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A Century of Leadership in Mathematics and Its Teaching

Beliefs and Perceptions of Learners and Teachers in Mathematics Education

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AIMS AND SCOPE

The *Journal of Mathematics Education at Teachers College (JMETC)* is a recreation of an earlier publication by the Program in Mathematics Education at Teachers College, Columbia University. As a peer-reviewed, semiannual journal, it is intended to provide dissemination opportunities for writers of practice-based or research contributions to the general field of mathematics education. Although many of the past issues of *JMETC* focused on a theme, the journal accepts articles related to any current topic in mathematics education, from which pertinent themes for future issues may be developed.

JMETC readers are educators from pre-kindergarten through twelfth-grade teachers, principals, superintendents, professors of education, and other leaders in education. Articles appearing in the *JMETC* include research reports, commentaries on practice, historical analyses, and responses to issues and recommendations of professional interest.

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PREFACE

What beliefs and perceptions about mathematics do individuals hold, and how do they affect their learning and teaching? The Fall 2023 edition of the *Journal of Mathematics Education at Teachers College* presents three research papers that delve into these questions in distinct contexts. The two following short reports carry on the theme through to teaching arithmetic, one from a historical perspective and the other from a classroom teacher's perspective.

Parker-Holliman and Maina open this issue by investigating the lived experiences of Black girls in accessing advanced secondary mathematics courses while in middle school. Through qualitative methods, the research highlights the impact of their perceptions of societal messaging and identifies protective factors that empower them against the odds.

Transitioning toward pre-service teachers' beliefs, Phelps-Gregory et al. explore the definitions of good teaching and learning. This qualitative case study reveals the diverse beliefs held by elementary pre-service teachers, shedding light on the complexity of their beliefs while providing insights into how their beliefs relate to their self-efficacy and teacher efficacy.

Tsami et al. provide a quantitative report from Greece on gender-based differences in student perceptions around the use of educational technology. It examines college students' perceptions regarding their comprehension and performance in learning probability theory.

Kaori Yamamoto
Jimmy Giff

Guest Editors

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Protective Factors that Yield Empowerment for Black Girls' Mathematical Brilliance

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ABSTRACT Black girls are marginalized and often experience barriers to accessing advanced mathematics, which affects their socialization and identity. Little is known about the experiences of Black girls who have gained access to advanced mathematics programs. The participants in this study were 11 middle school Black females enrolled in advanced mathematics, a course with a curriculum at a higher grade level and a faster pace compared to their same-age peers. Using a qualitative methodology, we use collective memory writings, individual and focus group interviews, and the researcher's journal data to examine how girls' perceptions of societal messages work to impact and empower Black girls enrolled in advanced mathematics coursework and extend current research on this topic. We conclude that Black girls have various protective factors--innate characteristics that yield positive outcomes, influencing their self-efficacy. The themes uncovered as a result were that Black girls are motivated by engaging in valuable mathematics that is meaningful to them; their perceived mathematical identity represents a protective factor. This research study illuminates that Black girls are brilliant, but only those with prominent protective factors are often recognized in educational institutions for their merit.

KEYWORDS *Black, girls, advanced, mathematics, identity, protective factors*

Introduction

This study aimed to seek a deeper understanding of how messages, relationships, and real-life experiences contribute to the empowerment, self-consciousness, and self-efficacy of middle school Black girls who participated in advanced mathematics coursework. This study examined how micro-messages are delivered intentionally through society and impact students' academic trajectories. Micro-messages are small and semi-conscious messages that become apparent in daily interactions, conversations, and instruction. Delivered when interacting with others, micro-messages are presented in five forms that include facial expression, tone of voice, hand gestures, choice of words, and eye contact; these messages can exude either a negative or positive connotation and shape every relationship, allowing educators to either damage student relationships or forge better ones (Young, 2016b). Brilliance is used as an anchor to illuminate resiliency among Black girls as they endure and overcome obstacles along their educational journey. Martin (2018) argues that we initially begin asserting that Black children are brilliant, not as a conjecture but as a self-evident starting point, logical statement, and axiomatic truth. Resilience refers to coping and returning from all challenges (Tugade & Fredrickson, 2004).

The challenges and struggles often encountered by Black females as learners and doers of mathematics have taken on many descriptive terms in education. Explicitly referring to the mathematical challenges experienced by Black female students, one must consider the marginalization due to the intersections of gender and race (Evans-Winters & Esposito, 2010; Young, 2016; Young et al., 2017). Consider coupling this marginalization with other transitional factors middle school students confront in adolescence. The multifaceted experiences of racism and sexism of being Black and female in the mathematical sciences have been described as "double-blind," "double disadvantage," "double and multiple jeopardies," and "gendernoir" (O'Brien et al., 2015; Young et al., 2017; Hotchkins, 2017).

Students are equipped with protective factors that are linked to their identities. Protective factors for adolescents are the feelings of belonging or being connected to a school and are advantageous when students have positive and successful interactions with peers and teachers (Catalano et al., 2004). These factors include academic, emotional, and mental support and connecting with peers as friends. These protective factors are essential to the child's well-being and academic trajectory during adolescence (Center for the Study of Social Policy, 2023). Protective factors are the mechanisms that allow people to be resilient despite adverse circumstances in their lives. These constructs, considered attributes, are shaped by their experiences in educational settings and the greater community in which they live. Direct or subtle messages in learning communities can either work to build students up as confident learners or tear them down, contributing to feelings of despair and discouragement related to mathematics.

The brilliance and resilience of Black girls in this study are examined through the lens of their protective factors. Protective factors, both internal and external, work as the strengths of individuals and their communities that mitigate risks and are characterized to contribute to healthy and positive development (Center for The Study of Social Policy, 2023). We often see them as attributes that help students navigate difficult situations successfully. "The protective factor model suggests that promotive assets or resources modify the relationship between a risk and promotive factors and outcomes" (Zimmerman, 2013, p. 382). This research examines barriers that impact Black girls along their educational journey. We desire to amplify Black girls' voices to share how they overcame obstacles and persist with superior achievement and consistent success.

Literature review

McGee and Pearman (2014) examine protective factors in two categories. The internal factors are associated with intrinsic motivation, strategic agency, and drive; external influences mediate internal drive. External factors refer to a more resource-oriented outcome. These external factors include support of solid family socialization, community support of education, and early at-home mathematical development. This study focused on Black girls in an advanced math program to identify the protective factors that supported their academic success. This success extends from their home environments and contributes to their mathematical performance.

Berry (2008) investigated the protective factors for middle school Black males. He identified five themes that support Black middle school male students' achievement in mathematics: (a) successful early educational experiences in elementary school that worked to circumvent potential problems common among Black boys, (b) identification of academically gifted abilities by the school staff with advocacy efforts from their parents, (c) active support systems, (d) self-identification as a good mathematics learner, and (e) identities associated with other non-academic activities. With these supports, he argued, African American middle school males are more likely to succeed in navigating urban education systems.

On the other hand, black girls' challenges in mathematics education have multiple layers, more so than Black boys, offering a lens into their reality of oppressive sexism and racism. The literature suggests emerging themes of unpacking stereotype threat and addressing bias and subtle messages, such as micro-messages, that accompany high levels of learning.

Stereotype Threat

Steele (1997) mentions that schooling experiences for Black and white children appear to be the same in curriculum and instruction. However, it encourages us to consider how these two racial student groups may still experience the classroom so differently that Black children's schooling experience impacts their achievement significantly. Social-psychological threats exist in classrooms that negatively judge groups of students and cause unjust treatment to them. These "threats in the air" are stereotypical and exist as situational threats that work to impact group members negatively. In the school environment, groups can fear being diminished by the stereotypes imposed on them, causing a self-threatening disposition that hampers their achievement.

Steele shares that stereotype threat is a situational pressure that affects a sub-portion of the stereotyped group. He finds that in schools, this threat often affects the more confident students in each subgroup, making stereotype threat more opposing to students who are high achievers or performers compared to others. Stereotype threat affects intellectual performance, especially among women. His studies examine mathematics and suggest women experience society-disseminated stereotype threats associated with their mathematics ability in mathematics-performance environments. This stereotype confirming implication threatens their belongingness and acceptance in mathematics spaces. Although men could be equally threatened, women carry additional pressure given the historical stereotype confirmations in our society related to women's roles and their capabilities or the lack thereof related to their abilities in science, technology, engineering, and mathematics.

McGee and Martin (2011) define stereotype threat as a "type of confirmation bias in which the threat of being viewed through the lens of a negative stereotype suppresses academic performance among Black students at all academic levels" (p. 1348). McGee and Martin (2011) used the personal narratives of 23 high-achieving Black college students to provide insight and detail into the outcomes of stereotyping. The narratives provided feedback on Black peoples' innate ability to perform at high levels in mathematics. Further, McGee and Martin (2011) remind us that students recognize that stereotypes exist in schools and classroom contexts. The participants in this study felt a need to justify that they were not constantly mathematically inferior. Although stereotypes significantly impact the students who experience them, it is essential to realize that in this study, the participants' constant exposure to negative cultural views of Black learners' ability in mathematics contributed to a pattern of resilience that worked to assist them in being powerful beyond belief amid discouraging messages of inequity. Over time, the participants managed stereotypes by incorporating their complex protective tactics and unique identities, which helped them deal with the burden of functioning in a radically stressful and frequently emotionally debilitating environment. This study revealed that the participants were vigilant to verify these stereotypes as wrong, served as role models and mentors to their Black peers and family members, and always felt pressure to be competent in their studies.

Bias and Subtle Messages

According to Covington Clarkson and Contreras Gullickson (2020), Black girl magic in the media is ubiquitous, establishing a positive narrative for Black women with a history of curriculum, societal, and career marginalization. They encourage us to initially work to understand Black females' challenges to examine how they break through barriers. This qualitative study revealed themes of confidence, notions of connection, competence, culture, community, and communication. Findings suggest that Black girls want to be included in rich and rigorous mathematics learning, and when they receive high expectations, they work toward them. The participants sought opportunities to communicate about challenging mathematics and desired to be acknowledged with micro-affirmations for their success. This study sought to establish sources of "agency and validation for students of color" (Covington Clarkson & Contreras Gullickson, 2020, p. 65) while opposing white normed constructs. Encouraging and mentorship support Black girls while navigating the racial and social issues in mathematical spaces.

Research Questions

This study centered on seventh and eighth-grade middle school Black girls aged 11-13, enrolled in advanced mathematics, Algebra I, a high school mathematics course designed for ninth-grade students. The following questions guide this study:

1) How do intrinsic and extrinsic factors affect Black middle school girls' performance in advanced mathematics classes?
2) How do societal messages about Black girls' and their mathematics abilities impact them, and how do they combat damaging stereotype threats?

Methodology

This study was conducted at Katherine Johnson Middle School – KJMS (pseudonym), located in the southern region of the United States. We chose this site to conduct this study since it was the most significant urban middle school in the state where the lead researcher resided and enrolled the highest number of students. We wanted to attract as many diverse potential participants as possible. The sixth to eighth-grade population at the research site was approximately 1,764 middle school students, with 930 students enrolled in seventh to eighth grades. Among the population, 150 students were enrolled in Algebra I, the most advanced course available. Of the 150 students enrolled, 23 students self-identified as Black and female.

Participants

Black girls enrolled in Algebra I as middle school students were the most suitable for this research study because they met the following criteria: 1) mathematical advanced placement, 2) race identity as Black, and 3) gender designation as female. The mathematics placement criteria required students to be slated for or enrolled in Algebra I before or during their eighth-grade school year.

Among the 23 students eligible for the study, 11 agreed and provided appropriate permissions to engage in this study. All eligible and willing students were invited to participate. The 11 participants (denoted by pseudonyms) were Alexandria, who was in seventh-grade and considered twice accelerated in mathematics since she was a seventh-grade student taking a ninth-grade mathematics course; musically talented Allison; Aaliyah, who has a passion for social action related to race and gender; Amari, who uses technology to assist her in learning when it becomes difficult; Elizabeth, an energetic seventh-grade student who enjoys engaging in art and music; Kayla, an inter-district student who travels from a nearby city to attend school and intrinsically motivated, introverted, and reserved; May, who is academically competitive with her friends; McKenzie, a confident student and avid reader; and Penelope, Trinity, and Zoe all student-athletes who also engage in several school activities as leaders.

