *initiating* and *connecting* is comparable to "develop new mathematical insights." However, while Lamberg (2019) describes the three levels of sense making primarily as distinct, sequential phases, our data shows that the PSTs interweave discourse moves with various purposes throughout the discussion. In this regard, our findings are more similar to that of Sherin (2002) who analyzed the discussions of a middle school teacher over the course of a school year and found that while the teacher included three structures similar to Lamberg (2019), the teacher progressed through the structures in a fluid, often cyclical manner.

While the PSTs in our study used a range of productive discourse moves, we did not see instances of teachers *modeling* or *pointing to specific aspects of discourse.* Ghousseini (2008) describes this meta-level discourse move as the teacher stepping outside of the conversation and commenting on it for the purpose of developing students' skills for participating in discussions. We hypothesize that students in our study did not do this for one of two reasons. First, it may be that this discourse move develops later after PSTs become more familiar with other discourse moves and facilitating whole class discussions. Second, it may be that the particular approximation of practice the PSTs were engaged in (i.e. teaching their peers in a methods class) removed the need for them to engage in this move because the PSTs acting in the role of students already had strong discussion skills. Given that Sherin (2002) describes the challenge of teachers navigating the tension between supporting the discussion process and ensuring students learn the content, it may be beneficial for PSTs to engage in approximations of practice similar to the one in our study prior to attempting to facilitate whole class discussions in an actual classroom.

We hope that the framework presented here will be helpful to both teacher educators and PSTs teaching and learning how to facilitate productive discussions. The work of leading discussions is complex, and by decomposing this practice, we hope to demystify the process. As we move forward with our research, we plan to examine how teacher educators can use this framework as a teaching tool to help PSTs develop their abilities to facilitate whole class mathematical discussions.

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Challenges in Using African Languages as the Language of Learning and Teaching (LoLT) for Mathematics in Rural Schools: Foundation Phase Teachers' Perspectives

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ABSTRACT The South African Language in Education Policy (LiEP) advocates using African languages in the Foundation Phase (FP) which is the first phase of formal schooling in South Africa. This paper seeks to identify the challenges in implementing the language provisions prescribed in the LiEP in South Africa and within the region. Using the challenge-based learning (CBL) approach, this paper provides a glimpse into the challenges that 50 FP teachers based in six rural schools in South Africa face when using African languages as LoLT when teaching mathematics. Data collected through questionnaires, focus-group interviews, and observations in mathematics classrooms were used to illuminate these challenges. The results indicate that teachers experienced three main challenges: multilingual learners and learners who were taught in English in pre-school, English scripted lesson plans, and teacher training that was only provided in English. Importantly this paper identified a language disjuncture between policy and practice where the policy requires teachers to teach in African languages while the teaching material is written in English.

KEYWORDS *African languages, challenge-based learning, mathematics teaching, language of learning and teaching, rural schools*

Introduction

This paper draws on community-engaged research conducted from 2018 to 2020 by academics in an open and distance learning university, the University of South Africa (UNISA), in the rural communities of three provinces of South Africa: North West (NW), Limpopo, and KwaZulu-Natal (KZN). These three provinces were selected for this study because most of the population residing in these provinces is located in rural settlements, and the pre-service teachers trained in this ODL university are employed in rural schools. Some of the academic responsibilities in an ODL university are to engage with communities to develop strong relationships and share resources while advancing and enhancing scholarship in what is usually called community engagement. In response to UNISA's community engagement mandate, this study aims to identify challenges experienced when using African languages as a language of learning and teaching (LoLT) for mathematics as well as sharing reading and mathematics resources with identified schools in rural communities.

Permission to collaborate with the schools was requested from the three District Directors, who, together with their team members, granted us permission and selected two schools in each province to participate in this community engagement project. Furthermore, ethical clearance was granted from UNISA, followed by the research request application at each provincial office of education. We conducted a needs analysis using an open-ended questionnaire to set up specific, measurable, and relevant deliverables for the selected schools, many of which focused on challenges teachers faced when using African languages to teach mathematics. Based on these initial findings, we, together with the beneficiaries and district officials, jointly decided to undertake a study to explore the challenges experienced by teachers when teaching mathematics using African Languages and to uncover possible solutions.

Naziev (2018) highlights that mathematics requires language to build, discover, understand, teach, and learn. However, there are different views about the specific language that needs to be used. Evidence from the literature suggests a significant relationship between language use and mathematics learning (Smith, 2017). Peng, Lin, Unal, and Lee (2020) argue that African languages as the LoLT of mathematics leads to high quality teaching and learning. Peng, et al., conducted a meta-analysis of 344 studies involving 393 independent samples to determine correlations between using LoLT in mathematics and its success in the mathematics classroom. The main finding from Peng et al.'s (2020) study suggests that language should be used to communicate, represent, and retrieve mathematics knowledge as well as to facilitate working memory and reasoning during mathematics performance and learning. They further established that the use of language to retrieve mathematics knowledge may be more important for foundational mathematics skills.

The South African Language in Education Policy (LiEP) (Department of Education, 1997) advocates the use of African languages in the Foundation Phase (FP) in accordance with Section 6 of the Constitution of the Republic of South Africa which promotes the use of all 11 official languages that were historically neglected in the Apartheid era. Section 29 (2) of the Constitution states that everyone has the right to receive education in the official language or languages of their choice in public educational institutions where such education is reasonably practicable. This background is supported by Stoop (2017), who encourages teaching learners in their preferred language, usually their mother tongue. FP is the first phase of formal schooling in South Africa. It involves teaching from the Reception Year, which is called Grade R (for children between 5-6 years of age). The phase ends with Grade 3 teaching children up to the age of 9 years.

