Polyrhythmic Cognition and Metric Spaces: A Compositional Framework

Louis-Michel Tougas

Abstract

This paper introduces a novel approach to rhythm-based contemporary music composition. I propose that recent insights into the cognitive limits of rhythm and meter can be translated into a compositional framework that considers not only the structural and abstract properties of rhythm structures, but also their perceptual impact on an audience. This method enables the preservation of a high degree of structural complexity while enhancing its perceptual effectiveness and minimizing notational intricacy. The approach is facilitated by the use of OpenMusic, a computerassisted composition software, alongside a visual representation of metric modulation networks that I call a *rhythm lattice*. To illustrate this approach, I present two examples from my recent chamber music works.

Keywords: Metric Modulation; Ars Subtilior; Polyrhythm; Computer-Assisted Composition; Music Notation

Introduction

The abstraction that music notation offers—especially since the advent of notation software and computer-aided and algorithmic composition tools—also has the potential drawback of ignoring many important considerations, including the performability, the perceptibility, and the semantic and cultural ties that seemingly abstract musical structures may bear.

While the abstraction of rhythms into purely numeric structures is a powerful tool for organizing and developing musical material, these structures don't necessarily take into account how they will translate back into sound. Moreover, music notation is not a style-agnostic language for the representation of sound. It is a culturally loaded system of conventions that was developed with the goal of preserving the music of the historical and cultural context in which it was developed. However, approaching notated music composition by first forgetting the constraints of standard music notation can serve as a powerful way to explore less obvious aesthetic paths.

In this paper, I will present my personal polyrhythm-focused compositional approach, informed by three main fields of research. The first one is the study of music and music notation prior to the Western common practice, which allowed for different possibilities of temporal expression. The second perspective comes from the field of music perception and cognition. Many critiques have been formulated against strictly structuralist approaches to music composition throughout the second half of the twentieth century. However, integrating knowledge from research on music cognition can help reconcile the need for structural complexity and the perceptibility of such structures without resorting to outdated forms of expression. The third aspect concerns the use of computer-aided tools to synthesize structural and perceptual constraints into a usable model for music composition. As part of this work, I will present excerpts from two of my recent pieces to demonstrate the alternative approach to notation I am using and the aesthetic potential it unfolds.

Notation

The modern music notation system in the Western world crystallized around the end of the Renaissance (Bent 1998). Since then, there have been few structural changes regarding how pitch or rhythm functions within the notation itself. However, prior to this crystallization and standardization, many alternative possibilities for the representation of music had been developed (Maria and Berger 2002). Perhaps the most striking examples of this rich diversity of representational practices come from the period called *ars subtilior* (more subtle art), transitioning between the Middle Ages and the Renaissance in the context of the Great Schism that divided the Catholic Church into two rival factions with their own pope, one based in Rome and the other in Avignon (Apel 1946; Taruskin 2009).

The *ars subtilior* saw the development of many different approaches to music notation, but perhaps the most complex and obscure one is what Willi Apel calls "mannerist notation," which he discusses extensively in his *Notation of Polyphonic Music 900-1600*. The use of hollowed and full noteheads, along with different notehead colors (black, red, and blue), allowed for very complex possibilities of rhythmic intricacies. The difficulty which the modern transcriber faces is of particular interest to me, since it shows that notation is not merely a representation of a more abstract process but the musical content itself, which cannot easily be translated into any other language. As Apel describes, "It is in this period that musical notation far exceeds its natural limitations as a servant to music, but rather becomes its master, a goal in itself and an arena for intellectual sophistries" (1949).

Just as the *ars subtilior* composers have adapted and expanded the notational innovations of the *ars nova* for their own expressive needs, I believe that today's music notation standards can be adapted to better fit contemporary views of time as a multi-dimensional parameter.

Metrical Structures, Rhythm Perception, and Cognition

In their *Generative Theory of Tonal Music* (GTTM), Lerdahl and Jackendoff propose that the brain structures isochronous streams of pulses hierarchically, and they define metrical structure as "the regular, hierarchical pattern of beats to which the listener relates musical events" (1983). However, they also note that not all music exhibits metrical structure in this sense:

> "In fact, though all music groups into units of various kinds, some music does not have metrical structure at all, in the specific sense that the listener is unable to extrapolate from the musical signal a hierarchy of beats. Examples that come immediately to mind are Gregorian chant […] and much contemporary music […]" (1983, 17).

Numerous intermediary possibilities exist between a strictly hierarchical pulse structure and completely arhythmic music. In particular, the simultaneity of multiple isochronous streams—which form a polyrhythm—frequently produces perceptually ambiguous stimuli, as each stream suggests more than one potential underlying metrical structure, as described by Lerdahl and Jackendoff. Polyrhythms are perceptually intricate objects that can be interpreted in a variety of ways, depending on the listener's focus. For this reason, I consider polyrhythms a powerful tool for artistic expression, as they embody both rational structure and perceptual openness.

While polyrhythms can exist within a purely hierarchical framework of groupings and subdivisions, twenty-first-century musicians have access to a wider array of possibilities. These often require a more decentralized approach to meter, one that may not adhere to the predictable, well-ordered hierarchies of common-practice music. The concept of perceptibility is slippery and can easily lead to over-simplifying musical structures, but integrating knowledge related to how we perceive sound stimuli and how our brain treats and organizes them can serve as a very powerful artistic approach. My personal approach to polyrhythm incorporates not only strictly hierarchical pulses and integer-related subdivisions but also second-order metric modulations and other complex techniques. These approaches often blur the boundary between metricity and ametricity. I use this knowledge not as a set of limitations but as a means to explore alternative paths to temporal complexity that are ideally going to be heard and felt by an audience.

In his seminal work *Hearing in Time: Psychological Aspects of Musical Meter*, Justin London summarized the research of a number of previous research on limits regarding the cognition and perception of pulse and meter and gave a detailed account of similar boundaries found by multiple researchers over the years (2004, 28–30).

The first set of boundaries indicates that isochronous sound stimuli are perceived as *pulsations* if their constituent inter-onset interval is approximately between 100 ms and 2000 ms (Bolton 1894; Hirsh 1959; Monahan and Hirsh 1990; Repp 2005). The inter-onset interval (IOI) is the time between two sound events and corresponds to the inverse of the frequency; for example, IOI of 1000 ms is equivalent to a frequency of 1 Hz or to a metronomic value of 60 beats per minute (bpm). As such, IOI values between 100 and 2000 ms correspond to an upper limit of 600 bpm and a minimum of 30 bpm. While these values were found in ideal laboratory conditions, many factors regarding the nature of the sound stimuli themselves affect the perceptibility of their metricity. Parameters such as the amplitude envelope of each pulse, their frequency spectra, or an eventual masking effect caused by a competing sound source may affect the clarity with which one can interpret a pulse train as metric or not. For this reason, I use these values not as hard boundaries, but as good indicators of what seems a reasonable temporal space to develop abstract polyrhythmic structures that will translate into perceptually meaningful stimuli once used in a musical composition and played by performers in a concert hall. In all cases, the mere consciousness of speed limits for the perceptibility of pulses provides a useful basis for the categorization of different polyrhythms based on whether or not they might be perceived as rhythmic stimuli at different moments during their emission.