Data Sources

This research study took place during the Fall 2019 semester. As a qualitative study, four types of data collection mechanisms were employed: 1) individual interviews, 2) focus-group interviews, 3) collective memory writing, and 4) the researcher's journal.

Interviews

Interviews were designed to foster interactivity between the researcher and participants. Each participant engaged in individual and focus group interviews in October 2019. Thus, they yielded data that elicited in-depth descriptions, context-rich personal accounts, and perceptions of their educational and, specifically, mathematical experiences. The focus group interviews were employed to engage the research participants in different collaborative interview questions. The open dialogue centered on the participants' experiences and parallels among the participants' lived realities. The semi-structured individual interview encouraged rich and detailed data concerning how they perceived mathematical experiences from elementary to middle school, ranging between twenty-five and forty minutes. The two focus groups comprised approximately half the participants and engaged in different collaborative questions centered on the parallels in their lived experiences. The individual and focus group interviews took place on the school's campus in a welcoming location familiar to all participants, which gave them privacy from students not participating and an opportunity to participate honestly without restrictions.

Collective Memory Work

Written memories through collective memory work (Haug,1999), a feminist research methodology, was employed non-traditionally. Collective memory work, implemented through journaling, gives participants a solidarity voice that is unfiltered, uniquely their own, and without constraint that speaks to their individual histories and allows their written expression to speak accurately to their mathematical experiences. Parents and participants were introduced to the collective memory work procedure during the informal meeting, and it was reinforced as assent and consent forms. The collective memory work in this study was presented as writing response prompts on a Google form with an opportunity to respond on a desktop computer. The participants engaged in this process twice, once after their interview and another after the focus group opportunity.

Researcher's Journal

The primary researcher kept a record of changing thoughts, new ideas and connections, and details related to the literature and its connection to this study. The researcher's journal also included participants' non-verbal expressions relevant to the study. These reflective and reflexive accounts were also derived from occurrences outside of the research agenda and timeframe in their natural environments as they occurred.

Data collection and procedure

The interviews for this study were audio-recorded to provide an accurate, verbatim account of the participants' ideas. As a benefit to the researcher, the artifacts allowed for the researcher's word-to-word dictation and transcription. These data collections were stored in the researcher's locked personal computer and journal. Data collection and a thematic analysis were systematically done to determine themes and patterns from the collected data using an inductive approach to coding.

Role of the Researcher

As an educational advocate within the KJMS community, the lead researcher has explicitly worked with students through supplemental programming adjacent to middle schools in a non-profit organization within the community. This organization works with Black girls in the community after school and during the summer but is not connected directly with the school. For six years of this study, the lead researcher has focused on the educational empowerment of Black girls in science, technology, engineering, and mathematics. This role has enabled her to build relationships with the stakeholders in the community. This opportunity provides an advantage in this research, given that the participants had already acquired trust with the lead researcher. This connectedness strengthened a partnership that became advantageous for the research agenda since the lead researcher is familiar with the community and a few parents of prospective participants who were ideal for this study since they met the requirements to engage.

Results

The participants in this research study shared aspects of their experiences and focused on accountable relationships with their parents, coaches, and school personnel. These accountability relationships ensured the participants that their achievement in the classroom was expected and supported in their homes, non-academic spaces such as community centers or churches, and on the basketball court. The prevalent themes that arose were motivation from the engagement in activities involving enjoyable mathematics that they found valuable in their real world, examining the mathematics perceptions held by society about them, and their own perceived mathematics identity.

Valuable Mathematics Outside of School

The participants in this study shared how they used mathematics outside of the classroom. Shopping with parents was mentioned by more than half of the participants in response to how they use mathematics outside of school. Zoe and Trinity also shared how they use mathematics as student-athletes. Zoe reflected on conversations with her basketball coach. It usually takes others an extremely long time to complete the statistical sport record books, but her coach noticed she finished the task in a fraction of the time and told her, "Hello! You are good at math." These statistical record books use data from sporting practices or events to collect and track results from championships, tournaments, and player record holders' performances. Trinity wrote about the same coach but in a different context: "Our coach limits how many turnovers we have and tells us not to go over a certain number, which brings us (to) inequalities."

McKenzie, who conducts scientific experiments, mentioned, "My dad likes to look up different chemical reactions that would happen with certain household objects. And then we make the experiments and just have fun doing that." McKenzie uses mathematics to measure appropriate chemical reactants to ensure safe and successful experiments. Other participants connected their mathematics experiences to the arts. Allison and Elizabeth reflected on how musical note lengths have particular purposes; for example, the slow and soft notes in songs "really bring emotion to people," mentioned Allison. Participants also mentioned using mathematics when cooking, playing games, or keeping scores for quiz bowl competitions.

Valuable Mathematics Inside of School

Many participants identified their elementary experiences as their starting point for becoming advanced in mathematics through enrichment opportunities. Aaliyah realized that her engagement of advanced academic skills in enrichment programs at the elementary level allowed her to "escape class to work on a project or do something fun." In gifted and talented and mathematics enrichment activities, Aaliyah engaged in learning that extended the regular classroom curricula. "My Math Olympiad™ teacher already taught us algebra concepts like variables, math formulas, and more. We also played math games to help us understand the concepts. These things helped; on the test, I was already ahead." Engagement and valuable mathematics activities such as the stock market game, mathematics Olympiads, and gifted and talented mathematics activities prepared them to excel in mathematics now and in the future, representing each participant's desire.

Perceptions and Self-Perceptions of their Mathematics Identity

The girls in this study were asked, "How do you view yourself in mathematics?" They viewed themselves as "brilliant mathematicians." However, they believed that perception differs in society. Trinity recognized that some people believe Black girls cannot do high-level mathematics "because we didn't have the same education they did at some point. So [they think] we will be behind." She reiterated, "They always had a low, you know, way of looking at Black people."

Aaliyah elaborated,

> It mostly originated with White people, but then, you know, Black people started to think it themselves; I guess we're seen as lower level, as a downgrade. They think because some people are poor, and I guess when you live in those areas, you act differently, so they don't expect you to do higher math.

This sentiment was reinforced by Allison, who shared:

> Since many people are in the minority group, people expect lower than us. They don't have

> high expectations for us because we're minorities in American society. It's because there are stereotypes. They expect you to act a certain way for being a certain color.

These brilliant mathematicians identified several advocates in their network. Allison reflected on the mathematical mindset her mother encourages her to have that will lead to desirable thoughts to do mathematics.

Elizabeth's older brothers made mathematics fun for her. She elaborated:

> [My brothers] said if I were smart enough to be in the advanced class, I would understand the work. If I can't understand it, I'm smart enough to go ask the teacher or an adult to help me with it. I think it's their personalities that helped me with math because they're both really good at math. So, with them being really good in math and also being hyper and energetic and weird and quirky, all those things tied up into one, with math, it brings this new environment for me just to learn it at home.

Zoe shared how her dad showed up for her and her mathematical development even while living on another continent due to his military duties. As a protective factor and a source of accountability, Zoe understood that her academic performance was not an accomplishment of her own; it was one that she shared with her family.

The brilliant mathematicians in this research study used their quotes as sources of motivation. Some words they choose to motivate themselves and their peers despite societal perceptions include: "Don't doubt how good you are in math. Because when you doubt [yourself], you believe you can't do advanced things and won't try." Amari echoed this sentiment: "Push past what you think your limitations are; if you go beyond what you think you can do, you might discover that you are good at advanced math." Aaliyah said, "Try it, and then be willing to do it." Alexandria suggests, "If you say you cannot do it, you will not. And if you say you can, you will."

Discussion

This research study investigated the protective factors that led to the empowerment, self-consciousness, and self-efficacy of mathematically advanced Black girls in middle school. With a critical lens on the protective factors, this study sought to find motivators associated with their affiliations and accountability relationships that contributed to their heightened mathematics success.

The key findings referenced the value of mathematics in and out of the classroom for the participants and served as a motivator for them to push forward. Black girls in this study were eager to learn mathematics in meaningful ways that contributed to their curiosity in the real world. They sought opportunities to do science experiments involving mathematics and opportunities to escape the regular classroom to engage in fun enrichment and gifted and talented spaces that allowed for exploration of the content in exciting spaces of learning. Their self-perceptions as mathematics students differed from their perception of societal views of Black girls as lower-level individuals with little or no mathematics competencies. Given these stereotypes, they found it necessary to demonstrate their worth while rejecting the opinions of others. Their protective factors worked as accountable validation systems at the school, community, and home levels, with the family offering the most support. Despite societal views of their intellectual ability, the participants persisted in achieving at the highest level. Fathers, mothers, and brothers (in that order) held the most accountability value to the participants in this research study. People of accountability helped to cultivate their brilliance as mathematicians. Other supports, such as accountability partners, were mentioned and the participants mentioned athletic coaches, teachers, peers, and tutors were members.

Limitations

Black girls from one school within a single school district participated in this research study, thus limiting the focus to a single community. The collective memory instrumentation, which required the students to reflect and narrate their past experiences, was the most challenging activity for the participants. This challenge of completing both writing prompts may be age-related or effort-based. The quality of the writing was superior for the ten participants who did engage in the memory writing activity. Most participants completed one of the two writing prompts. One participant was checked out of school for a family emergency and did not have an opportunity to engage fully with either collective memory writing prompts.

Study in Context

The findings examine the implicit biases in our society and how the brilliant Black girls in this research openly acknowledge the presence of racism, sexism, and stereotype threats in our country. Achievement gap narratives in education are supported by policy reports that label Black students as mathematically illiterate

(Martin, 2009; National Research Council, 1989). Many of the participants in this research study know that society sees them as "lower level" or incompetent in doing high-level mathematics, which supports the literature that Black students, especially those who face multiple jeopardies due to the intersections of race and gender experience discouraging educational and societal narratives (Evans-Winters & Esposito, 2010; Tang, 1997; Young, 2016; Young et al., 2017; Hotchkins, 2017). Meso-affiliated resources, those linked to the student's identity and shaped by their community and neighborhoods (Martin, 2012), can push Black girls forward toward success and combat these narratives so they may push past them toward success.

The participants in this study shed light on the empowerment and resilience of Black girls enrolled in advanced mathematics as they reflect on the valuable ways, they use mathematics. This research study mirrors the claims aspects of McGee and Martin's research in illuminating Black girls' success in mathematics.

McGee and Martin (2011) concluded that "racial stereotypes are powerful, but not deterministic" (p. 1380). These messages speak volumes about the empowerment the participants in this study maintain while encouraging others. Using their quotes, many reflected positively with encouragement, suggesting that if you push past your limits, you can achieve, and if you say you can do something, you will accomplish the goal.

Implications

Our findings suggest that all students are vulnerable, and protective factors can propel them forward. Students with more protective factors are better supported in the educational system to engage in higher-level learning. Engagement in extracurricular activities allowed the brilliant mathematicians in this study to explore mathematics in relevant ways. Due to this value, the participants could grapple with mathematics through practical means. These relationships are valuable for Black girls and add a sense of accountability to their mathematics learning.

Examining the compounded effects of stereotypical messages presents an opportunity to engage in dialogue centered on this phenomenon of placing Black girls in a limited mathematical box that is not true to their identity or abilities. Ultimately, there is value added when educators, policymakers, and stakeholders creatively produce opportunities and examine bias by unpacking Black children's resilience, agency, and tenacity to communicate their brilliance as normative and not as societal outliers. Black children are less likely to be identified as gifted or to participate in advanced placement courses due to internal school policies that act as barriers that limit them (Anderson, 2020). Programs such as Black Girls Code©, GIRLS WHO CODE, GIRLS-wSTEAM®, Black Girl MathGic™, and MathCounts© may help spark these students' interests in mathematics and create new, innovative, fun, challenging, and engaging programming to enhance the education of brilliant Black girls.

Conclusion

Experiences during elementary school equip our students with a mindset for mathematics. Although this research is focused on participants at the middle school level, it emerges at the intersection of early and intermediate elementary levels, a time when they begin cultivating a solid mathematics identity. Black girls are brilliant mathematicians and seek opportunities to enjoy mathematics in the real world. Narratives focused on how students use mathematics in valuable ways became a significant theme uncovered in this study. Each participant in this study was engaged in mathematics outside of school and found those experiences to be the most memorable and valuable.