Theoretical Framework

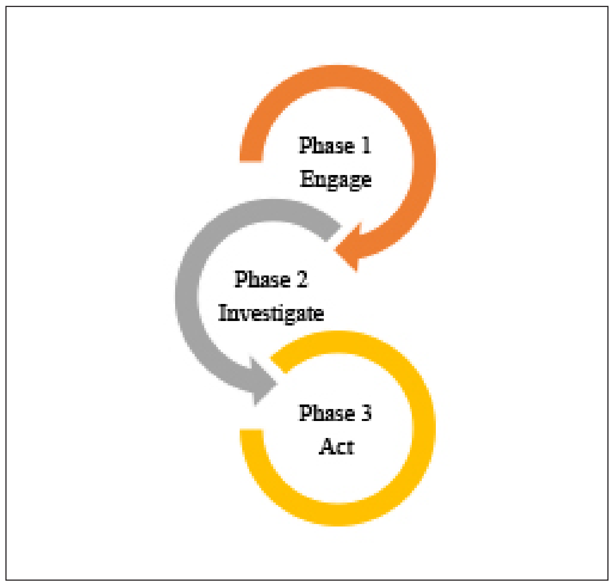
This study is underpinned by the challenge-based learning (CBL) framework adapted from Nichols et al. (2016). According to Nichols et al. (2016), the CBL framework emerged from the 'Apple Classrooms of Tomorrow Today' project initiated in 2008 to identify the essential design principles of a twenty-first-century learning environment. We found CBL to be a suitable theoretical framework for this paper because it is used in universities, schools and institutions worldwide to empower learners, pre-service teachers, administrators and community members to address local and global challenges.

As highlighted by Johnson et al. (2009), considering problems of global importance is a defining characteristic of challenge-based learning. Thus, this paper's literature review reflects on studies conducted nationally and globally. Another reason for identifying CBL as the relevant theoretical framework for this paper is that it frequently utilizes collaborative learning experiences. In this study, university academics, teachers and department officials worked together to learn about challenges experienced by teachers when using African languages as LoLT in a rural school and proposed solutions. Nichols et al. (2016) expanded on implementing CBL on challenge activities using the three phases illustrated in Figure 1.

We used these three interconnected phases for the CBL framework in Figure 1, developed by Nichols et al. (2016), to establish a community engagement project, collect data and design intervention strategies.

Figure 1

Challenge-Based Learning Phases



Literature Review

The literature reviewed in this section focuses on studies aimed at promoting the use of African languages in teaching mathematics. We include studies that are focused on promoting the use of Indigenous languages in teaching mathematics to align the current study to international trends.

Experiences of using Indigenous Languages as the LoLT for Mathematics in Africa

Nkonde et al. (2018) used a case study in Zambia's primary schools to do a preliminary evaluation of the impact of using African languages as the LoLT for mathematics. Surveys, interviews, and focus-group discussions were used to gather data from 30 primary school teachers in one of Zambia's provinces. Despite the challenges associated with using African languages as the LoLT for mathematics, the teachers were still in favor of this practice (Nkonde et al., 2018).

Nahole and Haimbodi (2022) conducted a study to explore challenges that pre-service teachers faced in teaching mathematics using indigenous languages. They administered a questionnaire to 90 pre-service teachers from the Rundu campus of the University of Namibia. Their findings revealed that pre-service teachers faced difficulties teaching some mathematical concepts that were literally translated from English to indigenous languages. Nahole and Haimbodi (2022) recommended that further studies be undertaken to explore how teachers teach mathematical concepts with no corresponding words in indigenous languages.

International Views on the Use of Indigenous Languages as LoLT for Mathematics

Turning now to the international context, Edmonds-Wathen, Owens, and Bino (2019) report on a Papua New Guinea project whose purpose was to support elementary teachers in integrating i ndigenous cultural practices and learners' home language in the implementation of the mathematics syllabus. Through a one-week workshop, Edmond-Wathen et al. (2019) found that teachers responded positively to the language resources and showed a willingness to use their own knowledge of indigenous culture and language to develop and teach mathematics curriculum. These findings become relevant in this study as the participating schools are in rural communities.

In the context of the Philippines, Perez and Alieto (2018) determined the correlation between mother tongue proficiency and mathematics success. A descriptive, correlational, non-experimental, cross-sectional research design was used with 71 Grade 2 Filipino learners. They found that learners with high mother tongue proficiency also achieved high scores in mathematics. Perez and Alieto (2018) concluded that learners taught in their mother tongue can make sense of mathematical concepts discussed in the classroom. This evidence suggests that there is no need to promote the English language as a LoLT of mathematics over the mother tongue.

Challenges Experienced when using African Languages in Teaching and Learning of Mathematics

In the South African context, the LiEP encourages the use of the African languages (mother tongue in other contexts), in FP teaching. Essien (2018) undertook a study to review the role of language in the early years in Kenya, Malawi, and South Africa. The study aimed to evaluate research conducted over 10 years on using African languages as the LoLT of mathematics and the findings pointed to the following challenges:

- No source of reference when using African languages as LoLT for mathematics due to the lack of longitudinal studies that investigate the impact of language on the teaching and learning of mathematics
- Many African languages lack the mathematical vocabulary necessary to teach mathematics
- Teachers' preference for using English over African languages when teaching mathematics due to a need to translate most of the mathematical concepts from English to an African language and the fact that they were trained to teach in English.

These challenges strengthen the need to explore the use of African languages as the LoLT of mathematics in primary school, which is the focus of this study.