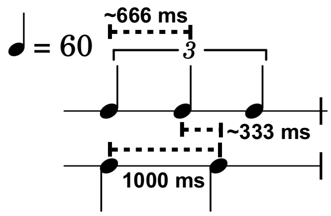

Figure 1: 3:2 polyrhythm

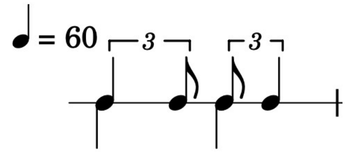

Figure 2: 3:2 compound rhythm

For example, Figure 1 presents a simple 3:2 polyrhythm, where the bottom voice is a quarter note pulse and the top voice a quarter note triplet. The written metronome mark in quarter note equals 60, indicating that the IOI of the bottom voice is 1000 ms. As the top voice is one and a half times faster (a 3-to-2 ratio) than the bottom one, we can calculate that its IOI will be of $1000 \text{ ms} \times 2/3 = {\sim}666 \text{ ms}$. The distance between the second note of the top voice and the second note of the bottom voice is $1000 \text{ ms} - {\sim}666 \text{ ms} = {\sim}333 \text{ ms}$.

The common pulse denominator between two isochronous streams corresponds to the slowest subdivision shared by both streams. Numeric values can prove useful to find this common denominator, especially in the case of more complex polyrhythms. For Figure 1, the bottom voice is a succession of quarter notes (i.e., one-quarter of a whole note), which can be represented in numeric form as $1/4$. In the top voice, one single pulse of the quarter note triplet is equal to a sixth of a whole note, or $1/6$. The common pulse denominator can be found by dividing the greatest common factor (GCF) of both numerators by the least common multiple (LCM) of the denominators. This gives the value $1/12$, or one pulse of an eighth note triplet, which corresponds to the difference between the second beat of both voices and has an IOI of about 333 ms, or frequency of 180 bpm.

All these values fall well within the range found by London for the perception of a pulse, as the common IOI denominator between the two voices is ${\sim}333$ ms, or a tempo of 180 bpm. This also means that this polyrhythm can be reduced into the compound rhythm shown in Figure 2, where the eighth note triplet equals 180 bpm.

However, more complex examples and different metronomic values can very rapidly lead to crossing these cognitive boundaries. Figure 3 shows a 5:4 polyrhythm with a metronomic value of quarter note equals 130 bpm. The IOI of the bottom voice is,

$$60/130 \times 1000 = {\sim}462 \text{ ms},$$

and the IOI of the top voice four-fifths of it is,

$$4/5 \times 60/130 \times 1000 = {\sim}369 \text{ ms}.$$

The time interval between the second beat of each voice is equal to the difference between these two values, and the time interval between the third beat of each voice is twice that last value:

$$462 \text{ ms} - 369 \text{ ms} = 91 \text{ ms},$$
$$91 \times 2 = 182 \text{ ms}.$$

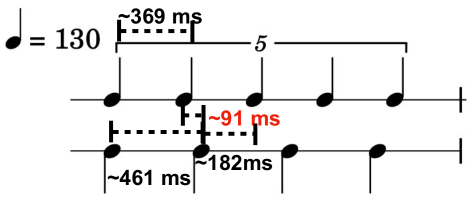

Figure 3: 5:4 polyrhythm

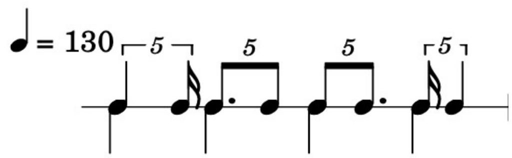

Figure 4: 5:4 compound rhythm

While almost all values are comprised between 100 and 600 ms, making the relation between all successive pulses metrical, the time interval of 91 ms is slightly too short to be interpreted as strictly metrical; therefore, the second pulse of each voice will rhythmically conflict with each other. As all polyrhythms are inherently symmetrical, this will also be the case between the fifth beat of the top voice and the fourth beat of the bottom voice.

The common denominator between the two voices of Figure 3 is arithmetically the sixteenth note quintuplet (see Figure 4), but because its tempo is out of our cognitive range, the next-slowest common denominator will be used to perform and mentally represent this polyrhythm metrically, which is the eighth note quintuplet with an IOI of ~ 182 ms. This shows that a polyrhythm is not entirely perceptually metrical or ametrical but that different types of relations exist between all consecutive pulses of the corresponding compound rhythm.

This perspective allows many possibilities for the construction of different polyrhythms that present phases that are metrical and other phases where metricity is blurred. In that sense, because the chosen tempo of 130 bpm puts the common denominator of the two streams just slightly below our perceptual range, the 5:4 polyrhythm of Figure 3 becomes a complex perceptual object presenting both metrical and ametrical phases rather than a strictly rhythmic structure.

As discussed earlier, a performer aiming to interpret Figure 3 as accurately as possible may encounter challenges if they attempt to deconstruct the rhythmic structure by finding a common denominator between the two parts. One alternative strategy frequently used by performers is to re-notate complex rhythms in a way that remains as perceptually faithful to the original as possible while facilitating the cognitive processing and interpretation of the music. In the case of Figure 3, a performer might begin by identifying the common denominator between the two streams, resulting in Figure 4. However, upon realizing that the IOIs between the second beat of each voice are too short to be performed metrically, as well as between the fourth and fifth beats, the performer

Figure 5: 4-beat reading of 5:4 polyrhythm

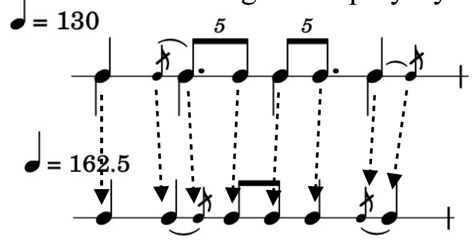

Figure 6: 5-beat reading of 5:4 polyrhythm

could then re-notate the metrically conflicting notes in one of the voices as grace notes, while preserving the onsets that are metrically feasible. One possible outcome of this re-notation is shown in Figure 5.