The counter-narratives of stereotypical messages must be pushed forward, illuminated, and shared widely. Enrichment activities like Gifted and Talented programs and the Stock Market Game, taken from the participants' perceptions, helped prepare the theme for advanced mathematics curricula. Communities that advocate for Black children must push the agenda to unlock the chains of confinement to provide access to challenging curriculums, including those of advanced mathematics, to Black girls.

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How Pre-Service Teachers Define Good Mathematics Teaching and Learning

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ABSTRACT Mathematics self-efficacy (beliefs about oneself as a learner of mathematics) and mathematics teaching efficacy (beliefs about oneself as a mathematics teacher) are important constructs that influence pre-service teachers' (PTs') learning and teaching (Bandura 1986). However, less is known qualitatively about how PTs define good teaching and learning when they make efficacy judgments such as, "I am good at learning mathematics" or "I am good at teaching mathematics." This qualitative case study used journaling to examine 23 elementary PTs' definitions of being good at doing and teaching mathematics. Our findings suggest PTs define being good at mathematics in a variety of ways, including receiving good grades, being fluid (quickly and successfully doing procedures), and having the ability to apply mathematics to new contexts. PTs also held a variety of definitions of good mathematics teaching, including focusing on student understanding, using group work and manipulatives, and having passion. These results have implications for researchers studying self-efficacy and teaching efficacy as well as for teacher educators hoping to engage PTs fully in their classrooms.

KEYWORDS *Beliefs; mathematics self-efficacy; mathematics teaching efficacy; pre-service teachers; teacher education*

Introduction

"I am bad at learning mathematics." "I am a good mathematics teacher." We have all heard such statements, that describe being good or bad at doing or teaching mathematics. But what do such statements mean? What do they mean to students? To researchers? To future teachers? Do these different groups hold similar definitions of good at learning, doing, and teaching mathematics?

The statements above are mathematics self-efficacy and mathematics teaching efficacy assertions. Mathematics self-efficacy is a student's belief about their ability to do, perform, or learn mathematics (Bandura, 1986). Mathematics teaching efficacy is a teacher's belief about their ability to teach mathematics, particularly to bring about student learning and engagement (Tschannen-Moran & Hoy, 2001). Much of the research on efficacy in mathematics learning and teaching has been quantitative (e.g., Enochs et al., 2000; Midgley et al. 2000). While valuable, such work often aggregates in a way that can lose nuance. Because of this, researchers understand much less about how students, particularly elementary mathematics pre-service teachers (PTs), define good teaching and learning when they say, "I am good at mathematics" or "I am good at teaching mathematics." Understanding PTs' definitions is important so teacher educators can better target their teaching to support PTs. In addition, recognizing PTs' definitions is important to help us better understand the existing quantitative research and what PTs mean when they respond to Likert surveys on efficacy. In this paper, we present a qualitative case study that used journaling to examine 23 PTs' definitions of being good at doing and teaching mathematics.

Conceptual Framework

This study centers on PTs' beliefs; PTs' beliefs are their judgments or notions about various ideas related to teaching and learning, including their beliefs about themselves as teachers and learners (Fives & Buehl, 2012; Gill & Fives, 2015). PTs' beliefs are important because teachers' beliefs guide their actions in teaching, influence their practice, and influence their students' outcomes and beliefs (Buehl & Beck, 2015; Fives & Buehl, 2012; Tschannen-Moran & Hoy, 2001). PTs' beliefs also influence their mathematical understanding and their future development as teachers (McLeod, 1994). And, importantly, teacher educators can influence PTs' beliefs during their undergraduate classes (Fives & Buehl, 2012). We will refer to PTs' beliefs about what it means to be good at doing or teaching mathematics as their "definitions."

PTs' definitions of being good at doing mathematics are related to their mathematics self-efficacy, their beliefs about their ability to do or learn mathematics (Bandura, 1986). PTs' definitions of good mathematics teaching are related to their mathematics teaching efficacy. Mathematics teaching efficacy refers to teachers' or PTs' beliefs about their ability to teach mathematics (Tschannen-Moran & Hoy, 2001). For example, a PT could believe "I am good at mathematics" (high self-efficacy) and "I am good at teaching mathematics" (high teaching efficacy). However, in both statements, PTs' definitions of good affect how we should interpret the meaning of their claims. "I am good at quickly solving procedures" is different from "I understand the main concepts" and yet both could be captured by "I am good at mathematics."

Previous Research

Much work has been done on mathematics self-efficacy and mathematics teaching efficacy, generally showing that believing one is good at something (i.e., high mathematics self-efficacy and high teaching efficacy) leads to beneficial outcomes on effort, persistence, and achievement (e.g., MacPhee et al., 2013; Pendergast et al., 2011; Zeldin et al., 2008). High mathematics self-efficacy has been linked to higher achievement and a growth mindset, a belief that you can improve your mathematics skills (Komarraju & Nadler, 2013; Usher et al., 2019). High mathematics self-efficacy has also been linked to perseverance, grit (the ability to work to overcome obstacles), and self-regulation (Muenks et al., 2018; Tanner & Jones, 2003; Usher, 2009; Usher et al., 2019). In general, researchers agree that high self-efficacy is better for learning, though some researchers suggest that self-efficacy needs to be calibrated (that is, one should not be overly optimistic or pessimistic) (Pajares & Miller, 1994: Russell & Phelps-Gregory, 2022).

Research on elementary PTs finds they may have more mathematics anxiety, more negative attitudes, and more mathematics avoidance than other groups (e.g., Rech et al., 1993; Stoehr, 2017). This suggests that elementary PTs may have lower mathematics self-efficacy, however, little research has examined this specifically. Some research suggests both elementary and secondary PTs have high mathematics self-efficacy (Phelps, 2009; Zuya et al., 2016). Other research has found elementary PTs who take advanced mathematics classes have high mathematics self-efficacy, but those who do not take advanced mathematics classes have low self-efficacy (Xenofontos & Andrews, 2020). More work on understanding PTs' mathematics self-efficacy beliefs is needed.

High teaching efficacy for mathematics teachers has been linked to their students' achievement as well as their students' own mathematics self-efficacy (Althauser, 2015; Chang, 2015). High teaching efficacy is also linked to teachers' instructional practices, their willingness to implement challenging teaching strategies, and their beliefs in their ability to effect change in students (Bates et al., 2011; Bruce & Ross, 2008; Depaepe & König, 2018). Mathematics self-efficacy is positively correlated with mathematics teaching efficacy (Bates et al., 2011).

Most research on mathematics teaching efficacy has examined in-service teachers, with less focus on PTs. Some of the limited research has found PTs have high mathematics teaching efficacy (Zuya et al., 2016). In a qualitative study, Xenofontos and Andrews (2020) found that the majority of elementary PTs in their study expressed high mathematics teaching efficacy, stating they were confident they could fulfill their visions for teaching mathematics.

Unfortunately, previous research has often defined efficacy differently, using constructs ranging from mathematical competence to a belief in one's ability to fulfill visions. Previous work has often been quantitative, using Likert scales to measure mathematics self-efficacy and teaching efficacy (Enochs et al., 2000; Midgley et al. 2000). These include survey items such as "I know how to teach mathematics effectively," "I will typically be able to answer students' questions," or "I'm certain I can master the skills taught in class this year" (Enochs et al., 2000, pp. 200 – 201; Midgley et al., 2000, p. 19). While reliable and valid, these instruments assume PTs define success in mathematics and mathematics teaching in the ways measured by the instrument. Quantitative studies may thus fail to capture PTs' own definitions of teaching or being good at mathematics. To address this, our small-scale, qualitative study sought to capture PTs' definitions using their own words. The work presented here addresses the following research question: What do elementary PTs believe it means to be good at mathematics and to be a good mathematics teacher?

Methods

To study the phenomenon of PTs' beliefs about good learning and teaching in mathematics, we conducted a qualitative case study. Qualitative case studies involve the researchers studying one case, in this study a class of elementary PTs, in depth. A case study allowed us to examine PTs' definitions of being good at mathematics and good at mathematics teaching in detail.

Participants

Participants in this study were elementary PTs who were majoring or minoring in mathematics and were enrolled in an elective mathematics class for their major, which served as the case for this study. Two of the researchers served as instructors for this class. All PTs in the class were invited to participate in the study; 23 PTs agreed. Participants had previously completed one general mathematics class and at least two mathematics content courses for elementary teachers. Their responses in this study reflect those of mid-program PTs, who have completed some relevant mathematics and education classes but do not have significant teaching experiences. All participant names are pseudonyms.

Data Collection

This study used a journaling approach to explore PTs' beliefs. Using journals is a common research practice because it offers an opportunity to explore past and present experiences in relation to beliefs (Bullough Jr., 2015). As part of the course, PTs were asked to write regular journal entries; the journal prompts were written collaboratively by the instructors based on research and readings from the course syllabus. This study focuses on the first two journal entries, completed during weeks one and two of the semester. Journal 1 broadly asked, "What does it mean to be a good math teacher?" Journal 2 first asked participants to name a time they were good at math and then asked, "What does it mean to be successful at learning mathematics?" Both journals also had optional sub-questions such as, "What makes being a good math teacher different from/similar to being a good teacher of another subject?" The goal of the sub-questions was to prompt PTs who might otherwise write less; however, participants were explicitly told they did not have to answer every sub-question. PTs were given the journal entries before instructors shared their own opinions (to avoid bias from instructor opinions) and PTs completed the journal entries on their own outside of class (to avoid peer opinion bias). PTs were given points for completing the journals but were not graded on their ideas, to encourage them to write freely. Journal responses were submitted electronically; each was generally one to two single-spaced pages.

Analysis and Reliability

Recall that our goal was to understand PTs' definitions of being good at mathematics and teaching mathematics. We were interested in understanding what a PT meant when they said, "I am good at mathematics," recognizing that this could mean a variety of ideas including understanding concepts, being better than peers at mathematics, or having procedural fluency. Our goal in coding was to capture PTs' definitions of being good at mathematics and teaching mathematics.

To capture these definitions, the first author read all journal responses and then used a randomly chosen subset of the data to develop codes inductively. The use of an initial, smaller subset of data allowed us to withhold a portion of the data for checking the initial coding; after such checks, the first author coded the remaining data. Codes were created to capture PTs' definitions, and thus we had codes such as, "good teaching means using group work," "being good at mathematics means getting the right answer," and, "being good at mathematics means you can help others with their work."

We followed Campbell and colleagues' (2013) analysis and reliability process. The units for analysis were meaning units; we unitized the data based on PTs' main ideas. Codes could be applied to a single phrase or to several sentences to better capture participants' meaning. To ensure reliability and to check against coding drift, the first author randomly chose three participants (13%) and stripped the codes from the analysis, leaving the unitization (to ensure consistent unitizing) (Campbell et al., 2013). The second author then coded the unitized data. The first two authors reached 80% reliability on their first coding, an acceptable level of reliability (Campbell et al., 2013).

Results

Our findings suggest PTs define being good at mathematics in a variety of ways, including the ability to apply it to new contexts, to get good grades, and to do mathematics fluidly. PTs also showed variation in their definitions of being good at mathematics teaching with definitions including using group work and manipulatives, having passion, and focusing on understanding.

I was good at mathematics when…

Since past research has often shown some elementary PTs may have mathematics anxiety, negative math attitudes, and mathematics avoidance (Stoehr, 2017), we asked PTs to identify a time they were good at math, to help them think about what this meant to them. The majority of PTs (n = 13 PTs, 57%) responded that a time when they were good at mathematics was in algebra, with PTs citing both secondary and post-secondary algebra but with secondary algebra being the most common. Some PTs (n = 5, 22%) identified an experience in their university teacher education mathematics classes or their elementary school experiences (n = 3, 13%). Finally, one PT identified a secondary geometry course, and one said a secondary probability course.

Participants often described their positive mathematics experience in detail and connected it to their definitions of being good at mathematics. For example, Lucas wrote:

> Sixth grade was when I was first introduced to algebraic equations, and it was a math concept that I really understood… I knew that I could always get the right answer with an algebraic equation given to me because I had exceptional basic math skills that always helped lead me to the solution.