Strategies for Teaching Mathematics using African Languages

In Kenya (Njoroge (2017) used a pre-and post-test to measure the efficacy of using an African language in mathematics and science teaching. He divided Grade 1 learners into two groups: one group was taught in Gĩkũyũ (Kenya's predominant home language), and the other group was taught in English. The findings indicated that using an African language as the LoLT for mathematics and science was effective. The group taught in Gĩkũyũ recorded better performance. The mean score obtained in the post test examination is higher than that in the pretest on the experimental group in the two subjects than the group taught in English (Njoroge, 2017). These findings imply that in Kenya, using African languages in mathematics teaching has the potential to improve learner performance. However, Njoroge recommended the following strategies to realize that potential:

- Avoiding the use of English as a sole language to improve classroom engagement and discussion.
- Using and adopting the African language as a legitimate language of mathematical and scientific communication to create opportunities for learning.
- Using home language (which is African language) as the LoLT.

Lastly Njoroge (2017, p. 142) calls for "a deliberate, proactive and strategic use of the learners' home languages as a transparent resource in the teaching and learning of Mathematics and Science in multilingual environments."

Method

This study employed a qualitative exploratory case study design which is described by Turner (2019). This process requires researchers to embed themselves in the study context for a long period. Turner further asserts that the exploratory case study design enables the researcher to thoroughly observe the participant and the research sites in order to learn about them. Teachers' views and experiences were collected through questionnaires, focus group discussions and classroom observations in community-engaged research, which adapted the CBL's three phases illustrated in Figure 1 by Nichols et al. (2016). Permission was obtained from the Provincial Departments of Education, teachers, and parents of learners for classroom observation (Engagement phase). Data was collected during the investigation phase from the 50 teachers who were purposefully sampled from the six schools using questionnaires, focus-group discussions, and observations while sharing knowledge, expertise, and resources to develop support strategies/material to address the identified needs. In the last phase (Act phase), we conducted workshops to support the FP teachers. Data from the questionnaire were teachers' perspectives, and focus group interviews were used to gather their views on the challenges they experience when teaching mathematics in African languages as the LoLT. Observations were used to confirm the views and perceptions gathered from the teachers.

Data Analysis

All the data collected were prepared and grouped according to the methods used in collecting the data: questionnaire, focus group discussions and observations. From the questionnaire, the first part of the data analysis focused on the participants' profile which is presented in Figure 2. Secondly, we identified challenges to determine similar and different challenges across the schools. Data from focus-group discussions were transcribed, and the transcriptions were combined with classroom observation reports and loaded into Atlas.ti for analysis. We coded all challenges we identified in the data and afterwards categorized them according to their sources. Example categories are challenges emanating from LoLT, teacher training, lesson preparation and presentation. From the categories, the following themes emerged: the LoLT of mathematics, English scripted lesson plans, and teacher training.

Results

To maintain the teachers' and the schools' anonymity, we used letters of the alphabet to name the schools, for example, Schools A to F. For the teachers, they created pseudonyms with reference to the schools they come from; for example, SAT1 stands for Teacher Number 1 from School A. Figure 1 presents the number of participants per school. The pseudonyms are outlined in Table 1.

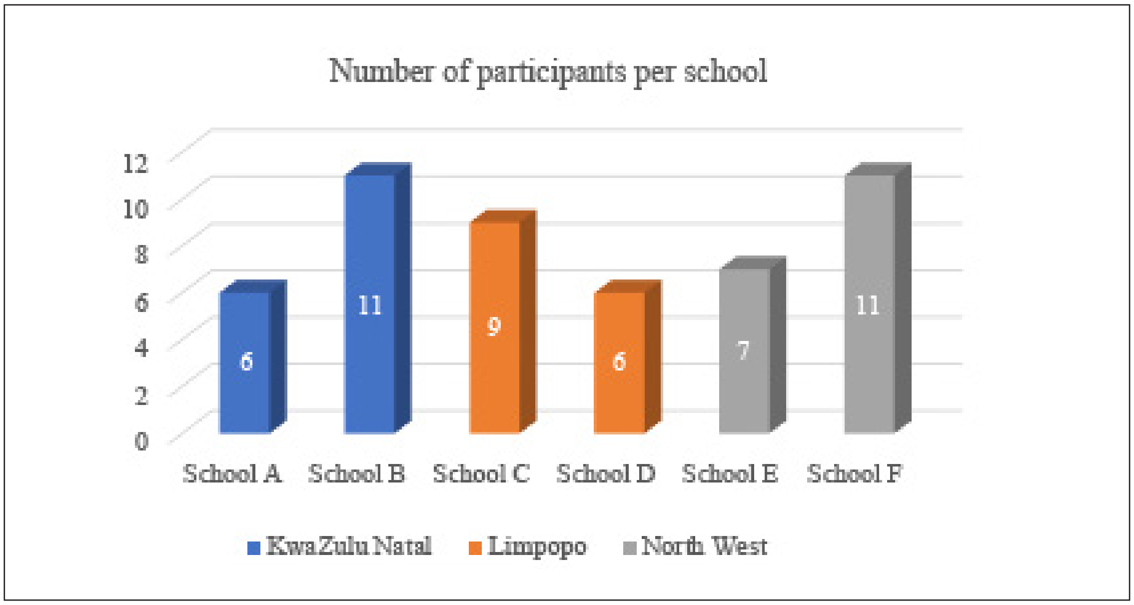
Figure 2 shows that the schools with the highest number of participants were in NW with 18 teachers (seven in School E and eleven in school F). KZN came second with 17 teachers (six in School A and 11 in School B). Limpopo schools had the lowest number of participants (nine in school C and six in school D).