Figure 6 takes this approach further by altering the notated tempo while maintaining the same rhythmic proportions. This adjustment allows the primary voice to be written as quarter notes at 162 bpm, rather than as uneven quintuplets, simplifying the reading process even more.

In his doctoral dissertation on the performance of the music of Györgi Ligeti, pianist Imri Talgam developed a similar re-notation strategy to reduce the cognitive load required to perform multiple polyrhythmic voices simultaneously and maximize the perceptual differentiation between auditory streams (2019). In a subsequent collaborative work on the performance of Xenakis' highly complex *Mists*, Talgam and I developed a perceptual grid[1] (Figure 7) for the reduction of multiple polyrhythmic streams. Basing our work on his experience as a performer of rhythmically complex music and the data provided by London, we determined a number of perceptual categories for all events occurring inside the 0 to 110 ms IOI range that could allow for more precision than just grace notes before or after. This perceptual grid was used in a prototype of an automated rhythmic reduction algorithm I programmed in the *OpenMusic* software (Bresson, Agon, and Assayag 2011) developed at IRCAM.

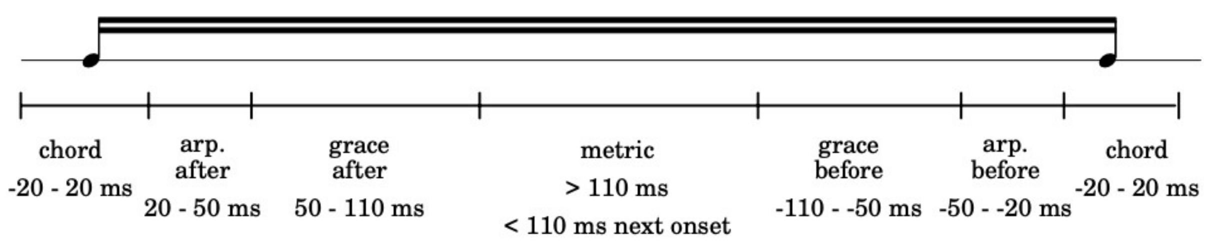

Figure 7: Talgam & Tougas: Rhythmic grid for the perceptually-driven re-notation of small IOIs

[1] The grid is partially inspired by representations of similar data found in London (2004). See for example Fig. 2.1, p. 36.

In his essay *Le feuilleté du tempo*, French composer and music theorist François Nicolas defined a metric modulation as "a change of tempo and meter achieved through a pivot on a stable unit of duration" (1990). Perhaps the composer the most well-known for his use of metric modulations is Elliott Carter, up to the point where we sometimes hear the term modulation "à la Carter." Nicolas also proposed an original and very comprehensive description of metric modulations, which he presents as multi-layered processes rather than simple tempo equivalence. Even though he gives them the names *impulse, pulse,* and *measure*, the multiple layers Nicolas uses to produce his model of meter is reminiscent of the *prolatio, tempo,* and *modus* used by Middle Age and Renaissance composers and seem to indicate a certain filiation between the temporal conceptions developed in the second half of the XXth century and those of the late Middle Age. I have used Nicolas's model for metric modulation for the programming of a few functions in *OpenMusic*, and it has also served as the starting point for the development of my concept of decentralized modulations.

Computer-assisted Composition

In order to assist in the cognitively-informed manipulation of complex polyrhythms and metric modulations, I have developed a library of tools for the computer-aided composition software *OpenMusic*. I will present two that I find most useful for my compositional works.

1. Calculation of all metrically related rhythms at a given tempo

Given a certain metronomic value and a rhythmic figure, the first tool allows the calculation of all possible metrically related rhythms that fall within a certain speed range in beats per minute (bpm). Using a minimum tempo limit of 30 bpm and a maximum of 600 bpm makes the range correspond to the cognitive boundaries found by London. However, if for any compositional reason I feel the need to restrain that range further, I only have to change one number in the patch. These reasons usually have to do with musical context: a 600 bpm pulse train may be realistic to perform or perceive accurately as a metric rhythm in laboratory conditions or in the case of instruments that "speak" very fast— such as some percussions or piano —but the attack of bowed string instruments, for example, is so slow that the sound barely has the time to come out and the second pulse is already gone. Asking performers to play intricate compound rhythms at that speed, therefore, seems unreasonable and of little perceptual use.

For that reason, I often use IOI values between 125 and 1500 ms. 125 ms corresponds to a thirtysecond note at 60 bpm, which is an ample range for polyrhythmic structures. It does not mean that I would never use any faster values in my music, but I believe it to be more than enough for the elaboration of a perceivable metric grid. As for the slower boundary, pulses can theoretically be perceived at tempi down to 30 bpm (or 2000 ms IOI), but I personally find 40 bpm to be a reasonable, usable limit. Not all metronomes go below that range, and in my experience, performers tend to count in subdivisions anyway if the music is written with a very slow metronome mark.

There are three inputs to the patch shown in Figure 8: the tempo, in bpm, of the initial rhythmic stream, and the start and end of the IOI boundaries. Some results at the bottom of the patch are reproduced in Table 1 in the appendix.

The tool comprises a series of smaller functions that can be organized into three categories. The first one generates a list of ratios to be used, with each ratio corresponding to a certain subdivision in relation to the given initial tempo. The second category is a series of filters that remove certain subdivisions if they fall out of the given cognitive boundaries. The third category allows the sorting of subdivisions according to their corresponding IOI or tempo bpm and the grouping of subdivisions that share a common denominator.

I can either decide to use a list of arbitrary numbers or calculate the maximum subdivision the given tempo allows, taking into account the limits I determined. For example, if I want a section to only use quintuplets and septuplets but not triplets or sixteenth notes, I can make a list with the corresponding ratios and input it into the next filtering function. I often prefer to calculate all possibilities first and then filter at a later stage, as I feel that it allows me to consider possibilities I wouldn't necessarily have thought of in the first place.

The function *max-sub* takes the initial tempo of a quarter note and a maximum tempo limit as inputs and returns the fastest subdivision of the quarter note below the maximum tempo. In the case of 60 bpm and a minimum IOI ♩ 125 ms (480 bpm), the function returns 8, or an eighth of a quarter note, equivalent to a thirty-second note,

$$= 60 \text{ bpm}, 1000 \text{ ms [IOI]}$$
$$100 \;/x \geq 1 \; \text{ms}$$
$$x = \{1, \;, 3, 4, 5, \;, 7, 8\} \; \max(x)$$
$$= 8$$

$$/ \quad 8 =$$

A second version of *max-sub* uses a subdivision of the whole note instead of the quarter note and outputs 32 as an upper limit for a tempo of 60 bpm. This produces a much greater number of subdivisions, as any equal division of the whole note can serve as a first division instead of just the quarter note. However, this approach requires an additional filtering function later on in the process, which will be described below.