Lucas went on to say that being good at mathematics meant having right answers and being fluid (quickly and successfully using procedures), both of which could be seen in his description of his positive experience.

I am good at mathematics when…

We now further explore PTs' definitions of being good at mathematics. PTs had multiple meanings of being good at mathematics, some of which fit together and some of which were contradictory. PTs also often held multiple meanings at once, as Lucas did above.

I apply it to new contexts and situations.

The most expressed definition was that a person was good at mathematics if they could apply their learning to new problems or new contexts (n = 17, 74%). For example, Olivia said, "A successful learner can apply the concepts to different situations and can solve problems using a bank of different strategies." Lily wrote:

> There was a time when I realized I really understood a math concept. The concept was using algebra to figure out what "x" is. ... I learned this concept in a deeper way because I understood how it could be used to solve bigger problems and I understood how the smaller problems fit into this bigger concept.

One of the contexts mentioned by multiple PTs was applying mathematics to real-life contexts. For example, Charlotte express ed:

> I learned it in a deeper way because I was able to relate it back to my life and things that I actually did in real life. I used algebra when I was selling pop and vegetables in high school, doing banking, managing time, you name it.

For PTs like Olivia, Lily, and Charlotte, being good at mathematics partially meant being able to use mathematics in new situations and for new problems.

I explain it to others.

Another common definition was that someone was good at mathematics when they could explain it to others (n = 16, 70%). For example, Mia said, "I can always tell how well I understand a concept by how well I can explain or teach it to another person." And Hannah said:

> As a future educator, I think that [to] know if a student was successful in mathematics [we can use] their ability to explain it. If a student can do more than demonstrate a problem, and [can] actually explain why the answer is the answer and how they came on that answer then they will actually fully grasp the concept.

For PTs like Mia and Hannah, being good at mathematics partially meant being able to explain it well to others.

I get the right answers and good grades.

Some PTs (n = 13, 57%) expressed a belief that right answers and good grades showed you were good at mathematics. For example, Maya expressed, "I was enlightened when I wrote all my work out and got the right answer. That is when I decided math was my favorite subject because there is always a right answer and no what ifs." In addition, some PTs (n = 5, 22%) described a belief that both right answers and understanding mattered. For example, Lucas wrote:

> I felt that I was successful at learning in that math concept because I was good at it. In other words, I was performing well on my homework, quizzes, and tests. Based on my own experiences in learning, I believe that being successful at learning means not only performing well on classwork, quizzes, and tests but also showing confidence and understanding in the concept being taught.

However, an almost equal number of PTs (n = 11, 48%) expressed doubt that right answers or high grades alone meant you were good at mathematics. Mia said:

> The grade that I receive is not always a reflection of how successful I was. I have completed many math courses and received an A without actually mastering the material because I was able to get good grades on exams by memorizing formulas and procedures.

Samantha wrote, "A student can solve every problem correctly and still not be proficient in math because he or she does not understand what each step means and why their method works." Thus, for PTs like Avery or Lucas being good at mathematics partially meant getting the right answers and good grades. However, it also appeared as though some of the PTs doubted this definition of being good at mathematics.

I can do mathematics fluidly.

Some PTs (n = 12, 52%) also defined being good at mathematics as doing it "fluidly" or quickly and successfully, particularly for procedures. Olivia said that successful learning in mathematics is, "Fluid. If a person is successful at learning mathematics, then their 'math actions' are fluid." Avery wrote, "In my elementary years I was drilled over and over with adding, subtracting, multiplying, and dividing numbers so that drilling and practice made me able to become fluent in those areas." And Emma said, "I know I'm successful at learning mathematics when I don't have to think about a problem before completing it and just automatically know what to do." Thus, for PTs like Avery and Emma, being good at mathematics meant being able to do procedures and problems quickly and successfully.

I work hard at it.

Finally, some PTs (n = 6, 26%) expressed a definition that being good at mathematics is based on time, effort, and work. For example, Layla wrote, "Being successful at learning mathematics means you …want to put the time into studying it. Without putting in the time and effort, you will not be a successful learner in any subject." And Charlotte said, "Finally, I feel that being successful learning math means sticking with it because hard work does pay off in that subject area compared to others in my opinion." For these PTs, being good at mathematics partially meant having the ability and willingness to study and learn.

I am a good mathematics teacher when…

PTs also had multiple meanings for being a good mathematics teacher, often holding several at once. Each definition of good mathematics teaching will be explored below.

I encourage students to work independently.

A common definition (n = 14, 61%) was that good mathematics teaching is when students are encouraged to do their own learning. Lily said, "Students best learn math when they are able to discover how to do problems in their own way." And Anna wrote:

> In a good math classroom, students are given problems that they have to figure out how to solve. They should use the tools they have been given and push themselves a little bit farther… Students learn best by doing, they can use problem-solving skills to enhance their higher-order thinking skills.

For PTs like Anna and Lily, being good at teaching mathematics meant encouraging students to problem-solve independently.

I use group work and manipulatives.

PTs also said good mathematics teaching involved the use of group work (n = 14, 61%) and manipulatives (n = 16, 70%). Charlotte said:

> Because math should not simply be the teacher lecturing but rather students cooperatively learning in groups or students are teaching other students. These strategies/resources encourage students to share and find what will not work as well as what might/does work for solving the answer…

For PTs like Charlotte, being good at teaching mathematics partially meant using group work and manipulatives.

I am passionate.

A common belief about good teaching was that good teachers, especially mathematics teachers, must be passionate (n = 17, 74%). Avery said, "Teachers need to always keep that positive vibe in the classroom for the students because if the teacher is not excited about the subject, the students will not be either." Interestingly, many PTs mentioned that passion was especially important in mathematics because of the nature of mathematics. Samantha wrote, "Most importantly, a great math teacher remains positive. Math is giving a 'scary' characteristic that remains with students throughout most of their education. Love math and show students that they too can love math!" Charlotte said, "Finally, a good math teacher differs from other subject areas by being able to make their subject fun due to the bad reputation math holds… so to combat that stereotype, teachers must try to make it fun…" For PTs like Avery and Charlotte, being good at teaching mathematics partially meant being passionate about mathematics and making your teaching interesting.

I connect mathematics to real life.

PTs (n = 15, 65%) also talked about how good mathematics teachers make real-life connections. Avery wrote, "Explaining how math is used outside the classroom will help students to realize that math is an important concept to learn in life." Samantha said:

> One idea would be to have my students create a project connecting the math concept we are learning in class to a real-life experience by actually carrying out the activity… Then when my students leave my classroom they can engage in math during their daily lives, easily and freely.

For PTs like Avery and Samantha, being good at teaching mathematics partially meant connecting mathematics to real-life concepts.

I teach different strategies.

Another common belief was that good teachers did not use only the best strategy for students to learn and memorize (n = 16, 70%). This was often tied to students and using different strategies with different students depending on their needs. For example, Emma wrote, "Not all students learn the same so accepting and showing to the class many different ways to find answers will be beneficial in ensuring you reach all students." And Henry wrote:

> [Good teaching] requires that you be open to different ideas and interpretations of problems. You need to understand that there is not always going to be one right way to get an answer. It also means creating different ways of getting to answers to best fit all the students.

For PTs like Harper, Emma, and Henry, being good at teaching mathematics partially meant showing students different ways to solve problems and allowing students to pick their own solution method.

I focus on understanding.

PTs also believed that good mathematics teachers focus on understanding (n = 9, 39%) and did not move on if students did not understand (n = 8, 35%). For example, Mia said, "Math teachers specifically have to be good at recognizing when students are struggling and when students are ready to move on to a more challenging concept." Harper wrote, "When I become a teacher, I want to make sure my students are all understanding the concept before I move on with the lesson." For PTs like Mia and Harper, being good at teaching mathematics partially meant helping students understand before moving on.

Discussion

In previous studies, researchers often define being good at doing or teaching mathematics and then design instruments based on these definitions. In contrast, this study asked PTs about their own definitions. This flip allowed us to examine PTs' unique definitions of mathematics self-efficacy and teaching efficacy. Our findings show PTs do define being good at mathematics teaching and learning in a variety of different ways. As a result of this finding, we suggest that, as teachers and researchers, we should not assume PTs mean the same things when they make statements like "I am good at mathematics" or "I am good at mathematics teaching."

There are obviously several limitations to the study. First, it was a small-scale case study of a single classroom and thus the results are not generalizable. This small scale allowed us to collect detailed qualitative data but more work would be needed to see if these PTs' definitions of good teaching and being good at mathematics were held by other groups of PTs. In addition, the study used journals to examine PTs' beliefs. Journals allowed PTs time to think and write at home. However, they also prevented us from asking follow-up questions. Future studies using interviews that allow for follow-up questions could yield additional results.

Our findings have implications for researchers. Common quantitative scales and instruments may be measuring only one piece of mathematics self-efficacy or teaching efficacy. For example, our work found many PTs believe in the importance of group work, manipulatives, passion, and connecting mathematics to real life and define doing things like these as good mathematics teaching. However, the most commonly used mathematics teaching efficacy instrument, the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI), rarely addresses these definitions (Enochs, 2000). While PTs' definitions of being good at mathematics included concepts measured on instruments like the Motivated Strategies for Learning Questionnaire (MSLQ) (Pintrich, 1991), such as good grades, their definitions also included topics not commonly measured, such as the ability to explain mathematics or use it in new contexts and situations. These common scales and instruments may need to be adapted if it is found in larger studies that PTs and teachers define good teaching differently.

Our findings also have implications for teacher educators. PTs may have high mathematics teaching efficacy or self-efficacy but define it differently from us. Knowing this, we can make more informed instructional decisions. For example, knowing PTs may value real-life connections, we can include more of these in our instruction. By doing so, we can engage PTs more fully while also helping them build their self-efficacy and teaching efficacy.

A final implication for teacher educators is that PTs will bring these definitions of good learning and teaching with them when working with their future students. Teacher educators can build on this to help PTs construct lesson plans that further student learning. For example, since our findings suggest that PTs want to connect mathematics to real life and they believe in the importance of explanation, teacher educators could help them build lessons that do this in their student teaching and show them sources to draw on in their future teaching practice. This will hopefully translate to positive classroom experiences for their future students.

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New Technologies and the Gender Factor in the Comprehension of Probabilities: Evidence from the Perceptions of Students

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ABSTRACT Mathematical education in Greece is constantly evolving in the pursuit of optimal learning outcomes for students despite their cognitive differences. This study seeks to gain insight into the use of new technologies in teaching probability theory and the gender differences in the comprehension of probability theory. To this end, a survey was conducted involving 500 students of the Department of Statistics and Insurance Science at the University of Piraeus. The respective questionnaire involves questions of self-reported results and employs the Likert scale to obtain the students' perceptions. Our data demonstrate no difference among the genders regarding the use of new technologies or their performance (i.e., the test scores) in the relevant courses.

KEYWORDS *Probabilities, gender, new technologies, pedagogy*

Introduction

Female-identifying individuals have been reported to be heavily underrepresented in most mathematically intensive areas (Holman et al., 2018; Thelwall et al., 2019), which has stirred the discussion on gender stereotypes in education and the corresponding implications in the occupational realm. Furthermore, modern education methods are embracing electronic means (e.g., electronic platforms, educational software, digital games, etc.), which have strengthened the efforts to transmit knowledge in these fields. However, the following questions have arisen: "Are the new teaching approaches that employ mathematics educational technologies achieving their purpose of improving the level of understanding and helping students pursue their professional goals?" and "Does the ability to understand mathematics and its branches depend on gender?". We acknowledge that similar questions have been addressed before regarding various education systems. Nevertheless, such issues remain largely overlooked with regard to Greek education, and we aim to fill this gap.

The purpose of the present study is twofold. Firstly, we investigate whether electronic educational means contribute to a deeper and more effective comprehension of the notions traditionally included in Probabilities courses. Secondly, we assess whether gender affects the ability to understand and learn the relevant notions. To this end, a questionnaire consisting of thirty questions was developed and distributed to students at the Department of Statistics and Insurance Science at the University of Piraeus. The relevant questions addressed various aspects relating to the application of electronic educational means, the learning outcomes as perceived by the students, and their interest in the field of Probabilities and its potential for further study. Moreover, we employed statistical tests to assess whether gender relates to the answers given, thus serving both our purposes. We obtained a sample of 500 responses from students who have been taught the course in probability theory or courses involving substantial probabilistic contents. Finally, we acknowledge that both the research questions entail a vast range of important aspects and implications, which are unlikely to be covered in 30 questions. However, the present study, albeit not fully covering these topics, provides the foundation for a promising strand of research.