Table 1
Pseudonyms

Pseudonym	Whom does it represent?
SAT1	Teacher Number 1 from School A
SAT5	Teacher number 5 from school A
SAT4	Teacher number 4 from school A
SBT1	Teacher number 1 from school B
SCT1	Teacher number 1 from school C
SCT3	Teacher number 3 from school C
SCT4	Teacher number 4 from school C
SET1	Teacher number 1 from school E
SET2	Teacher number 2 from school E
SET4	Teacher number 4 from school E
SET5	Teacher number 5 from school E
SFT1	Teacher number 1 from school F

Figure 2

Number of participants per school



The first set of questions aimed to profile the languages of the schools; we looked at the home languages of teachers and their learners and the LoLT of each school. Schools E and F's LoLT is Setswana; Schools C and D's LoLT is Tshivenda, and Schools A and B's LoLT is isiZulu. It was evident that in KZN both teachers' and learners' Home Language (HL) is isiZulu, and they use isiZulu as LoLT. A similar finding was apparent in Limpopo schools where both teachers' and learners' HL is Tshivenda and the LoLT is Tshivenda. The findings in Northwest schools were different, as evidenced by the fact that both teachers and students spoke 5 specific languages.

From the data in Figure 3, it is apparent that NW schools are multilingual. The most striking result from

Figure 3

NW Schools Various Languages

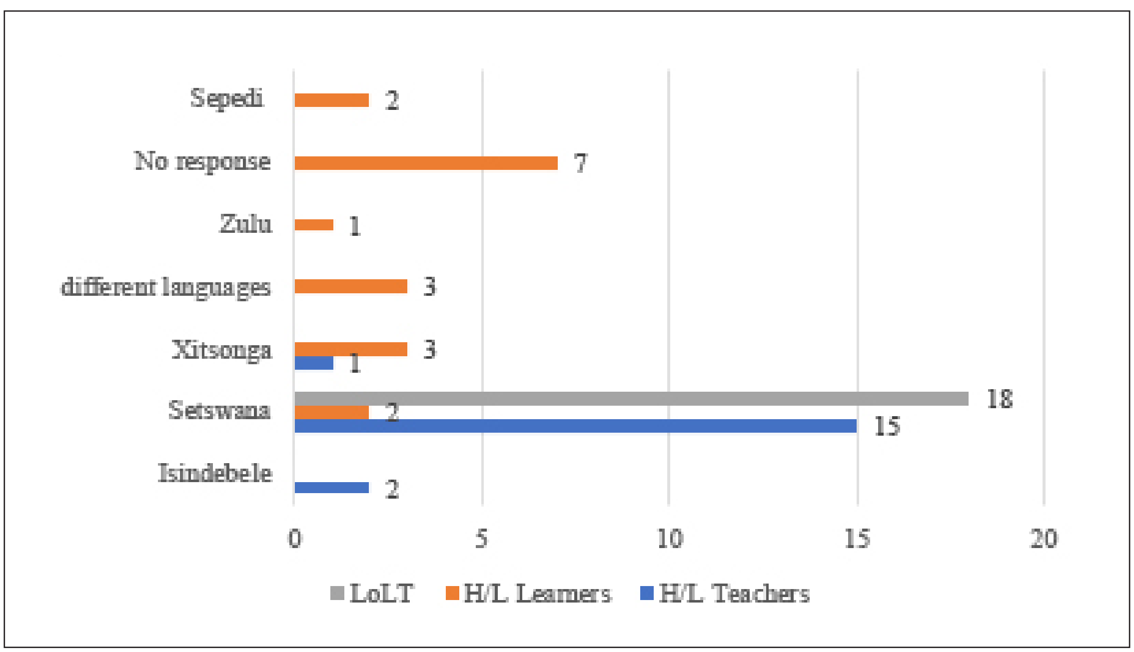


Table 2

Competency to Use LoLT to Teach Mathematics in FP

Province	EXCELLENT Use LoLT through the lesson	GOOD Use LoLT but switch to English	FAIR Use a bit of LoLT and English dominantly	INCOMPETENT Use English throughout the lesson	NO RESPONSE
KZN	2	13	1	0	0
Limpopo	0	15	0	0	0
NW	0	5	9	0	1

the data in Figure 3 is that only two participants indicated that the learners' home language was the same as LoLT which translates to only 11% of learners. However, most teachers are Setswana speaking (15 out of 18). There are 7 teachers who did not specify their HL, that could be because their HL was not listed in the questionnaire, or they omitted the question by mistake. The single most striking observation to emerge from the data is the NW language profile which links to Rivera and Ward's (2017) conclusion that learners in these schools tend to have difficulty in understanding mathematical concepts as they are presented in African languages other than their own.

After profiling the schools' languages, we focused on this question: "How would you describe your competency in using the LoLT for teaching mathematics?" The responses to this question are summarized in Table 1.

Table 2 shows a positive relationship between Figures 3 and Table 1. From Table 1, NW teachers seem not to be confident using the LoLT to teach mathematics even though 15 out of 18 were Setswana-speaking.

The questionnaire, focus group interviews, and classroom observations data presented the following key challenges affecting FP teachers in the six rural schools when teaching mathematics using African languages: the LoLT of mathematics, English scripted lesson plans and teacher training. These are described in more detail in the following section.

Key Challenges Affecting FP Teachers

The LoLT in Mathematics

The findings indicated that classes consisted of multilingual learners and learners who were taught in English in pre-school. Data from the questionnaire revealed that in School A, where the LoLT was Setswana, the learners could not read Setswana, and the teacher read and interpreted for them. "Not easy, for learners, they cannot read, they understand when interpreted by the teacher". "Some speak isiZulu, Xitsonga, and Sepedi at home and at school they use Setswana to learn" (SBT1). In Schools C and D, teachers were competent in teaching mathematics in Tshivenda; however, they found it difficult to teach certain mathematical concepts in the African languages. "Some of the mathematics concepts do not have terminology in Venda" (SCT3). In Schools E and F, the LoLT was isiZulu. Most teachers from school F indicated that teaching mathematics in isiZulu was confusing. Furthermore, teachers from School E explained that lesson plans for mathematics were written in English while the LoLT was isiZulu.