The next function is called *gen-ratios* and simply produces the two-by-two combinations of all numbers between 1 and the input, in this case 8. The output result is a list of ratios, starting with ratios faster than the initial pulse (1/8, 1/7, 1/5, etc.) all the way until the given pulse (8/8, or 1/1), and then ratios that are slower than the pulse: 8/7, 7/6, 6/5, etc., until 6, 7, and finally 8. This list of over forty ratios will then be filtered according to various constraints related to our cognitive limits.

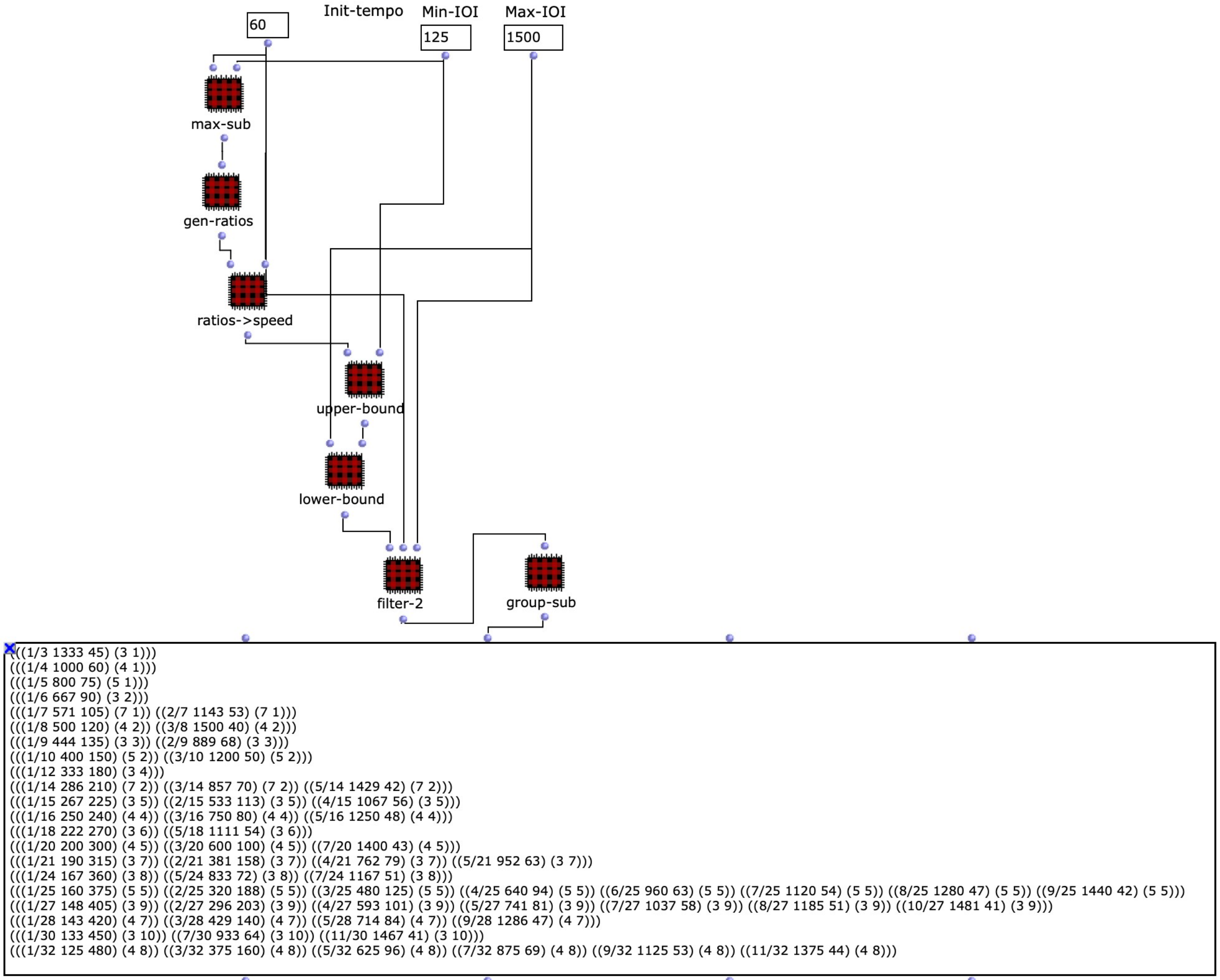

Figure 8: finding all metrically-related rhythms at a certain tempo

The output of *gen-ratio* then goes to the function *ratios->speed*, which calculates the IOI and tempo of each subdivision in the given tempo. Since $1/8$ means an eighth of a quarter note, we can divide the initial tempo by that ratio to obtain the frequency of the subdivision:

$$60 \text{ bpm} \div 1/8 = 480 \text{ bpm} \Rightarrow 125 \text{ ms IOI}$$

The first value is then multiplied by $1/4$ to represent a ratio of a whole note instead of a quarter note, and the three values (subdivision, tempo in bpm, and IOI in ms) are stored together for each of the inputted subdivisions. The list of all ratios and corresponding IOIs and tempi then goes through two functions that remove subdivisions if they are either faster than the maximum limit determined limit at the given tempo or slower than the minimum.

If the second version of *max-sub* has been used, a third filtering function is required to remove subdivisions that fall within the given range but require *thinking* outside of that range to be performed. For example, the subdivision of $1/31$, or 31 equal pulses inside a whole note, has a tempo of 465 bpm if the quarter note is equal to 60 bpm, which is lower than my limit of 480 and of the absolute 600 bpm as well. However, because 31 is a prime number, accurately performing 31 equal pulses at 60 bpm requires to be able to first have in mind a tempo of 15 bpm, or the whole note, and then subdivide it into 31 equal parts. While this is certainly not completely impossible, I would rather rely on other, more practical solutions, at least in the context of a real-world performance by human beings. The case is very different for 1/32 since it can itself be subdivided into four equal parts, thus only requiring the performer to subdivide quarter notes at 60 bpm into eight parts each. In other words, the function checks if a certain subdivision can be achieved through a nested tuplet, the levels of which would be metrically related to each other.

This is achieved by first finding if the given subdivision can be grouped into larger values that would allow the subdivision of a pulse faster than the minimum tempo. In this case, the goal is to determine whether the subdivision can itself be divided into a certain number that corresponds to a faster tempo than 1500 ms, or 40 bpm. Since 31 is a prime number, there is no possibility of regrouping this subdivision into an even number of parts that would each have a duration comprised between 125 and 1500 ms. A test function called *metrically-related* calculates if the ratio has a common denominator subdivision with 1/4 (the quarter note) that is slower than a 125 ms IOI, or 480 bpm.