Theoretical Background

Probability theory provides a systematic approach to studying randomness and uncertainty (Ramachandran & Tsokos, 2021). Central notions in probability theory include random variables, distribution functions, stochastic processes, and events, which are the mathematical abstractions of non-deterministic events, either occurring once or evolving over time. These notions commonly exhibit increased complexity, thus posing various challenges on the learner's part and the teacher who is expected to guide them through the learning process.

Many students need help comprehending probability and statistical concepts in various educational contexts (Chiesi & Primi, 2010). Traditionally, education systems address the respective challenges via the mechanisms of probabilistic algebra. Pratt & Ainley (2014) argue that for secondary school students, probabilistic algebra is most often learned as a set of rules to be followed in order to get the correct answer, which leaves the notions themselves distant and meaningless. In this context, Biehler (1991) defines the "concept-tool" gap as the lack of integration between the meanings of uncertainty students hold and the performance demands made in curricula and examinations.

The notions of Probability theory offer a powerful foundation for many mathematical techniques, from statistical methods in science and social policy to technological risk analysis and economic decision theory (Fox, 2003). Thus, subjects in Probability theory have gained their place in modern curricula. Furthermore, Probability theory provides valuable tools for almost all sciences, and this is why the relevant introductory notions are included in secondary education as well.

Gender Diversity

The idea that males outnumber females in fields within science, technology, engineering, and mathematics (STEM) has become the conventional wisdom over the past few decades, both in education and in the occupational realm (Stewart-Williams & Halsey, 2021). The study by Stewart-Williams & Halsey (2021) emphasizes that the underlying claim that males outnumber females in quantitative subjects is commonly associated with the notion that males are more object-oriented (i.e., interested in things) and females are more socially oriented (i.e., interested in people). The same study attributes the STEM gender gap to workplace discrimination, gender preferences, within-gender variability, socialization, culture, and biology.

Similar traditional views have been supported in several studies focusing on the performance on standardized tests of mathematical reasoning, such as SAT-M and GRE. For example, Halpern et al. (2007) and Tsui et al. (2011) report that females score lower than males on the mathematics section of the standardized tests for admission to colleges and graduate schools. Furthermore, Gallagher and Kaufman (2005) report that males score higher than females on these tests. However, males and females attend equally demanding classes in high school-level mathematics, and females earn higher grades. In line with this research, several studies (e.g., OECD, 2018; Mostafa, 2019) highlight that the gap between male and female participants in STEM courses is still relevant. Hango (2013) attributes the STEM gaps to gender differences in mathematical ability and differences in values and preferences, such as labor market expectations, including family and work balance, differences in motivation and interest, and other influences. Moreover, Fryer and Levitt (2010) assess various factors potentially causing the gender gap. These factors involve time invested in preparation for mathematics courses, lower parental expectations for females, and biased tests, with the results, however, providing little support for the aforementioned hypotheses. Contrary to the previous beliefs, a significant body of literature suggests that very few cognitive measures support the notion of gender differences. For instance, the paper by Spelke (2005) reviews experimental studies finding no difference in the primary abilities for mathematics and, thus, concludes that mathematical and scientific reasoning relate to biological cognitive capacities, which are equally shared between males and females. This latter school of thought has existed since as early as the 1970s, with the seminal work of Maccoby & Jacklin (1974, p. 349) arguing that the notion of objected-oriented males as opposed to socially-oriented females is one of the "unfounded beliefs about sex differences." We note again the distinction between sex and gender, which, however, during the 1970s, was more of an academic discussion than a self-identifying matter.

In this context, the report by the United Nations Educational, Scientific and Cultural Organization (UNESCO, 2019) advocates that gender bias is still prevalent and, in some settings, relevant to the entire education system. This bias is met in various forms, including the absence of women as leaders in textbooks, differential expectations of males and females by teachers, school policies that put pregnant girls at the door rather than respecting, protecting, and fulfilling their educational aspirations, etc. Furthermore, the same report claims that progress toward gender equality requires complementary and collective actions that promote female rights and empowerment, given the institutional, societal, political, and legal barriers that have historically restricted their participation in complete education, including their equal participation in higher education within STEM fields. The economic aspects of increasing female participation in STEM education are discussed in the report by the European Institute for Gender Equality report (EIGE, 2017), which forecasts the employment and economic benefits of closing the gender gap in STEM for the European Union (EU), and finds a considerable rise in employment, estimated between 850000 and 1200000 jobs and an increase between 2.2 and 3.0% in the GDP per capita, by 2050. This evidence substantiates the targets of gender equality with regard to STEM participation, which has become essential to the EU.

The use of New Technologies in Education

We increasingly observe the great influence that new technologies exert in the field of education. Online education platforms have been gaining momentum during the past two decades and, by now, have become an integral part of modern education systems. The Greek schools seem to be grasping this impetus and have intensified their efforts to engage electronic means in teaching conduct. Images, videos, and apps are arguably more appealing than conventional tools such as books and blackboards, especially for younger cohorts, and thus, such tools become highly useful during the lessons. Thus, technology-aided teaching seems to provide a very promising avenue for class conduct, capable of leading to more elaborate and comprehensive courses. Furthermore, substituting conventional with computer-based tools, which allow for course experiments, differentiates the perception of the students for the course and may be argued to lead to a more enjoyable and creative educational experience.

One important aspect of the education shift toward technology relates to bridging the concept-tool gap, discussed concerning probability courses in Batanero et al. (2005) and statistics in Biehler & Hoffman (2011). Such approaches design integrated learning programs utilizing digital simulations to overcome the stochastic misconceptions on the part of the students and create a more efficient learning environment.

The students' perceptions regarding technology-aided teaching provide another strand for empirical research. The study by Davidovitch and Yavich (2018) investigates the perceptions of Israeli students in Grades 9 and 10 on the use of tablets for education purposes. The specific study differentiates between STEM and non-STEM subjects and considers both the cognitive and the affective dimensions. The study finds no difference between STEM and non-STEM subjects both in the cognitive and the affective dimensions. Furthermore, the study finds no correlation between the age of the students and their tendency to weigh more, either the cognitive or the affective dimension. On the other hand, the study finds significant gender differences in the weighting of the cognitive and affective dimensions regarding the use of tablets for education. More precisely, this study finds that males attribute more weight to the cognitive dimension, which is consistent with the traditional school of thought, and to the affective dimension, which contrasts with the traditional views. Overall, these results suggest that males exhibit better affinity than females toward technology means.

Moreover, Mezhennaya & Pugachev (2018) assess the effects of using interactive computer-based methods with regard to the subject Probability theory and mathematical statistics offered to third-year engineering students at the Bauman Moscow State Technical University. The results of this study demonstrate that students achieve higher education outcomes when computer-based tasks are incorporated into the conventional lectures and seminars of the course regarding the first (i.e., Probability Theory) of the three teaching modules of the course. These findings reveal the positive effects of interactive computer-based education methods on the learning outcomes in probability theory.

The role of computer-assisted teaching was examined by Gürbüz & Birgin (2012) with regard to detecting and remedying probability misconceptions, such as the representativeness heuristic, the positive and negative recency, and the equiprobability bias. The specific study showcases that both the instructional approaches under consideration (i.e., traditional instruction and computer-assisted teaching) improve the students' understanding of probability contents and reduce their misconceptions. However, when the improvements in the groups are compared, the authors find that the intervention in the computer-assisted group was more effective in remedying the misconceptions. These findings demonstrate the effectiveness of computer-based technology against conventional teaching regarding elaborate notions in probability theory.

The purpose of the present study is to investigate (i) whether electronic means contribute to a better understanding of probability theory and its applications and (ii) whether gender plays a role in the learner's ability to understand and assimilate the relevant notions. This study contributes to the mathematics education literature regarding the Greek educational system, wherein relevant studies are scarce.

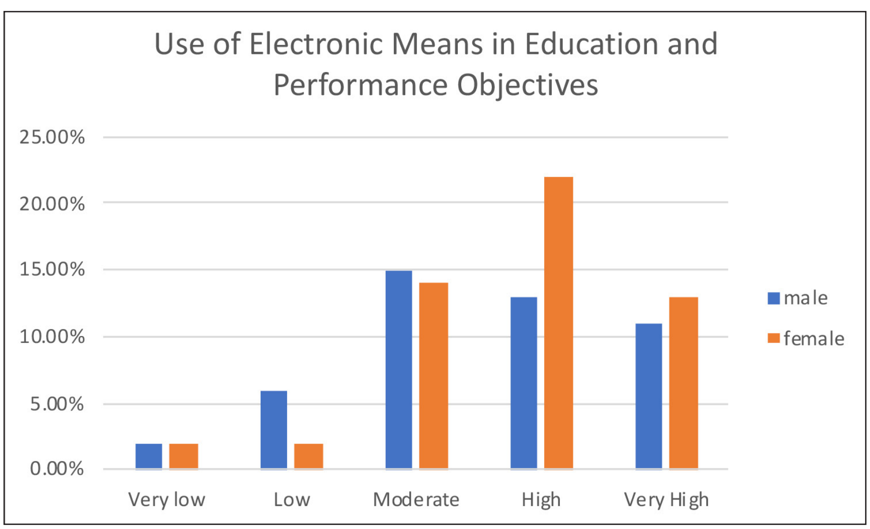
Statistical Analysis

In order to assess the effects of the application of new technologies in the teaching of probability theory and the gender-based differences in the comprehension of probability theory, we conducted a survey. The relevant questionnaire was developed by the teaching staff in the Department of Statistics and Insurance Science at the University of Piraeus and was administered to the students of the department through electronic communications. The target group involved students who, by that time, had attended the course Probability Theory I via MS-TEAMS and were currently attending the course Probability Theory II via conventional lectures. Therefore, these students possessed experience in attending relevant content both virtually and conventionally. The questionnaires were distributed between December 2021 and January 2022, and their responses were collected through the university's electronic platform. It comprised a total of 30 questions (i.e., demographic questions, multiple choice questions, questions of self-reported results, and Likert scale questions) assessing the students' perceptions. The questionnaire included choices for male, female, and non-binary respondents. We obtained responses from 500 participants, 47% (235) of whom were males and 53% (265) were females (no one identified as non-binary), all of whom were undergraduate students. Below, we present the statistical analysis of the questionnaire responses. We produce the frequency charts, discuss the results of the questions, and perform statistical tests to examine whether gender affects the students' perception regarding the application of new technologies for teaching purposes and the ability to understand and learn the notions of probabilities. Retaining the respondents' gender information enables us to test the various aspects of technology and education, taking into account the gender factor.

The first question considers the application of electronic means for educational purposes (e.g., electronic platforms, electronic educational material, e-books, e-exams, digital games, etc.) and their effectiveness regarding the learning objectives of the courses (i.e., the ability to comprehend and apply the basics of probability theory, set theory and combinatorics, and successfully address relevant real-life problems and applications), as perceived by the students. This is a 5-point Likert scale question, with the answers indicating the degree to which the respondent thinks that electronic means support the course performance objectives. Potential answers indicate the degree, which can be very low, low, moderate, high, or very high. We emphasize that this is a perception question that investigates the students' views on the effectiveness of the relevant means and directly relates to the first of our research questions. The respective answers are summarized in Figure 1.

Figure 1

Percentage by gender regarding the use of electronic means in education and performance objectives



Initial inspection of the specific results reveals a balanced picture regarding the use of electronic means for education purposes. Excluding the high degree where the females clearly surpass the males and the low degree where the males surpass the females, the rest of the answers indicate that there is no difference between males and females in their perception regarding the use of electronic means for

Table 1

Test results for the use of electronic means in education and performance objectives

	Test statistic	P-value	df
Pearson χ^2	4.170	0.383	0.383
Number of observations	500		

education purposes. Furthermore, a substantial part of the participants, both males (i.e., 24%) and females (i.e., 35%), seem to agree that the application of electronic means facilitates the learning objectives of the Probability courses.