We tried to clarify some of the questionnaire responses, such as "teaching mathematics in isiZulu is confusing, during the focus-group interviews. The teachers provided additional context and confirmed that using African languages as the LoLT for mathematics posed challenges, such as translating some mathematical concepts because they did not have the vocabulary to teach in the LoLT "Learners understand English math terminology better than isiZulu" (SCT1). "Our main challenge is that children do not know Setswana because they are taught in English in the pre-school. They know numbers and colors in English" (SAT1). In School A, teachers said, "Translation often causes confusion (how to teach shapes because they change sekwere [square] or khutlonne [rectangle] also confuses khutlonnetsepa [rectangular])" (SAT4). "Translation causes arner dependency on teachers" (SAT5). In School B, they indicated that it is difficult to translate some of the terminologies, leading to a delay when a teacher stops and thinks. Apart from the challenges raised regarding African languages as LoLT, the School F teachers felt that it would be better to teach in English. “English terms are simple and straightforward” (SFT1).

When we carried out classroom observations, they observed that the teachers tried to make sure that the materials displayed on the walls (e.g. posters) were written in African languages to enrich the learners’ vocabulary (see Figure 4).

Figure 4

Posters Displayed in the Classrooms

English Scripted Lesson Plans

Schools E and F teachers explained that the scripted lesson plans they received from the Department of Basic Education and the non-government organizations that supported them were written in English. “Lesson plans for mathematics is written in English while teaching and learning is in isiZulu” (SET1). SFT3 further explained, “The curriculum is supplied in English, yet we teach math in isiZulu.” The teachers wrote in the questionnaire that teaching materials were in English and African languages. SET2 stated, “Learners … confuse the language if it is written in English and isiZulu.”

During the focus group interviews, all teachers explained that translating the lesson plans into African languages was time-consuming and prevented them from completing the syllabus. This is confirmed in the following excerpt: “… this exercise is not only difficult but time-consuming” (SBT1). In School C, teachers mentioned that some concepts only had English terminology and they needed pictures to explain those concepts. “We struggle to understand terminologies in English, let alone translating it into isiZulu” (SET1). The language used in the policy and lesson plans for mathematics is difficult and confusing. When asked to elaborate, SCT4 explained: “Terminology in the policy is different from lesson plans. They use different terminology for one concept.”

Teacher Training

Teacher training also contributed to a lack of understanding of some of the mathematical concepts in their African languages as it was done in English. “I was trained to teach mathematics in mother tongue, and learners are struggling in writing number names and other concepts in isiZulu” (SET5). During the interviews, some of the responses indicated that the lack of constant use of African languages hindered them from effective teaching, especially when they had to use concepts that were not familiar to them. “The language used in lesson plans and policy to teach mathematics is difficult and confusing” (SET4).

Discussion

Under Results, beginning on page 22, we used the three interconnected phases for the CBL framework, namely: engage, investigate, and act. In this section, we detail how we used the CBL framework phases to discuss the findings of this study.

Engage

In this study, the ‘Engage’ phase was used to understand the FP teachers’ challenges when teaching mathematics using African languages. We identified the FP teachers’ challenges in using African languages when teaching mathematics through the questionnaire, focus-group interviews and classroom observations. Apart from collecting data from FP teachers, we consulted the literature focusing on teaching and learning mathematics using African languages. The challenges found in the literature linked with the findings are the African terminology for mathematical concepts, multilingual learners, and teacher training. Essien (2018) revealed the disadvantage of translating English mathematical concepts into African languages in that some concepts can only be in a phrasal form, not single words. The participants also raised concerns regarding translating mathematical concepts in African languages, leading to a delay in completing their teaching and learning activities.

Teachers from Schools A and B highlighted the challenge of having multilingual learners in one classroom, resulting in difficulty for other learners to understand mathematical concepts presented in different African languages. This challenge was also confirmed by Rivera and Ward (2017) and Umar (2018). In some instances, teachers code-switched for learners to understand what was being taught, but some teachers could not speak children's home languages in the classroom. Chitera et al. (2016) established that a lack of understanding of the mathematical concepts in other African languages while teaching affected the teachers' self-confidence.

Essien (2018) pointed out the challenge of some African languages that lack mathematics vocabulary. This challenge is seen as a contributing factor to the limitations of how teachers should be trained to teach mathematics using the African language in the FP. From the variety of challenges experienced in Schools A to F and those in the literature review, it seems that the LiEP advises schools to choose a LoLT, as per Section 8 of the LiEP (DoE, 1997) to redress the neglect of the historically disadvantaged languages in school education. However, based on the provincial government's failure to provide alternative language maintenance programs as mentioned earlier in this section and the South African government's provision of materials that are written in English only, one may conclude that the LiEP policy was implemented without a proper support mechanism and plan.

Investigate

In this phase, we and the participants explored the benefits of using African languages in mathematics teaching. Teachers did not mention any benefits of using African languages as the LoLT for mathematics. They described challenges they experienced in the classrooms, frustrations when translating, and fears of learner dependency. The benefits of using African languages in teaching mathematics were gathered from the literature. Some of the benefits identified by Nkonde et al. (2018) included improved participation and learner confidence. Since learners use the languages they understand, there will be no need to memorize the mathematical concepts. Teachers will have more time to focus on teaching and learning activities. Chitera et al. (2016) confirmed the benefit of using languages that both teachers and learners understand, emphasizing that there will be a flow of communication in the classroom leading to improved learner participation.

Act

The "Act phase" was used to solve the challenges for this study. There is some evidence to suggest that the teachers from the six schools tried to solve the challenges faced when using African languages during mathematics teaching. After realizing that learners did not understand mathematical concepts in African languages, the teachers tried to solve this challenge by translating the concepts using both English and African languages. Although the translation solved part of the challenge, it created other difficulties that brought frustrations to the teachers, such as time constraints in covering the curriculum. In School C, they solved the challenge of translating specific terms by using pictures to simplify the concepts for the learners.