The final step is to group all the rhythms that share the same subdivision denominator. For example, all multiples of 1/16 (a sixteenth note) will be put in the same list, and so on for all the rhythmic values. The final output of the patch gives all the rhythms that are metrically related to a quarter note at 60 bpm, as well as the way to group the subdivisions so that each sub-level is metrically related as well.

For example, this is the line concerning the 1/21 subdivision:

(((1/21 190 315) (3 7)) ((2/21 381 158) (3 7)) ((4/21 762 79) (3 7)) ((5/21 952 63) (3 7)))

The first number in the first set of parentheses gives the rhythm itself in relation to a whole note. The second number is the corresponding IOI in ms, while the third number is the tempo in bpm. The last pair of numbers indicates how to group the subdivision. In this case, the groupings would be three times seven pulses of 1/21.

2. Production of "Rhythm *Lattice*"

The second tool presented here allows the generation and manipulation of what I call a "rhythm lattice."

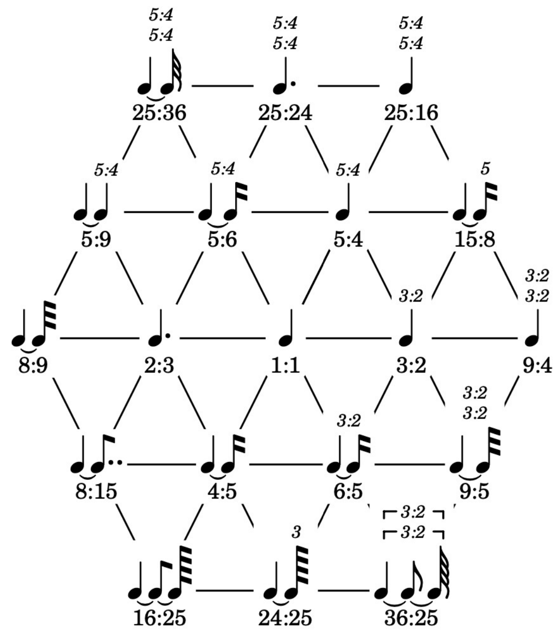

Figure 9: Louis-Michel Tougas - rhythm *lattice*

Composers interested in just intonation principles often use lattice diagrams to represent rational pitch-space. Each node of the graph corresponds to a harmonic ratio, and each movement on the graph corresponds to an interval expressed as a frequency ratio. I have adapted this mode of representation, transferring this pitch-space representation into time-space instead. The ratios used in my lattices still correspond to frequency relations, but the range is much slower, and the musical rationale becomes somewhat different. The rhythm lattice represents paths for metric modulations and polyrhythms that can be constrained, once again, by cognitive boundaries.

The representation was first inspired by work done in the field of just intonation by composers and theorists such as Harry Partch, Ben Johnston, and Erv Wilson, but it also intersects with music theorists Richard Cohn's "ski-hill graphs" and Justin London's "metric trees."

In his article *Complex Hemiolas*, *Ski-Hill Graphs and Metric Spaces*, Cohn introduced a visual tool he called the *ski-hill graph*, designed to represent metric spaces (2001). These spaces are more or less analogous to the definition Lerdahl and Jackendoff give of a metrical structure: that is, a hierarchical pattern of subdivisions and groupings of a pulse. The ski-hill graph depicts possible *integer* relations between a certain number of subdivisions that together form a metric space. It shows, for example, that a quarter note can be subdivided into either two eighth notes or a three eighth notes triplet. At the bottom of the graph, a unit pulse ties all the subdivisions together.

While the ski-hill graph was, to my knowledge, not specifically designed to illustrate metric modulations, it is a useful tool to visually show the multiple possible paths between an initial pulse and a related subdivision. For example, a possible path only made of integer relations between the initial quarter note pulse with ratio 1:1 and the quarter note triplet 3:2 would be to first subdivide the initial pulse in an eighth note triplet, and then group those eighth notes by two. Another path would be to do the opposite, or make groups of two quarter notes, and then subdivide them into three equal parts.

In *Hearing In Time*, London uses a similar representation but with the addition of cognitive boundaries relative to the absolute tempi that the subdivisions produce (2004, 39–41 see Fig. 2.24). This way, not only is it possible to see the possible integer relations between pulses but also their associated tempo, and therefore the potential to be perceived as a pulse or not.

While my "rhythmic lattice" may appear somewhat similar visually, it serves a slightly different goal than the representations described above and can therefore be seen as either a development or a variation of it.

The first difference is the inclusion of the graph of other subdivisions than 2 or 3, and therefore of more than two axes at a time. The addition of axes with higher prime numbers—5, 7, 11, etc.—allows the representation of more complex metric structures from outside the pre-twentieth century classical music canon, both temporally and culturally.

The second difference is somewhat more structural, as it concerns the goal of the representation itself but also the difference between rhythm and meter. The "link" between each node of my graph is not an integer relation but a frequency ratio; therefore, it does not depict a strictly hierarchical tree structure made of a series of subsequent groupings of a single initial pulse but rather a number of potential poly-metric spaces, made of a number of pulses whose corresponding tempi are closely related.

In a certain way, my rhythm lattices are somewhat of a second-order ski-hill graph, where all the integer relations forming the metric space are implicit. For example, the lattice does not show how to get from a quarter note to a quarter note quintuplet, but it indicates that modulating once to the quarter note triplet and then to the quarter note quintuplet produces the ratio 15:8 and that the corresponding tempo of that new pulse will be 112.5 bpm if the initial quarter note was 60 bpm. Each node itself is expandable into its own metric tree—which is often what happens in contemporary music. In that sense, the graph is not a representation of strictly metrical relations but rather of pulse relations. For that reason, the graph does not show subdivisions or groupings of any pulse, including by 2, since the operation does not produce a categorically different metric space, but just a movement inside an existing one. However, a different compositional approach could adapt the lattice to use 2:1, 3:2, and 5:4 as axes without a problem.

In any case, the rhythm lattice is to be read in the following way:

The starting point is the middle ratio 1:1, corresponding to a quarter note. Any horizontal axis gives access to either a modulation to a quarter note triplet (3:2) if going towards the right or to a dotted quarter note (2:3) if going towards the left. The diagonal axis that goes from bottom-left to top-right is the 5:4 / 4:5 axis, corresponding to modulations to the quarter note quintuplet and the quarter note tied to a sixteenth note, respectively. The other diagonal axis, going from bottom-right to the top-left, is the compound ratio of the two first ones, or ratios of 5:6 / 6:5, corresponding to six sixteenth notes of a quintuplet or five sixteenth notes of a triplet.