In order to assess the students' views regarding the effectiveness of electronic means in education, we conducted a χ^2 test. This test groups the students into two groups (i.e., one including those answering "very low" and "low" who do not believe that the application of electronic means supports the course learning objectives, and one including those answering "moderate", "high", and "very high" who support the other view), and can be formally stated as follows:

> Ho: "Gender is independent of the opinion that attending a course using electronic means is more efficient in terms of the learning objectives of the course."
>
> H1: "Gender is not independent of the opinion that attending a course using electronic means is more efficient in terms of the learning objectives of the course."

At the 5% level of significance, we cannot reject the null hypothesis (χ^2 statistic = 4.170; df = 4; p-value = 0.383), and thus, we can conclude that gender is independent of the opinion that attending a course using electronic means is more efficient in terms of the course learning objectives.

The second question investigates whether the students believe that the new technologies adopted within the scope of the course enhanced their interest in probability notions. Here, we use the term new technologies as a term broader than electronic means, which includes software tools potentially requiring programming skills. Such tools have not always been particularly user-friendly and thus may have caused a level of anxiety in students. Nevertheless, the use of relevant tools has become the norm, especially regarding statistics education and practice, and therefore, their application is of particular interest. This is a 5-point Likert scale question assuming a similar structure to the previous one. We note that the specific question focuses on the interest aspect of technology, which is a vital element of the learning process. The corresponding results are summarized in Figure 2.

The students' answers show that using new technologies within the scope of the Probability course increased their interest in the relevant notions to a moderate to very high degree in a percentage that exceeds 80%. No major difference is observed between males and females, excluding the moderate degree to which females clearly surpass males.

In order to investigate the relationship between gender and the interest in the Probability contents caused by the use of the new technologies, we perform a χ^2 test, which groups the answers into two groups (i.e., one including the answers "very low" and "low" which involves the students who do not think that their interest was enhanced by the use of the relevant tools, and one including "moderate", "high" and "very high" which involves the students assuming a contrasting view), and can be formally stated as follows:

> Ho: "Gender is independent of the view that interest in the course is enhanced by the use of new technologies such as specialized software."
>
> H1: "Gender is not independent of the view that interest in the course is enhanced by the use of new technologies such as specialized software."

Figure 2

Percentage by gender regarding the use of new technologies and interest in probability

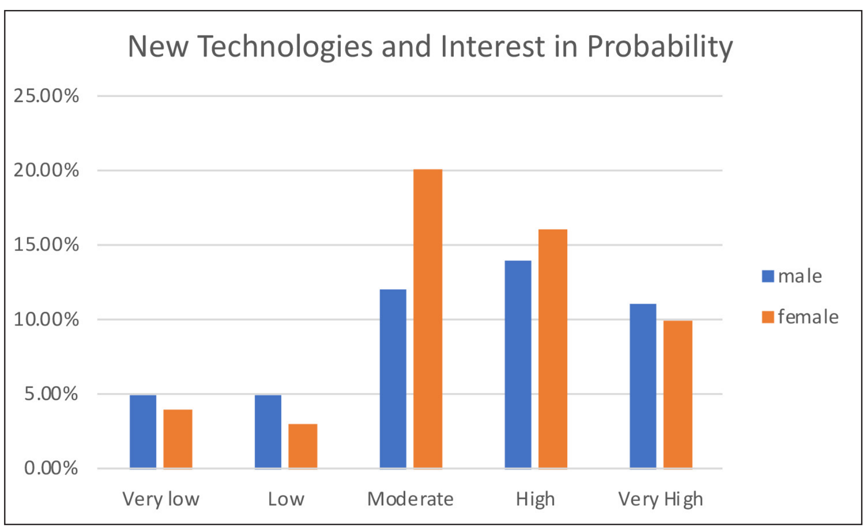


Table 2

Test results for the use of new technologies and interest in Probability

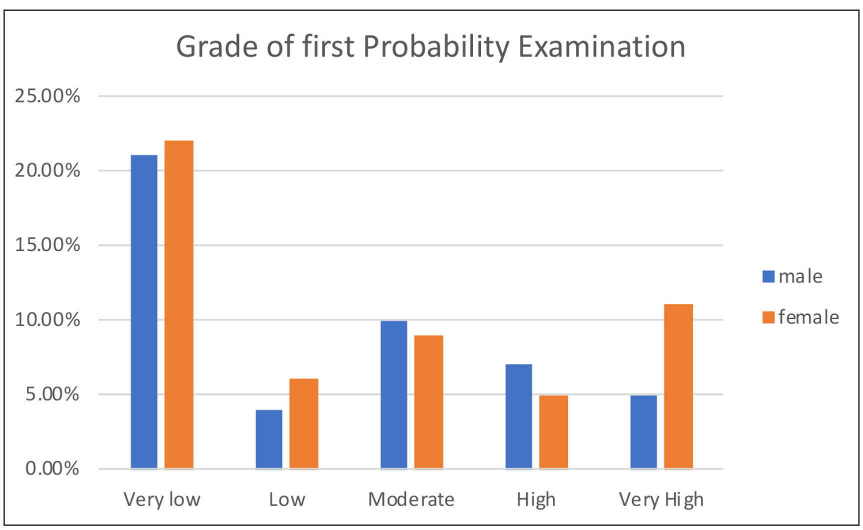
	Test statistic	P-value	df
Pearson χ^2	2.441	0.655	4
Number of observations	500		

At the 5% level of significance, we cannot reject the null hypothesis (χ^2 statistic = 2.441; df = 4; p-value = 0.655), and thus, we conclude that gender is independent of the view that using new technologies, such as specialized software increased their interest in the probability notions.

The next question focuses on the performance of the students in Probability, as indicated by their grades on the first examination. This is a self-reported results question utilizing the 10-grade scale adopted in Greek universities (i.e., grades 0-4 indicate insufficient performance, 5 indicates poor but sufficient performance, 6 indicates moderate performance, 7 indicates high performance, and 8-10 correspond to very high performance). This question is directly related to the learning outcomes of the course and is thus of utmost interest in the context of this study. The responses are summarized in Figure 3.

Figure 3

Percentage by gender regarding the grade in the first exam in Probability



The previous chart reveals that male students performed insufficiently and poorly but sufficiently in their first exam in Probabilities, about 25%, whereas female students performed about 28%. In other words, a total of 58% of the students performed poorly, receiving low and very low grades. Furthermore, there seems to be a slight differentiation regarding the very good performance, with 5% of males and 11% of females.

Subsequently, we seek to assess whether gender relates to performance in the first exam in Probability. To this end, we employ the χ^2 test formally stated:

Ho: "Gender is independent of the grade in the first exam in the Probability course."

H1: "Gender is not independent of the grade in the first exam in the Probability course."

At the 5% level of significance, we cannot reject the null hypothesis (χ^2 statistic = 2.709; df = 4; p-value = 0.608), and therefore, we conclude that gender is independent of the grade in the first exam in the Probability course.

The following question seeks to determine whether the survey participants possessed sufficient mathematics background prior to the commencement of their university studies. Sufficiency of their background is expected to have been reflected in their mathematics grades in the Greek State Exams. This is a question of self-reported results, using the grading scale of 20, which applies to Greek secondary education (i.e., grades under 10 indicate insufficient performance, 10-12.5 indicate poor but sufficient performance, 12.5-15 indicate moderate performance, 15-17.5 indicate good performance and 17.5-20 indicate very good performance). It is involved in the questionnaire to provide some insight into whether the results in the Probability exams relate to prior knowledge or to the class conduct of the course itself.

Table 3

Test results for the grade in the first exam in Probability

	Test statistic	P-value	df
Pearson χ^2	2.709	0.608	4
Number of observations	500		

The chart in Figure 4 reveals that the vast majority of the students had moderate to very high grades in high school level mathematics, that is, grades ranging between 12.5 and 15, which are considered moderate, grades between 15 and 17.5, which are considered high,

Figure 4

Percentage by gender regarding the school mathematics grade

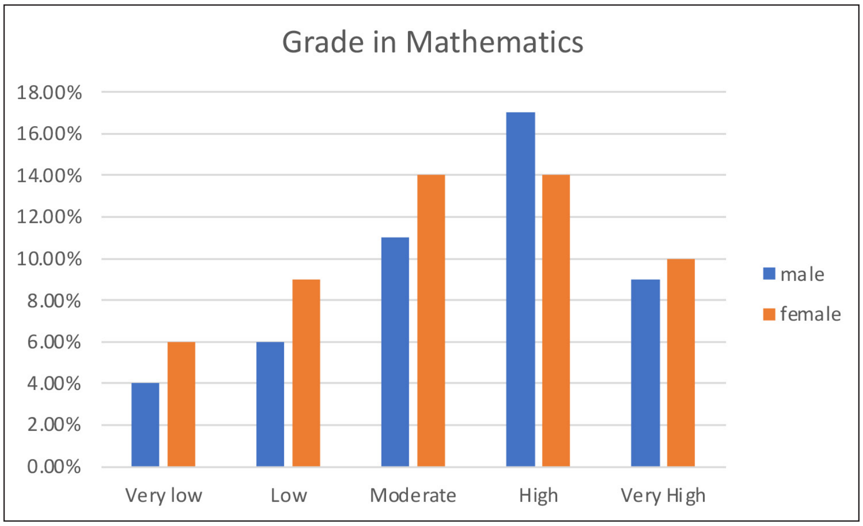


Table 4

Test results for the school mathematics grade

	Test statistic	P-value	df
Pearson χ^2	1.348	0.853	4
Number of observations	500		

and grades above 17.5, which are considered very high. Overall, these results suggest that the students have probably been sufficiently well-equipped with regard to mathematical contents prior to the commencement of their studies. This is to be expected, at least to some extent, given the quantitative nature of the specific department. Neither gender seems to stand out in the school mathematics performance.

Subsequently, we perform a χ^2 test to investigate whether the mathematics grade achieved at school is affected by the gender of the student. This test is formally stated as:

Ho: "Gender is independent of grade."

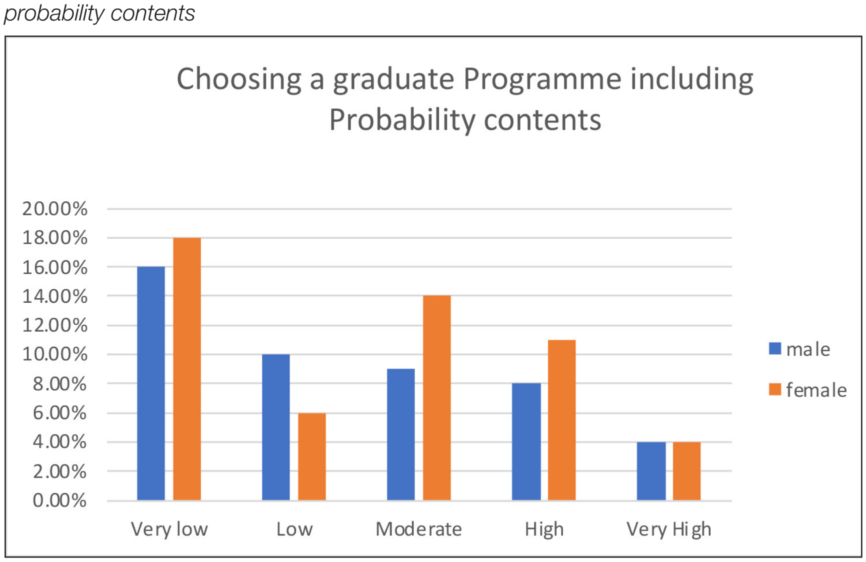
H1: "Gender is not independent of grade."

At the 5% level of significance, we cannot reject the null hypothesis (χ^2 statistic = 1.348; df = 4; p-value = 0.853), and we thus provide evidence that gender is independent of the school-level mathematics grade.

The final question we discuss is concerned with the intention of the students to pursue graduate-level studies that involve subjects entailing substantial probability content. This is again a 5-point Likert scale question with the answers indicating how likely the individual is to consider pursuing a relevant graduate program. The specific question seeks to identify whether the probability theory courses have enhanced the students' interest in the field and whether the relevant notions have been mastered enough for the individual to pursue advanced studies. The respective answers are summarized in Figure 5.