Conclusion

This paper found significant implications for understanding the challenges experienced by FP teachers when using African languages as LoLT for mathematics. The following conclusions can be drawn from the findings:

- Some FP teachers' home languages are not the LoLT at the schools where they teach.
- Some FP learners are taught in their second or third languages.
- There is a language disjuncture between policy and practice where the policy requires teachers to teach in African languages while the teaching material is written in English.
- The language used in the training of teachers in institutions of higher education, namely English, contributes to the language disjuncture.

Although this study focused on challenges experienced by FP teachers, the findings may well have a bearing on mathematics learners' performance which is assessed by the Trends in International Mathematics and Science Study (TIMSS) every four years. TIMMS is a study that compares reliable and timely trend data on the mathematics and science achievements of different countries including South Africa. Even though the FP teachers in the six schools attempted to tackle some of the challenges they experienced, their interventions did not fully address the challenges.

Recommendations

The findings from the literature review and empirical data informed the following recommendations:

- Provincial departments of education should provide alternative language maintenance programs through in-service training.
- A participatory approach should be used where teachers develop African language mathematical guides.
- A mathematics language laboratory should be established in each school where community volunteers can support and enrich learners' mathematics vocabulary of the African languages.

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NOTES FROM THE FIELD

The Fall 2022 issue features two *Notes from the Field* discussing methods for supporting students' academic and social needs both in and out of the classroom. First, Mei et al. examine the implementation and impacts of long-term partner study, called MathChavrusa, during the online and hybrid learning periods of the COVID-19 Pandemic. Duggan follows by discussing strategies to build and incorporate empathy in the mathematics classroom as a means of promoting a positive learning environment.

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NOTES FROM THE FIELD

Perspectives on the Evolution of MathChavrusa from Pre-COVID to Now

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Introduction

Chavrusa is a long-term, dyadic study partnership traditionally used in the context of Talmudic study. Chavrusa study includes pairs reading, discussing, and exploring texts together (Liebersohn, 2006). A guiding principle of chavrusa learning is that students interact with each other as both teachers and learners. While long a pedagogical model used in Jewish institutions of learning (Kent et al., 2012), chavrusa has recently made strides in other areas, such as medical education (e.g., Chung & Lee, 2019) and undergraduate coursework (e.g., Bergom et al., 2011). Since chavrusa promotes deep thinking in pair learning, we can naturally ask the question: can chavrusa-style learning positively impact the mathematics classroom?

One application of chavrusa learning is MathChavrusa, a learning model where students are paired and work with a partner for the full duration of a math content course (Flint, 2019). Differing from typical classroom groupwork where students collaborate only for the lifespan of a particular assignment, the student pairings in MathChavrusa last for the duration of the course. As with Chavrusa, text analysis is still central, but MathChavrusa expands texts beyond just a course textbook to include additional course materials like short articles, classwork, and problem sets. In cases where additional social and academic support were necessary, students were put into groups of 3 or 4 in a format called MathChabura (where "chabura" means "group of friends" in Hebrew).

Figure 1

MathChavrusa partners studying Topology in Spring 2019.

What we present is a glimpse into how MathChavrusa is being implemented in graduate mathematics classrooms through the perspectives of the three authors: one facilitating instructor and two participating students. We discuss MathChavrusa's impact on learning mathematics and the effect COVID-19 remote and hybrid learning had on this learning model. The format of this discussion consists of two overarching questions with written responses from all three participant authors.

Instructor and Students' Perception of MathChavrusa

Question 1: Impact

a) **(For Instructor) How is MathChavrusa intended to impact students' ability to learn mathematics?**
b) **(For Students) How did MathChavrusa impact your ability to learn mathematics?**

Instructor

The most telling way to know if a person has understood something is if they are able to teach it in such a way that the student grasps it. The litmus test is that the student is able to demonstrate their ability to actively solve problems. In the MathChavrusa structure students discuss and unpack what they are learning with a peer, a form of teaching one another. The model creates a comfortable learning environment, alleviating math anxiety and helping students to combat their reluctance to ask questions for fear of being judged (Stoehr, 2017). Through MathChavrusa interactions, one can identify which areas of students' understanding need reinforcement and additional learning opportunities.

When students are in the immersive MathChavrusa experience, they observe and discuss different approaches in solving problems, combating a misconception that there is a unique way to solve problems. Frequently, I witness breakthrough ("aha") moments during conversations between chavrusa partners. The relationship naturally spills over to outside classroom learning and collaboration. It builds a strong bond and trust, creating an overall safe and productive culture for exploring and identifying with mathematics.

Student 1

As an international student from a country whose medium of instruction is not English, I adapted to the American model and its associated mathematical terminologies quicker with my patient and kind MathChavrusa partners who came to understand my strengths and weaknesses and answered my questions. Even though I am confident in my math abilities, I would say that overcoming the challenges of the language and less familiar instructional methods was much easier with a long-time study partnership.

Furthermore, the MathChavrusa model encouraged me to meet my partners outside of the classes. I got more confident about organizing external study sessions with my MathChavrusa partner, which helped me learn the content more deeply.

Student 2

MathChavrusa's long-term partner-based learning provided an extra brain to trade ideas with, helping to hasten understanding of freshly introduced material by encouraging me to discuss mathematics with my partner and ask for guidance more readily than more traditional learning models. The varied assignments that my partner and I worked through were also useful for my mathematical development, as getting another person's thoughts on how to multimodally represent mathematics expanded my own repertoire of mathematical tools.

Question 2: What are the differences between in-person, online, and hybrid MathChavrusa during the COVID-19 pandemic?