This representation allows the design of metric modulation paths throughout longer sections while keeping the number of involved tuplets relatively small. This is more or less equivalent to the notion of *limit* in just intonation: in this context, ratios are built only from prime numbers below a certain threshold. For example, in limit 5, only the prime numbers 2, 3, and 5 are multiplied together to produce more complex ratios. The lattice above only uses prime numbers 3 and 5 as a modulation to twice or half of the duration, which constitutes a trivial case of metric modulation.

Trajectories in the network can also be expressed as an equation, such as,

$$(5/4)^{"} \times (3/2)^{\#},$$

where x is the number of modulations along the 5:4 axis, and y is the number of modulations along the 3:2 axis. A negative exponent would be a modulation in the opposite direction, so a series of 4:5 or 2:3 modulations. For example, starting from the 1:1 ratio, the trajectory going to 25:24 can be expressed as,

$$(5/4)^{\$} \times (3/2)^{\%\&}.$$

This represents two positive steps on the 5/4 axis and one negative step on the 3/2 axis.

The advantages of generating the lattice algorithmically are that the values on any axis can be changed to test many different possibilities and also that it makes it easy to verify if all the values fall within the 125-480 ms IOI range.

Figure 10 shows that, for example, using 3:2 and 5:4 again as starting ratios, the first-order rhythms—those connected directly to 1:1—can be generated and are the following ratios: 5:4, 3:2, 6:5, 4:5, 2:3, and 5:6. Then, all of these ratios can themselves be used recursively in the same function to generate the set of second-order metrically related ratios. This process corresponds to a secondary metric modulation where the center of the network becomes these ratios instead of 1:1.

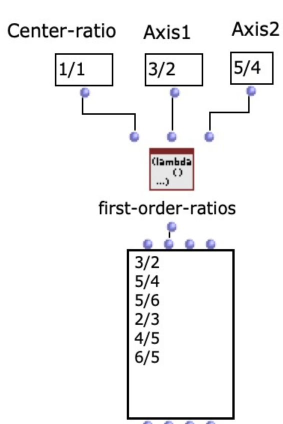

Figure 10: Calculation of first-order ratios in *OpenMusic*

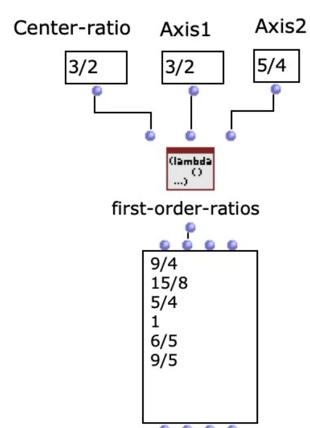

Figure 11: Calculation of ratios based on 3/2

Figure 11 shows the first-order ratios related to 3/2, which provides the second "web" around this ratio. This process can be repeated for all first-order ratios in order to generate the values for the complete lattice shown in Figure 9. Then, the function *metrically-related* used in the first tool presented above can be employed again to find out if any pair of two values share a common denominator inside the cognitive range and if it is possible to modulate from one to the other via a common pulse.

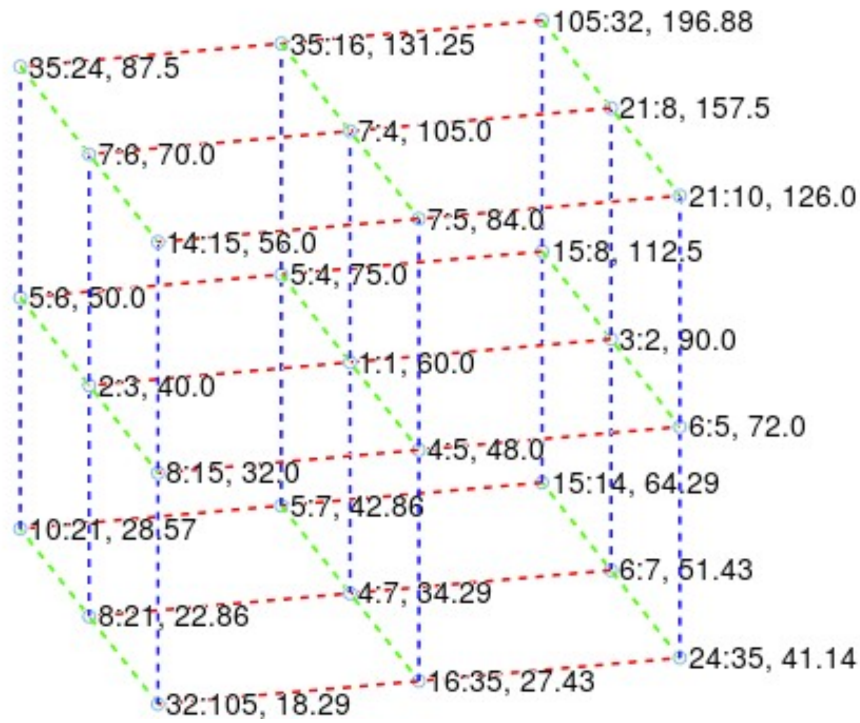

Figure 12: 3D rhythm lattice with axes 3:2, 5:4 and 7:4

In order to represent more than two axes at a time, a 3D visualization may be produced, each spatial dimension corresponding to a frequency ratio. Figure 12 gives an example of such a representation. The axes correspond to ratios 3:2, 5:4, and 7:4 respectively. Looking at the graph, one can see that starting with the 1:1 ratio at 60 bpm—which could be the quarter note or any other figure—the ratio 6:7, at 51.43 bpm, can be reached by modulating first to the quarter note triplet (one step towards the right on the red axis) and then to the double-dotted quarter note (one step down on the blue axis).

Decentralized Metric Modulations, Notational Innovations, and Musical Examples

Having presented the inspiration that led me to develop this compositional approach, the cognitive constraints I take into account, and my use of computer-assisted composition tools, I will now present two concrete musical examples that make use of alternative notation conventions that I developed and that are partly inspired by the notational practices of the *ars* *subtilior*. The innovation in this way of notating rhythm resides in the fact that any performer can start a related polyrhythm at any given time, regardless of the measure, time signature, etc., allowing for a complete interdependence of rhythmic voices. I use the word interdependence instead of independence since my goal is to work with complex yet perceivable metric relations between different pulse streams, not completely unrelated pulses. This approach has the advantage of reducing the notated rhythmic complexity of each individual stream, at least once the system is well understood by performers.