Figure 5

Percentage by gender regarding the choice of a graduate Program, including probability contents



This chart highlights that, in general, the students do not want to pursue graduate studies that include subjects with probability content. At first glance, male and female students are relatively close in their tendency to avoid such subjects at the graduate level.

In order to formally examine whether gender affects the choice of a graduate program that is relevant to Probabilities, we perform the following χ^2 test:

Ho: "Gender is independent of the choice of a graduate program, including Probability contents."

Table 5

Test results for the choice of a graduate programme which includes Probability contents

	Test statistic	P-value	df
Pearson χ^2	2.327	0.676	4
Number of observations	500		

H1: "Gender is not independent of the choice of a graduate program including Probability contents."

At the 5% level of significance, we cannot reject the specific null hypothesis (χ^2 statistic = 2.327; df = 4; p-value = 0.676), and thus, we provide statistical evidence that gender is independent of the choice of a graduate program, which includes substantial Probability contents.

Concluding Remarks

The purpose of this work is to assess whether the use of electronic means in education contributes to a deeper understanding of probability theory as well as whether gender affects the ability to learn the relevant content. Even though similar studies are not scarce in various contexts, we add to the corresponding body of literature with a survey addressed to the Greek academic community, bearing in mind the gender stereotypes still pertaining to Greek society and the dynamics of the rapid technological evolution, which tend to accelerate social change. To this end, we developed a questionnaire consisting of 30 questions, which was addressed to a sample of 500 students and provided the information necessary for the respective analysis.

Our results show that (i) students, regardless of their gender, believe that electronic means enhance the learning outcomes of the course, (ii) students exhibit a level of technology-driven interest for the Probability contents that is overwhelmingly moderate or above, and this is independent of their gender, (iii) students, irrespective of their gender, perform relatively poorly the first time they sit for the Probability exam, (iv) students possess a satisfactory level of prior knowledge regarding the Probability course, regardless of their gender, and (v) students, irrespective of their gender, are not very likely to include relevant courses in their graduate level studies. Overall, these findings are consistent with studies supporting the notion that affinity for quantitative studies is equally shared between males and females (e.g., Spelke, 2005).

The evidence provided in this study implies that efforts to intensify the utilization of electronic means for educational purposes are of vital interest to the educational community and should be strengthened. On the other hand, gender-based diversification of the teaching conduct, as well as the subsequent electronic means, do not seem to be of particular relevance here. However, further research is essential before it can be argued that such diversification efforts do not provide a substantial prospect. Finally, we note that the nature of quantitative courses such as Probability, which is intellectually demanding and may deviate from common experience, probably necessitates an appropriate combination between conventional lecturing and employing electronic means, which allow for effectively addressing problems of increased complexity. The boundaries between these two elements are subject to the course contents as well as the learners' background and potential. It is up to the teacher to assess and appropriately determine the exact setting for each course.

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Appendix

1=very little, 2=a little, 3=moderate, 4=a lot, 5=very much

1. Do you think that the application of electronic means in education has been more efficient in terms of the learning objectives of the course?

1	2	3	4	5
☐	☐	☐	☐	☐

2. Do you think the use of new technology increased your interest in the course?

1	2	3	4	5
☐	☐	☐	☐	☐

3. What was your grade in Mathematics in the Greek State Exams?

Under 10	10—12.5	12.5—15	15—17.5	17.5—20
☐	☐	☐	☐	☐

4. What grade did you get on your first exam in Probability theory?

Under 5	5	6	7	8—10
☐	☐	☐	☐	☐

5. How likely do you think you would choose a graduate program that includes substantial content in probability?

Under 5	5	6	7	8—10
☐	☐	☐	☐	☐

NOTES FROM THE FIELD

The Fall 2023 issue features two *Notes from the field* on the teaching of arithmetic. Darrow takes a historical perspective, focusing on the early teaching of arithmetic in America and providing connections to modern pedagogy. Continuing the discussion on modern arithmetic pedagogy, Witherspoon discusses a first account of using Number Talks in an elementary classroom.

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NOTES FROM THE FIELD

Briefly Recalling the Early Teaching and Learning of Arithmetic in America: Revisiting the Influence of Colburn's *First Lessons* Two Hundred Years Since its Publication

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KEYWORDS *history of mathematics education, curriculum, American education, arithmetic teaching*

Introduction

The first mathematical subject to enter school curriculum in the United States was arithmetic, which was later followed by algebra in secondary schools and colleges in subsequent years (da Ponte & Guimarães, 2014; Kilpatrick, 2014; Kaestle, 1983). In the earliest years of the United States, exact definitions of arithmetic were not generally or formally agreed upon or stated explicitly. However, several common elementary mathematical activities, such as numeration and calculation with the four operations, were ubiquitous in early writings on the subject (Slocomb, 1831; Colburn, 1821; Bjarnadóttir, 2014; Cohen, 2016). Additional components of common arithmetic included the calculations with, and the properties of fractions, decimal numbers, proportions, measurement, and elementary accounting (Bjarnadóttir, 2014; Cohen, 2016; Jones & Coxford, 1970; Karpinski, 1940). There also seems to have been a distinction made in school curriculum at the turn of the nineteenth century between arithmetic, which was characterized by concrete quantity and calculation, and algebra, which included the consideration of unknown and variable entities.

Early Teaching of Arithmetic in America

Consistent with the nature of education in the earliest years of the United States, the teaching of arithmetic began informally. Kaestle (1983) notes that in these early years, "Most children attended school at some time, but much education also came through the family, the church and the workplace" (p. 4) and that many others "did without schooling, remaining illiterate or picking up the three R's from parents or friends" (p. 4). The three R's (reading, writing, and arithmetic) formed the foundation of the curriculum in early informal and formal schooling in the early United States. However, information on the teaching of arithmetic has received less attention in historical texts due to the emphasis placed on developing students' ability to read and write, primarily to prepare them to engage with religious texts (Kaestle, 1983; Tyack, 1967).

The basic four operations (adding, subtracting, multiplying, and dividing) and related calculations formed the core of early arithmetic learning in America (Cohen, 2016). The teaching of more rigorous topics was generally subject to the widely varying proficiencies of individual educators, which ranged from family members, tutors, schoolteachers, to mentors (Cohen, 2016). Many students did not reach learning beyond calculations with the four operations, and most students' learning "ended with the Rule of Three" (Cohen, 2016, p. 122), which is a staple of early arithmetic teaching and refers to the solving of fractional proportions. The reasons for this ceiling of arithmetic learning include the public sentiment that arithmetic learning was "regarded as too difficult for children younger than ten or twelve to study" (Cohen, 2016, p. 118) and that mathematical learning was of little use to most of America's workers.

Although several of the founding fathers and early statesman such as Benjamin Franklin, Daniel Webster, and, perhaps most vocally, Thomas Jefferson, received instruction in arithmetic and advocated for its expansion in curriculum in the 1700's, widespread formal arithmetic instruction in schools was not realized until well into the 1800's (Cohen, 2016). However, during the early years of the United States and into the nineteenth century, formal arithmetic was rarely, if ever, taught to females and non-white males (Cohen, 2016). Females were generally excluded from mathematical learning except at the "most elementary level" (Cohen, 2016, p. 139), partly because girls did not progress in school past the first several years. Another factor was the prevailing and widely accepted notion that females could not comprehend arithmetic or other forms of mathematics (Cohen, 2016). For non-white males, the pervasive racist societal structure of the country and active institution of slavery prevented many from participating in any formalized learning whatsoever (Cohen, 2016; Kaestle, 1983).

The Prominent Pedagogy in Early American Arithmetic

The prominent pedagogy of the informal and formal teaching of arithmetic at the turn of the nineteenth century revolved around tenets of mental discipline theory. This theory, Kliebard (2004) notes, has its roots in antiquity and is characterized by the assertion that "certain subjects of study had the power to strengthen faculties such as memory, reasoning, will and imagination" and that "certain ways of teaching these subjects could further invigorate the mind and develop these powers" (p. 4). Kliebard continues by noting the famous analogy of the mind as a muscle; and this muscle is strengthened by "vigorous exercise" (p. 4) often in the form of "monotonous drill, harsh discipline and mindless verbatim recitation" (p. 5).

Historians and mathematics educators agree that this was quite visible in the teaching of arithmetic, and that the subject was particularly vulnerable to being naturally aligned with the theory. An instantiation of mental discipline in mathematics education practice was the "rules" or "rule method" of teaching, where students were required to repeatedly memorize and apply numerical facts and procedures (Bidwell & Clason, 1970; Cohen, 2016). Cohen (2016) notes that the learning "deliberately relied on memory, not on understanding" (p. 121). One pedagogical tool that illustrates this pedagogy was the copybook or ciphering book, which were "widely used in the eighteenth century as substitutes for textbooks, ever in short supply" (p. 120). The students were dictated rules and calculations that they dutifully copied into these books with little to no attention paid to the understanding of such mathematical work.

A Shift in Pedagogical Approach

The majority of school children experienced arithmetic under the "rule method" through the turn of the nineteenth century and well into the common school movement. Although it would take years, and in some cases decades, for schoolchildren and teachers in the United States to see it, a shift in mathematics education and research began taking place during the first fifth of the nineteenth century—one which focused on developing conceptual mathematical understanding.

Due to the delay in reaching the classroom, this shift was arguably most visible in the development of textbooks. The textbooks from the colonial period, such as the widely used *Cocker's Arithmetick* (1677), embodied the "rule method" and the associated characteristics of mental discipline (Bjarnadóttir, 2014). One of the earliest texts that contrasted this viewpoint, was Samuel Goodrich's *The Child's Arithmetic* (1818), which encouraged the use of "manipulatives" or tangible objects to be used in the development of arithmetic understanding. According to Cohen (2016), Goodrich argued "that learning by rules and rote actually prevented children from comprehending arithmetic" (p. 134).

Goodrich's text and his sentiments were significant, which were of the first to offer a consideration of pedagogical alternatives to the "rule method" which nearly completely characterized formal arithmetic teaching in the United States since the country's inception. Widely considered to be the most significant in this regard was the "inductive method" developed from the pedagogical theories of the Swiss philosopher Johann Pestalozzi and championed in America by textbook author Warren Colburn (Cohen, 2016; Kilpatrick, 2014; Karp & Furinghetti, 2016). Colburn's seminal text, *First Lessons in Arithmetic on the Plan of Pestalozzi, with Some Improvements*, was originally published in 1821, and was followed by the 1826 edition, the title of which was often adjoined with Colburn's name: Colburn's *First Lessons. Intellectual Arithmetic, Upon the Inductive Method of Instruction* (1826). The texts provided opportunities for students to discover arithmetic rules and develop an understanding of the related concepts. Also notable was the text's emphasis on "mental arithmetic," which encouraged computation without pencil and paper, and the complete omission of other classic elements of "rule method" pedagogy such as the "Rule of Three" (Bjarnadóttir, 2014).

The historical significance of this conceptual change in pedagogy was recognized immediately. Colburn's first and subsequent texts were an "instant sensation among educators in the 1820's" (Bjarnadóttir, 2014, p. 447). Bjarnadóttir (2014) notes that this shift in pedagogy was so profound that "the vast diffusion of numerical skills in the United States from the 1820's to 1900 is owed to [Colburn's] influence" (p. 447). It is important to note that Colburn's work is viewed historically as marking "the beginning of widespread concern with pedagogy in arithmetic teaching" (Bidwell & Clason, 1970, p. 1). Cohen (2016) notes that shortly after its publication, the *North American Review* "prophesied 'We have no doubt that Mr. Colburn's book will do much to effect an important change in the common mode of teaching arithmetic" (p. 134). Additionally Bjarnadóttir (2014) noted that it "electrified educators with the startling notion that children could learn arithmetic basics even before they could read and write" (p. 447), which stood in direct contrast to what had been the fundamental educational arrangement in American schooling since its inception.

This moment in the history of mathematics education characterizes one of the first widespread considerations of how students develop understanding and how pedagogy contributes to this. Moreover, it marks a shift in the aims of mathematics education. For it had been implicitly assumed that students should understand the mathematical content of arithmetic; however, the explicit statement of this as a goal and the academic treatment of how it should be attained was groundbreakingly new (Bjarnadóttir, 2014; Cohen, 2016). Another metric of the significance of this development is the fact that despite its beginning just over two hundred years ago, it "continues to resonate in educational theory and practice in the twenty first century" (Bjarnadóttir, 2014, p. 447; Cohen, 2016). Therefore, one may trace the roots of many contemporary pedagogical debates in mathematics education to this moment in time, particularly with respect to the debate of best practices regarding the interplay between mathematical skills and procedures and the development of conceptual understanding.