Instructor

At its core, MathChavrusa is amenable to online learning. The key is that two partners are working together. Logistically, MathChavrusa required a significant shift when we were catapulted into online learning. Instead of pairing students (MathChavrusa), groups of 3-4 members (MathChabura) were created. This way, if one's partner was absent or distracted (something we frequently contended with during the pandemic) no one would be without a study partner.

At first, the feel that is developed through interpersonal interactions of being in-person over a course of several months was challenging to replicate in online settings. But the intrinsic concept of MathChavrusa as a social learning experience was pivotal in helping my students as we shifted to remote connections, since the MathChavrusa/Chabura model ensured that students had close connections with their classmates to navigate the new and tricky terrain of online learning. In fact, some students related that MathChavrusa time was their only opportunity to meet and engage with other students in all their courses during the semester.

The Zoom breakout room feature was particularly critical in facilitating the MathChavrusa model in synchronous classes. Although breakout rooms disrupted the culture of multiple groups working concurrently in real time, an advantage was that each room had the solitude of their individual chabura without the background distractions of other groups. Once chabura discussions concluded, pooling multiple groups' ideas together was done in a larger group session as opposed to in brick-and-mortar classrooms, where one can turn to a different group to ask questions and compare ideas as situations arise. To compensate for the lag time caused by breakout rooms, we had shared MathChavrusa Google documents that enabled students and the instructor to share materials and ideas in real time.

The hybrid model of combined in-person and online learning saw students meet with their chavrusas via either modality as circumstances dictated. We incorporated MathChavrusa to be embedded in MathChabura in order to accommodate absences.

Student 1

Considering the key aspects of the MathChavrusa model, my experience was that reading aloud with a partner was not central in online teaching because extenuating circumstances sometimes made it difficult for students to speak. Additionally, Zoom breakout rooms felt more isolating than in-person groups, since I could only share ideas with my MathChavrusa partners rather than everyone else around me. Despite the more restricted nature of remote learning, MathChavrusa made student interactions possible during the pandemic.

The Covid-19 pandemic also resulted in some students being unable to join some classes, and their Chavrusa partners would be left alone. With online classes, one MathChavrusa breakout room tended to include more than two students (two being the number seen as ideal when in-person). Thus, the adaptation to the chabura model functioned to overcome this challenge. The fact that there were more partners was helpful especially when not everyone had access to the same technology. For example, the members with drawing tools could more easily show their work and write mathematical expressions while those without still contributed by typing the presented work to their shared documents. That way, everybody had a chance to contribute to the learning environment.

Student 2

While partner interactions remained in online learning, it was more difficult to communicate some mathematical ideas due to the limitations of computer inputs (e.g. keyboards, mouse controlled drawings, etc.). Additionally, groupwork in Zoom breakout rooms resulted in fewer interactions amongst partners and the class and fewer opportunities for learning and exchanging ideas, something students appreciated during in-person MathChavrusa (Flint & Mei, 2020). However, this was partly compensated by grouping multiple MathChavrusa pairs together in one breakout room during collaborative study. On the flipside, remote learning made it easier to schedule times to meet with my partner, improving MathChavrusa's out of class social impact. At the macro level, that MathChavrusa mostly maintained its ethos and structure transitioning from in-person to online learning created a comforting familiarity to an otherwise tumultuous time.

Hybrid learning brought about the most fluid version of MathChavrusa. The class was separated into two discrete groups: those online and those in-person. The interesting part came when an online student attended in-person and vice versa. In these cases, the affected student had the benefit of working with both their MathChavrusa partner and other students in their learning setting. This was a novel experience since it organically allowed students to learn with a wide array of peers in an environment in which collaborative learning was encouraged by the culture set by the MathChavrusa learning model.

Figure 2

A student working on a Linear Algebra exercise with her virtual MathChavrusa during hybrid learning in Summer 2022

Conclusion

The three perspectives presented above provide insights into how the instructor and students perceived MathChavrusa's impact on learning mathematics in various learning modes: in-person, online, and hybrid. All participants noted the positive social impact of working with a long-term partner, with discussions between MathChavrusa partners a key factor in improving student learning outcomes. Online and hybrid MathChavrusa each have distinct qualities that promote social interaction, with the former emphasizing small group chabura discussions and the latter highlighting the social flexibility inherent in the MathChavrusa learning model. While these adaptations were crucial for bringing MathChavrusa into the digital realm, they also brought about fewer pair-to-pair interactions than in-person MathChavrusa, something that could perhaps be improved with future developments in digital technologies. Taken together, MathChavrusa is a flexible learning model that can be adapted to both in-person and online learning environments, and provides an especially strong base in growing students' interpersonal academic support networks.

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NOTES FROM THE FIELD

Empathy in the Math Classroom

Katherine Duggan
Teachers College, Columbia University

I will never forget what my mentor teacher told me when I was student teaching in Michigan: "You need to remember that you became a math teacher because you love math and you are good at math. Not all of your students are going to feel the same way." This advice has remained with me as I have begun my teaching career in New York, more so upon hearing countless grievances about math material. What seems to be a common thread throughout these complaints, however, is students often feel checked out and doubtful of their abilities in math. Moreover, they are avoiding feelings of embarrassment, frustration, or helplessness in front of their peers rather than taking intellectual risks. Thus, my mission as a math educator has been to foster classroom empathy and camaraderie among my students.

Discussions surrounding empathy in the math classroom are not new, but the more I read, the more I see a lack of tangible ways for educators to practice these skills. For teachers who have neither learned these approaches nor experienced this type of environment in their own school, it may feel unnatural to create it in their own classroom. While implementing empathy in the mathematics classroom may take many forms, these five suggestions are staples that I keep in mind when planning lessons throughout the year and are helpful to avoid that mid-year slump.