Example 1: Piece for Quartet (2023)

Figure 13 shows a short excerpt of the first composition where I made use of the idea of decentralized meters and tempi, for drums, keyboard, electric bass, and electric guitar. The notation conventions are as follows:

- There is no single common meter, so bar lines are only used to indicate points of simultaneity between two voices.
- There is no written time signature, as all voices are constantly changing meters.
- The metronome markings and metric modulations only apply to the instrument immediately below. For example, the quarter note tied to a sixteenth note over the guitar part at the beginning of the staff only applies to the guitar.
- The metric modulations always refer to a tempo previously heard in another voice. The modulation of the guitar is preceded by a D because the rhythmic figure is in relation to the tempo of the drums (B for bass, G for guitar, C for keyboard, and D for drums).
- Any instrument can start a motive at any point in relation to any other voice.
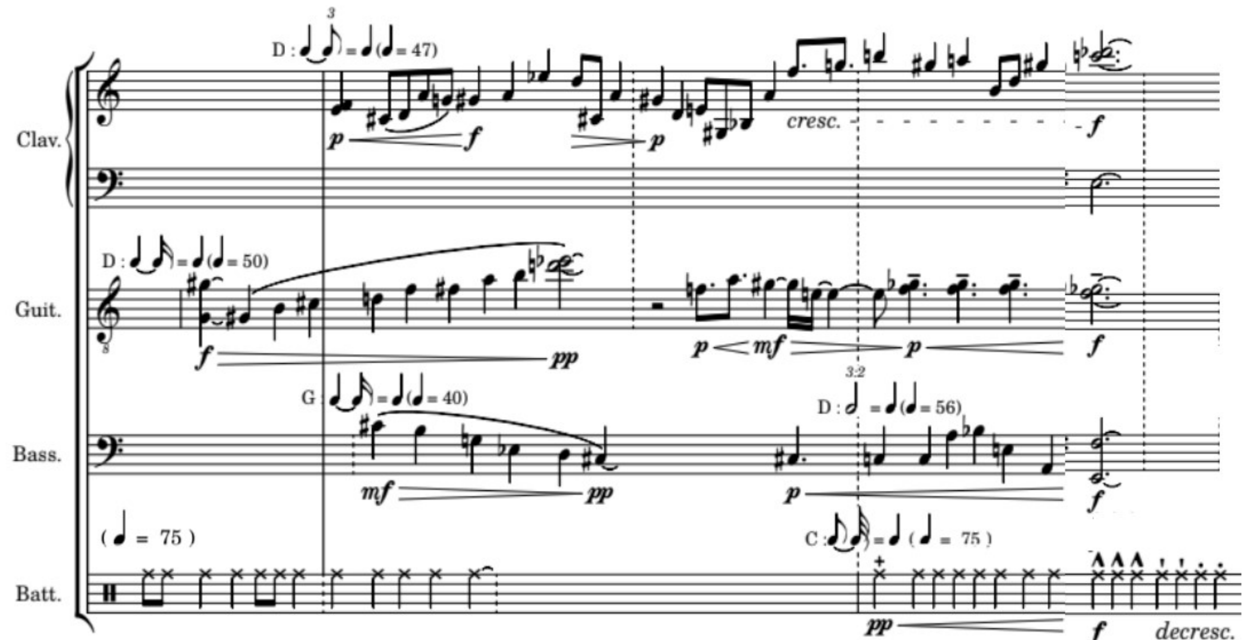
Figure 13: Louis-Michel Tougas - Piece for quartet (excerpt, 2023)

The series of metric modulations between the instruments can therefore be read as follows:

- The drums have a tempo of 62.5 bpm (written in the previous staff). On the second drum beat of the page, the guitar begins its line by modulating at the quarter note tied to a sixteenth note, corresponding to 4/5 times the original tempo, or 50 bpm. The resulting polyrhythm is a 5:4, equivalent to successive quarter notes tied to sixteenth notes, but the rhythm is written in quarter notes since the metronome mark is distinct for every instrument.
- The keyboard enters on the sixth beat of the drums (where the line crosses all staves), this time modulating from the drum's tempo through the quarter note tied to an eighth note triplet figure, or 3/4 of the initial tempo, corresponding to about 47 bpm. The next modulation comes from the electric bass, which starts on the fifth beat of the electric guitar, at a tempo that is 4/5 times slower than that of the guitar, or 40 bpm. The guitar then plays regular dotted eighth notes in the tempo of the keyboard. The drums begin a new pulse where the dotted line crosses all staves at a tempo of 75, or 5/8 of the keyboard's tempo. The electric bass starts as well, and after having played one beat in the tempo of the keyboard immediately modulates to the half note triplet of the drums, giving a tempo of about 56 bpm. Similar processes continue throughout the whole short piece, instruments coming in and disappearing, always in rhythms metrically related to a voice that is already present.

Example 2: Five pieces for string quartet (written for the Bozzini string quartet, 2023)