Connections to Today

In the two hundred years since Colburn's seminal text, the mathematics education community has reached a consensus that mathematics teaching which focuses solely on the memorization and the application of rules and procedures is generally bad practice. This is because tasks that focus solely on these elements of mathematics learning have been found to require low levels of cognitive demand and do not alone help students form rich connections among the mathematical concepts at hand (Stein et al., 2000). Although procedural learning should not be the sole focus or aim of mathematics learning, mathematics education research has also established that such learning is an essential component of mathematical understanding and proficiency (NCTM, 2000; NRC, 2001). Therefore, it is clear that the pedagogy of the colonial period did address some elements of mathematical learning; however, it did not contribute to the holistic development of conceptual understanding on which such a high pedagogical value has since been placed.

Pedagogy that develops mathematics knowledge through deductive logic and proof, problem solving, and discovery or inquiry-based learning tasks has been shown to require the highest levels of cognitive demand and develop rich conceptual understanding (NRC, 2001; NGA, 2010; NCTM, 2000; Stein et al., 2000). However, such learning is not always present in many American classrooms today. Most often criticized are pedagogies that still value elements of the mental disciplinarian model and do not provide such opportunities for rich conceptual learning. The reasons for why such teaching is still prevalent are numerous and complicated, but among the most common are restrictions of time, extensive curricular demands, inequitable access to instructional resources, and a resistance to changing traditional practice.

Although mathematics pedagogy has evolved substantially since the early nineteenth century, the field still wrestles with many of the issues that were present then. The impact of the mental disciplinarian approach on learning can still be seen in classrooms across America. Despite this, the last two hundred years have produced tremendous advances in the quality of mathematics teaching as well as our understanding of what it means to learn mathematics. Such a series of advances began in earnest with our forebears of the early 1800's.

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NOTES FROM THE FIELD

Developing Number Sense with Number Talks

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KEYWORDS *number talks, number sense, procedural fluency*

When I was a beginning fourth-grade teacher, Dr. Sherry Parrish, my math coach, introduced me to number talks. Number Talks are designed as computation problems for students to mentally solve, place fist on the chest with an extended thumb to show they figured out a solution, and then discuss their strategies for 5 to 15 minutes (Parrish, 2010). During these early years, I realized that I leaned towards being more teacher-centered when facilitating number talks. I primarily focused on students' ability to mentally solve various problems for the sake of solving rather than customizing number talks to address the specific needs of my students.

When I became a Math Coach, I realized the importance of crafting problems to support learning through a more student-centered approach. When classroom teachers used anecdotal notes to create carefully designed problems, I observed a significant improvement in their students' performance, which confirmed the effectiveness of the approach.

As an Assistant Professor, I have numerous opportunities to observe teacher candidates in classrooms across multiple districts. In these settings, I noticed (K-5) students counted every number starting from one with their fingers as their primary default strategy for addition and even multiplication. This motivated me to observe a second grade classroom in an urban setting with the intent to later facilitate number talks to the class for five consecutive days.

In this article, I provide examples of how strategic number talks implemented with purposeful questions can strengthen number sense and problem-solving strategies in an urban second-grade classroom. By using number talks to support students who employ limited and inefficient strategies to solve computation problems, I was able to anchor learning with conceptually based strategies rooted in understanding mathematical concepts, thereby enhancing their number sense.

Although quick images with small numbers as seen on dot cards or ten frames are great ways to introduce number talks, I decided to start with fluency within 10 by developing the strategy for doubles plus one (counting on one more from a doubles combination) because the classroom teacher was working on adding double digits with multiple addends. Over the years, I have successfully used doubles and doubles plus one number talks to help students to become fluent within 10 and then 20. I have also used this strategy to promote flexibility with numbers that can then be applied when adding larger numbers or multiple addends. The following sections provide a detailed explanation of each number talk employed throughout the week, with a summary also presented in Table 1.

Table 1

Number Talk Strings and Purposeful Questions

Day	Number Talk Strings and Purposeful Questions
Day I	Purposeful question posed, *"If I am looking for the sum, what type of problem do you think I will write?"* 5 + 5; 5 + 6 6 + 6; 6 + 7
Day II	4 + 4; 4 + 5 7+7; Purposeful question posed, *"If I continue with my mathematical pattern, what problem might I write next?"*
Day III	Purposeful question posed, "*What problem could we use to help us to mentally solve 8+9?"* 8+8 10+10; 10+11
Day IV	Review without a scaffold: 7+8; 6+7
Day V	Formative Assessment: 15+15; 15+16

Number Talks Goal: Doubles +1

Day I: During an initial observation when the second graders attempted to solve an addition problem with 4 addends (24 + 62 +70 + 33), it was noted that every student defaulted to counting on their fingers or drew inefficient models such as flats and lines, which resulted in several unreasonable answers such as 698. To reduce the need for counting while fostering mental strategies, I created a number string (a set of related math problems designed to teach strategies based on relationships between numbers) that I felt students would have access to while solving the problems. For immediate engagement, I posed the question, "If I am looking for the sum, what type of problem do you think I will write on the board?" After the students made several guesses, e.g., "plusses, minus, or times-ing" for addition, subtraction, and multiplication, I immediately wrote 5+5 and then 5+6 on the board. Although I did not explicitly confirm the vocabulary word, I was intentional about using the academic language at the beginning of the number talk, e.g., "What sum did you get for 5+6?" After I received an answer, I responded with the follow up question, "Did anyone get a different sum?" Most of the students knew 5+5 with automaticity and they all used the "counting all or counting on" strategies from 5 to solve the subsequent problem 5+6. For 6+6 and 6+7, the students solved in a similar manner, but one student counted on from the larger addend.

Day II. I posed 4+4 to tap into existing knowledge and then 4+5 to build upon previous learning. Again, many of the students knew the double (4+4) with automaticity, but all defaulted to using a version of the counting on strategy to solve the subsequent problem (4+5).

Unlike the responses to the smaller doubles where they knew with automaticity, the students resorted to counting on their fingers to accurately determine the total for 7+7. To promote critical thinking, I posed the question, "If I continue with my mathematical pattern, what problem might I write next?" One student immediately responded with 7+8. I asked them to consider why I might pose this question. In a pensive manner, one student calmly stated, "Well it is one more, so the answer is 15."

Day III. To further promote the strategy of "doubles plus 1," I asked the students, "What problem could I use to help us to find the sum of 8+9?" Several students exclaimed in unison "8+8." Even though the students were not asked to solve, many students excitedly shared that the sum of 8+9 is 17. For the next problem, the students solved 10+10 with automaticity. But when presented with 10+11, only a few added 1 more to 20 or knew with automaticity, while most counted on their fingers from 10 or 11. It is important to note that the students had more success in adding one when the teacher prompted them to think about a problem that could help them solve the problem at hand.

Day IV. As a review, the teacher posed problems without the doubles scaffold. Fortunately, the goal of doubles plus 1 resonated with most students as evidenced in the strategies they shared for 7+8 and then 6+7. The students who shared used the doubles combination to solve, for example, "7+7 = 14, so 7+8 = 15."

Day V. When the students were asked to find the sum for 15+15, most students used the "counting on" strategy to solve. As an indicator of great success, most students displayed their silent thumbs immediately when they were presented with the problem 15+16. Success was also deemed by students only having one answer (31) to defend for a double-digit addition problem. In addition to properly using the term throughout the week, the second graders were able to correctly indicate the sum as being the answer to an addition problem.

Reflecting on our Teaching and Learning

As I have shown in the number talk examples, the students were more successful in implementing mental strategies than their previous attempts in solving addition problems with inefficient counting all methods on their fingers or through drawn representations. According to Kamii (1993) "algorithms are harmful because they unteach place value and hinder children's development of number sense." Over the years I have noticed that if students are only presented with one way to solve a problem or traditional algorithms that are designed for memorization, number sense and problem solving can be daunting to students which is evidenced when students provide unreasonable answers. On the other hand, when teachers provide opportunities for students to engage in strategically planned number talks, students will have the opportunity to demonstrate at least two of the Common Core Standards for Mathematical Practice: Construct Viable Arguments and Critique the Reasoning of Others as well as Attend to Precision e.g., introducing, reinforcing, and applying mathematical vocabulary.

After engaging in five days of strategically crafted number talks, the second graders demonstrated flexibility of numbers and became more fluent within 20 and flexible with sums up to 31. The students began to transition from solely counting on their fingers to making use of the mental strategy doubles plus one. I solidified this strategy by inviting the students to predict and think about the problems being posed. To further strengthen number sense and promote procedural fluency, teachers could foster other mental strategies through strategically planned number talks such as making tens or decomposing and solving by place value. The teacher can continue reinforcing the concept of doubles plus one with the number string 25+25 followed by 25+26.

In this article, I have demonstrated the benefits of focusing on an efficient strategy for students to construct throughout the week. I encourage educators to consider using this model for fostering academic language and strategies such as subitizing with quick images that can be used to support students as they transition from counting to reasoning strategies.

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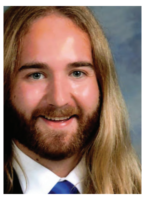

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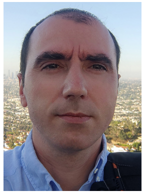

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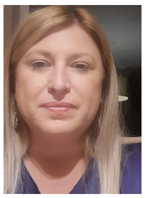

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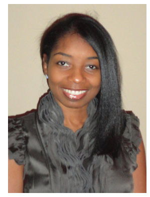

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ACKNOWLEDGEMENT OF REVIEWERS

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This call for papers is an invitation to mathematics education professionals, especially Teachers College students, alumni, and associates, to submit articles describing research, experiments, projects, innovations, or practices in mathematics education. The journal features full reports (approximately 3500 to 4500 words) and short reports (approximately 500 to 1500 words). Full reports describe findings from specific research, experiments, projects, innovations, or practices that contribute to advancing scholarly knowledge in mathematics education. Short reports ("Notes from the Field") provide examples, commentary, and/or dialogue about practices out in the field of mathematics education or mathematics teacher education; examples from classroom experience are encouraged. Although many past issues of *JMETC* focused around a theme, authors are encouraged to submit articles related to any current topic in mathematics education, from which pertinent themes for future issues may be developed. Articles must not have been submitted to or accepted for publication elsewhere. All manuscripts must include an abstract (approximately 150 words in length) and keywords. Manuscripts should be composed in Microsoft Word and follow APA format. Guest editors will send submitted articles to the review panel and facilitate the blind peer-review process. Articles for consideration should be submitted online at jmetc.columbia.edu, and are reviewed on a rolling basis; however, to be considered for the Spring issue, articles should be received by **January 31, 2024**.

CALL FOR REVIEWERS

This call for reviewers is an invitation to mathematics educators with experience in reading or writing professional papers to join the review panel for future issues of *JMETC.* Reviewers are expected to complete assigned reviews within three weeks of receipt of the manuscript in order to expedite the publication process. Reviewers are responsible for editorial suggestions, fact and citations review, and identification of similar works that may be helpful to contributors whose submissions appear appropriate for publication. Neither authors' nor reviewers' names and affiliations will be shared with one another; however, reviewers' comments may be sent to contributors of manuscripts to guide revision of manuscripts (without identifying the reviewer). If you wish to be considered for review assignments, please register and indicate your willingness to serve as a reviewer on the journal's website: jmetc.columbia.edu.

CALL FOR EDITOR NOMINATIONS

Do you know someone who would be a good candidate to serve as a guest editor of a future issue of *JMETC?* Students in the Program in Mathematics Education at Teachers College are invited to nominate (self-nominations accepted) current doctoral students for this position. Being asked to serve as a guest editor is a testament to the high quality and standards of the student's work and research. In particular, anyone nominated as a guest editor should be a current doctoral student whose scholarship is of the highest quality, whose writing skills are appropriate for editorial oversight, and whose dedication and responsibility will ensure timely publication of the journal issues. All nominations should be submitted to Ms. Juliana Fullon at jmf2213@tc.columbia.edu .