1. Ask students to address their feelings about math on day one.

On the first day of the school year, ask students to write a mathography, a personal account of their lived experience with mathematics. In addition, consider asking them questions such as: "What is your favorite memory that involves mathematics?" or "What does a mathematician mean to you?". This will encourage students to reflect more deeply on how math has affected them. Although these questions do not directly address their mathematical ability, they do provide insight into students' feelings and thoughts. Students can then slowly begin to see how math plays a different yet prominent role in their lives, and those of their peers. Analyzing their own biases and feelings about math can give students greater insight into their reactions to learning and sharing new ideas throughout the year.

2. Avoid immediately telling students their answer is incorrect.

For students who enter the math classroom with emotional baggage, immediately rejecting their answers not only discourages their participation, but also affects their self-confidence. Instead, it is essential to teach students the value of wrong answers. They provide you insight into how students are thinking; if one student shares their incorrect answer, there is a high probability that another student was thinking identically. Identifying the benefit of any student's answer or idea during discussions will then exemplify this for other students. If they are encouraged to participate and share their own growing ideas, they will encourage their classmates to do the same.

3. Build a culture of active listening.

As educators, we lead by example. If we want to create a positive classroom culture, then we need to exemplify this for our students. Giving students our full attention by facing them or using their words to extend the discussion, will encourage others to do the same, and continue to boost confidence. In addition, making eye contact and acknowledging other students in classroom discussions encourage empathy and give value to each voice in the conversation.

4. Grade homework for effort, not mastery.

Ongoing debates about the role and relevance of homework occupy several subject areas, but what seems most essential in mathematics is its extension of daily learning and additional practice of various topics. If homework is graded exclusively on mastery, the result is discouraged students who, after one class period of learning about a new concept/topic, feel as if they need to understand it completely. Rather, grading homework on effort and allowing students the opportunity to redo assignments conveys the idea that homework is an opportunity for growth. Furthermore, including more open-ended questions can allow students to reflect on their learning. For example, after a lesson on trigonometric identities, a homework question might be: "How would you explain what trig identities are to a third-grade student?." Answers to this question will often give you more insight into learning than a calculation.

5. Involve families in the conversation.

I have noticed that for most of my students with a fixed mathematical mindset, I often hear the same message when calling their parents to check-in. Parents often think that if they did not get high grades in math growing up, or if they are not a "math person", then their child will grow up the same way. By informing parents of how we are using empathy in the classroom and how discussions around math are centered, they can communicate the same message at home and build their child's confidence. Moreover, building this working partnership with families early on ensures that all students can cultivate a growth mindset. Fostering this classroom culture should be happening in and outside of the classroom.

Math educators often have a daunting task set before them. We need to teach our students about solving equations, derivatives, and the Pythagorean Theorem, to name a few of the hundreds of state-mandated topics. This task is often made more difficult by the mental hurdles students have placed in front of them, hurdles that have been building for years. If we can begin to break these hurdles and build a classroom culture full of empathy and a willingness to grow, students can gain lifelong skills and a greater appreciation for learning.

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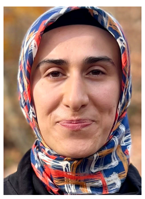

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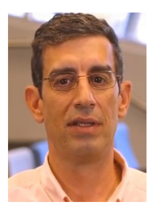

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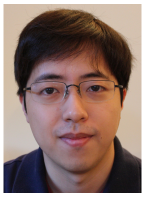

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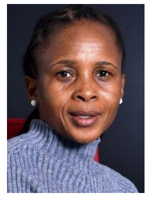

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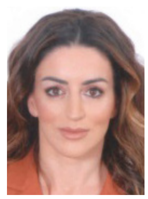

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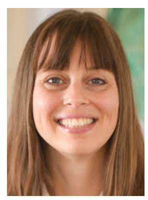

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ACKNOWLEDGEMENT OF REVIEWERS

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This call for papers is an invitation to mathematics education professionals, especially Teachers College students, alumni, and associates, to submit articles describing research, experiments, projects, innovations, or practices in mathematics education. The journal features full reports (approximately 3500 to 4500 words) and short reports (approximately 500 to 1500 words). Full reports describe findings from specific research, experiments, projects, innovations, or practices that contribute to advancing scholarly knowledge in mathematics education. Short reports ("Notes from the Field") provide examples, commentary, and/or dialogue about practices out in the field of mathematics education or mathematics teacher education; examples from classroom experience are encouraged. Although many past issues of *JMETC* focused around a theme, authors are encouraged to submit articles related to any current topic in mathematics education, from which pertinent themes for future issues may be developed. Articles must not have been submitted to or accepted for publication elsewhere. All manuscripts must include an abstract (approximately 150 words in length) and keywords. Manuscripts should be composed in Microsoft Word and follow APA format. Guest editors will send submitted articles to the review panel and facilitate the blind peer-review process. Articles for consideration should be submitted online at jmetc.columbia.edu, and are reviewed on a rolling basis; however, to be considered for the Spring issue, articles should be received by **February 1, 2023**.

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This call for reviewers is an invitation to mathematics educators with experience in reading or writing professional papers to join the review panel for future issues of *JMETC.* Reviewers are expected to complete assigned reviews within three weeks of receipt of the manuscript in order to expedite the publication process. Reviewers are responsible for editorial suggestions, fact and citations review, and identification of similar works that may be helpful to contributors whose submissions appear appropriate for publication. Neither authors' nor reviewers' names and affiliations will be shared with one another; however, reviewers' comments may be sent to contributors of manuscripts to guide revision of manuscripts (without identifying the reviewer). If you wish to be considered for review assignments, please register and indicate your willingness to serve as a reviewer on the journal's website: jmetc.columbia.edu.

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