The second example comes from my *Five pieces for string quartet*, written for the Bozzini Quartet and premiered at the Fall 2023 Gaudeamus Festival. This piece also makes use almost entirely of the idea of decentralized meter and tempo, on top of a more conventional just intonation system for the organization of pitches.
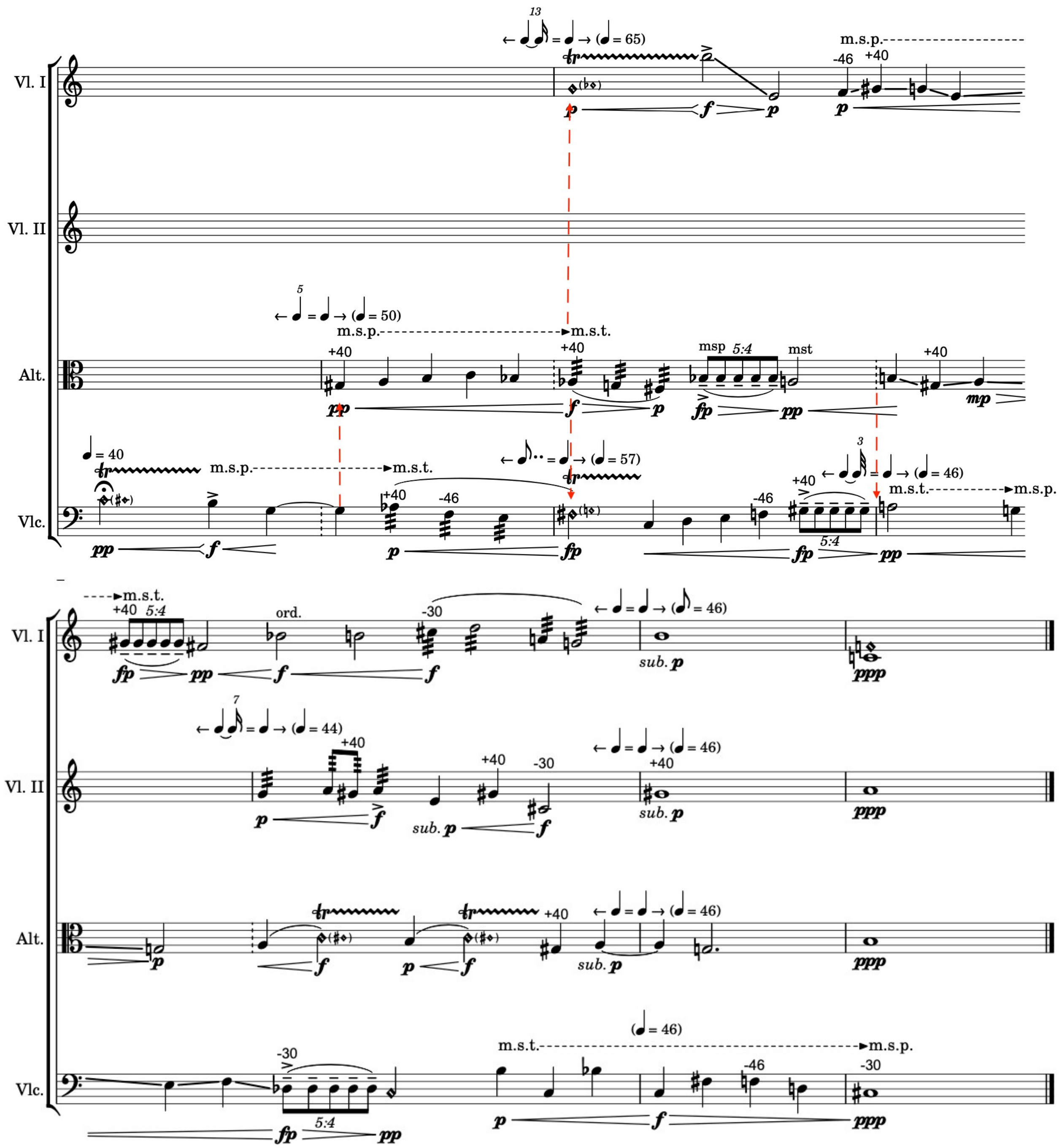
Figure 14: *Five pieces for string quartet* (IV)

The excerpt shown in Figure 14 is the fourth piece, which is the shortest of the five. Similar to the previous example, the cello begins at a tempo of 40 bpm, which serves as a temporal reference for the following modulations. Here, red arrows are used instead of letters to indicate which instrument constitutes the reference. The viola enters on the fifth beat of the cello at a tempo of 50, or 5/4 faster than the cello. The first violin then enters on the sixth beat of the viola at a tempo of 65, or 13/10 times faster than 50. At the same moment, the cello starts playing in a tempo that is 8/7 times that of the viola, or 57 bpm. Finally, the cello modulates again by taking the viola as a reference, this time using a 12/13 ratio, giving the metronome mark of 46.

Four of the five pieces were constructed using this same technique of decentralized metric modulations, which creates a complex fabric of interweaved pulsations that are all metrically related. While other parameters such as the amplitude envelope of each pulse or the proximity of two players may affect the perceptibility of the polyrhythmic relations, I believe that even the mere structural use of such a technique allows many possibilities otherwise impossible to write using conventional notation.

Conclusion

The ideas presented in the present paper are still at a preliminary stage, but they show that structural complexity does not need to be opposed to perceptibility. On the contrary, many avenues are still possible for the innovation of the temporal aspect of music in a way that is not merely abstract but also bears a strong perceptual impact. Moreover, the growing interest in cognition and understanding how the brain processes and treats external stimuli as information will surely open other paths for composers and artists in general who are interested not only in manipulating abstract structures but also taking into account how these structures might be perceived by an eventual audience. While a certain historical response to structuralism and complexity has been to adopt a more conservative approach through simplifying structures and finding refuge in more traditional idioms, taking into account perceptual and cognitive data at the very beginning of the structuring process opens a third way that has only started to reveal its potential.

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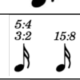
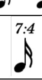
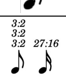
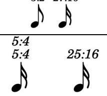
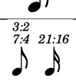
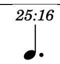
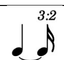
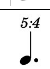
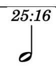
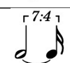
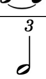
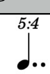
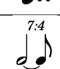
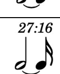
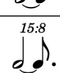
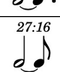
Appendix I: Table of all metrically related rhythms for quarter note = 60 bpm

Possible rhythmic figure(s)	Ratio (subdivision of the whole note)	IOI (ms)	Frequency (bpm)
	1/32	125	480
	1/30	133	450
	1/28	143	420
	1/27	148	405
	1/25	160	375
	1/24	167	360
	1/21	190	315
	1/20	200	300
	1/18	222	270

𝅘𝅥𝅯	1/16	250	240
15:8 ♪	1/15	267	225
7:4 ♪	1/14	286	210
27:16 ♪	2/27	296	203
25:16 ♪	2/25	320	188
3 ♪	1/12	333	180
𝅘𝅥𝅯.	3/32	375	160
21:16 ♪	2/21	381	158
5 ♪	1/10	400	150
7:4 ♪.	3/28	429	140
9:8 ♪	1/9	444	135
25:16 ♪.	3/25	480	125
♪	1/8	500	120
15:8 ♩	2/15	533	113
7:4 ♩	1/7	571	105
27:16 ♩	4/27	593	101

Possible rhythmic figure(s)	Ratio (subdivision of the whole note)	IOI (ms)	Frequency (bpm)
5:4 ♪.	3/20	600	100
♪‿𝅘𝅥𝅰	5/32	625	96
25:16 ♩	4/25	640	94
3 ♩	1/6	667	90
7:4 ♩‿𝅘𝅥𝅯	5/28	714	84
27:16 ♩‿𝅘𝅥𝅯	5/27	741	81
♪.	3/16	750	80
21:16 ♩	4/21	762	79

	1/5	800	75
	5/24	833	72
	3/14	857	70
	7/32	875	69
	2/9	889	68
	7/30	933	64
	5/21	952	63
	6/25	960	63
	**1/4**	**1000**	**60**
	2/7	1143	53
	7/24	1167	51
	8/27	1185	51
	3/10	1200	50
	5/16	1250	48
	8/25	1280	47
	9/28	1286	47
	1/3	1333	45
	11/32	1375	44
Possible rhythmic figure(s)	**Ratio (subdivision of the whole note)**	**IOI (ms)**	**Frequency (bpm)**
	7/20	1400	43
	5/14	1429	42
	9/25	1440	42
	11/30	1467	41
	10/27	1481	41
	3/8	1500	40